Correction to “Locally Trivial Outer Automorphisms of Finite Groups”

Math. Z. **114**, 173–179 (1970)

EVERETT C. DADE

Reinhard Lane has called my attention to an error appearing twice in the proofs on page 177 of [1]. It concerns a finite group U having a normal Hall subgroup H , and an automorphism α of U centralizing both H and U/H . I had assumed that α would then fix a complement to H in U and hence be trivial. The correct result, as Lane pointed out, is the

Proposition. *The automorphisms α of U having the above property are precisely the inner automorphisms by elements of the center $Z(H)$ of H .*

Proof. Obviously all these inner automorphisms have the property.

Suppose $\alpha \in \text{Aut}(U)$ centralizes both H and U/H . Let V be any complement to H in U . Then V is also a complement to the normal Hall subgroup $Z(H)$ in $C_U(H)$ $V = Z(H)V$. Since α centralizes H , it centralizes $U/C_U(H)$. It follows that

$$Z(H)V^\alpha = [Z(H)V]^\alpha = [C_U(H)V]^\alpha = C_U(H)V = Z(H)V.$$

So V^α is another complement to $Z(H)$ in $Z(H)V$. Because $Z(H)$ is an abelian normal Hall subgroup of $Z(H)V$, there exists $\sigma \in Z(H)$ such that $V^\alpha = V^\sigma$. If β is the inner automorphism of U determined by σ , then obviously $\alpha\beta^{-1}$ centralizes both U/H and H , and fixes $V \cong U/H$. So $\alpha\beta^{-1} = 1$ and $\alpha = \beta$, which proves the proposition.

Because of the above remark of Lane, we must change the proofs in [1] at two points. From now on, all numbers refer to statements in [1] and we adopt the notation of that paper.

Proof of Lemma (1.14). The original proof shows that K normalizes $L = N_G(C_G(K))$, and centralizes both the Hall subgroup $C_G(K)$ of L and the factor group $L/C_G(K)$. Setting $Z = Z(C_G(K))$ and using the subscript 1 to indicate images in $\text{Aut}(L)$, we obtain from the above proposition the inclusion:

$$K_1 \leq Z_1.$$

If $Z = 1$, then $K_1 = Z_1 = 1$. If $Z \neq 1$, then $K = K(Z)$ by Lemma (1.11). We know from (1.5)–(1.7) that $K = [C_S(ZC_G(Z))_p, C_T(Z)]$, where $C_S(ZC_G(Z))_p$ is the p -Sylow subgroup of the nilpotent group $C_S(ZC_G(Z))$. Obviously $C_G(Z)$

centralizes $C_S(ZC_G(Z))_p$. So $C_T(Z)$ acts on $C_S(ZC_G(Z))_p$ as the q -group $C_T(Z)/C_G(Z) \cong T^*$. Since $q \neq p$, it follows that $[K, C_T(Z)] = K$. Evidently $C_T(Z)$ normalizes K , and hence normalizes $L = N_G(C_G(K))$. So we can take images in $\text{Aut}(L)$, obtaining:

$$K_1 = [K_1, C_T(Z)_1] \leq [Z_1, C_T(Z)_1] = [Z, C_T(Z)]_1 = 1.$$

Therefore $K_1 = 1$ in all cases, and K centralizes L , which proves Lemma (1.14).

Proof of the Theorem. On line 10 of page 177 of [1] the group G is a Frobenius group with kernel N , and $K(N)$ centralizes both G/N and N . Since N is then a Hall subgroup of G , the above proposition implies that $K(N) \leq Z(N)$ (everything being already embedded in $\text{Aut}(G)$). This is impossible by Lemma (1.6), since $S/G = S^* \neq 1$. The contradiction proves the theorem.

Reference

1. Dade, E. C.: Locally trivial outer automorphisms of finite groups. *Math. Z.* **114**, 173–179 (1970).

Prof. E. C. Dade
Department of Mathematics
University of Illinois
Urbana, Illinois 61801
USA

(Received September 23, 1971)