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Adaptive Decorrelating Detectors for CDMA Systems *

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Abstract. Multi-user detection allows for the efficient use of bandwidth in Code-Division Multiple-Access (CDMA) channels through mitigation of near-far effects and multiple-access noise limitations. Due to its inherent noise and multipath immunity, CDMA multi-access is being considered as a platform for personal communication systems (PCS). As CDMA based digital communication networks proliferate, the need to determine the presence of a new user and integrate knowledge of this new user into the detection scheme becomes more important. The decorrelating detector is a linear multi-user detector that is asymptotically optimal in terms of near far resistance; however, in the presence of a new unknown user, performance of the decorrelator is severely degraded. Adaptive decorrelators are constructed which adaptively augment an existing conventional decorrelator to demodulate a new active user in addition to existing users. Several likelihood ratio based schemes are employed. Both synchronous and asynchronous communication are investigated.

Key words: Spread-spectrum communications, code-division multiple-access, adaptive receiver, decorrelating detector, likelihood ratio tests

1. Introduction

Code-division multiple-access (CDMA) implemented via direct-sequence spread-spectrum modulation is emerging as an important technique for implementing multi-user systems. In this form of CDMA, users are distinguished at the receiving end of a communications channel by their unique codes which they use to modulate the transmitted data. CDMA enables the number of potential users to be increased in bursty or fading channels with cellular topologies, making it particularly attractive for applications such as mobile telephony and personal communications systems (PCS) [15].

In a CDMA communication environment, demodulation requires the suppression of two forms of noise: ambient channel noise, which is often modeled as an additive Gaussian process, and multiple access interference (MAI), which is highly structured. The ambient channel noise can be treated using the classical methods of signal processing; however, the effective mitigation of MAI requires multi-user detection techniques. The *optimal multi-user detector* [18] (in the maximum-likelihood sense) suffers from a complexity that is exponential in the number of active users. An alternative detector is the *decorrelating detector* [7], which is a linear receiver (in operation and in complexity) that retains the so-called *near-far resistance* of the optimal multi-user detector. A detector is near-far resistant if it is insensitive to the effects of a large received power for the interferers relative to that of the desired user. Note that the conventional receiver, a receiver that is matched to the spreading code of the desired user, is severely near-far limited.

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Neither the optimal receiver, nor the conventional decorrelating detector, is adaptive. The use of a non-adaptive receiver can result in wasted resources, unnecessary computations, and potentially inferior performance if only a subset of the possible users is active. As CDMA based digital communication networks proliferate, the need to determine the presence of a new user and integrate knowledge of this new user into the detection scheme becomes more important. It is expected that multiple network providers will share the communication space; as a result, security and privacy issues will gain greater prominence. It will be desirable not to broadcast the network determined communication parameters of the new users; thus adaptive schemes for integrating the new users into communication will be necessary. While the option for setting aside a separate channel for the transmission of such side information is possible, this results in a waste of bandwidth due to the bursty nature of much telecommunications traffic. In addition, there is the possibility of error in transmission of such side information due the nature of the wireless channel. Another concern is the number of operations necessary at the mobile unit. The proposed scheme will reduce the number of operations and thus reduce power consumption at the mobile unit. These issues motivate the consideration of an adaptive multi-user receiver for the dynamic user set problem.

The decorrelating detector's performance, low complexity and near-far resistance make it a desirable detector if the signature sequences of *all* of the active users are known. To stress both its advantages and limitations, a simple example is provided. Figure 1 shows the probability of bit detection error as a function of the relative power between the desired user and the interfering users for decorrelators derived with different amounts of user set knowledge. The operation of the conventional detector is provided as a point of reference as is the single user bound, which is the optimal performance achievable when there is a single user and Gaussian noise only. The spreading codes were of length 31. In the case where the decorrelator is derived for three *known* users, the performance is quite close to that of the single user bound. If a fourth *unknown* user is introduced, the 3-user decorrelator's performance degrades severely, coming quite close to that of the clearly suboptimum matched filter. We also consider the performance of a decorrelator for 31 active users employed in the 3-user case. While this receiver is not affected by the relative powers of the interfering users, it is clear that there is performance reduction from over-engineering the receiver for all possible users.

In this paper, we propose an adaptive decorrelator that will operate in a varying communication environment. In a previous work [9] a constrained adaptive decorrelator was developed. In order to realize the detector in [9], the spreading codes of all active users were constrained to have the same cross-correlation value between pairs of distinct users. In addition, it was assumed that the decorrelating detector for $K-1$ existing, synchronous users was in operation, and only a single new synchronized user could become active. Using samples of the received signal, the receiver learnt the new user's communication parameters such that an augmented decorrelating detector could be determined. The current work generalizes the constrained adaptive decorrelator by removing the need for the cross-correlation constraint while retaining the other characteristics of the communication scenario. The relaxation of the cross-correlation constraints enables the reception of asynchronous communication.

In this work, a total system approach is taken to address the issue of learning and integrating knowledge of a new transmitting user into the receiver structure. Figure 2 depicts the system to be studied. Each portion of the system will be considered except for the determination of *whether* a new user has entered into communication. This problem is addressed in [10, 11]. Thus we shall assume that it is known to the receiver that a new user has begun transmission over the shared wireless channel. It should be noted that $R_K \to R_{K+1}$ refers

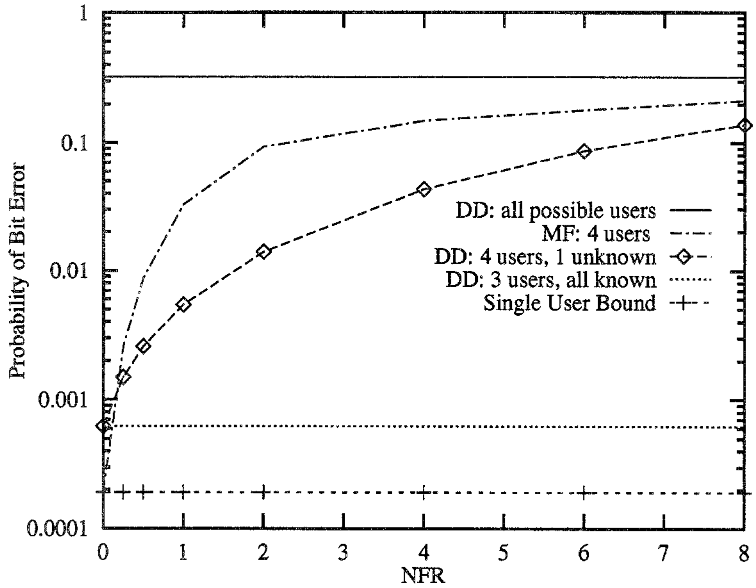

Fig. 1. Performance of several decorrelators derived under different amounts of user set knowledge.

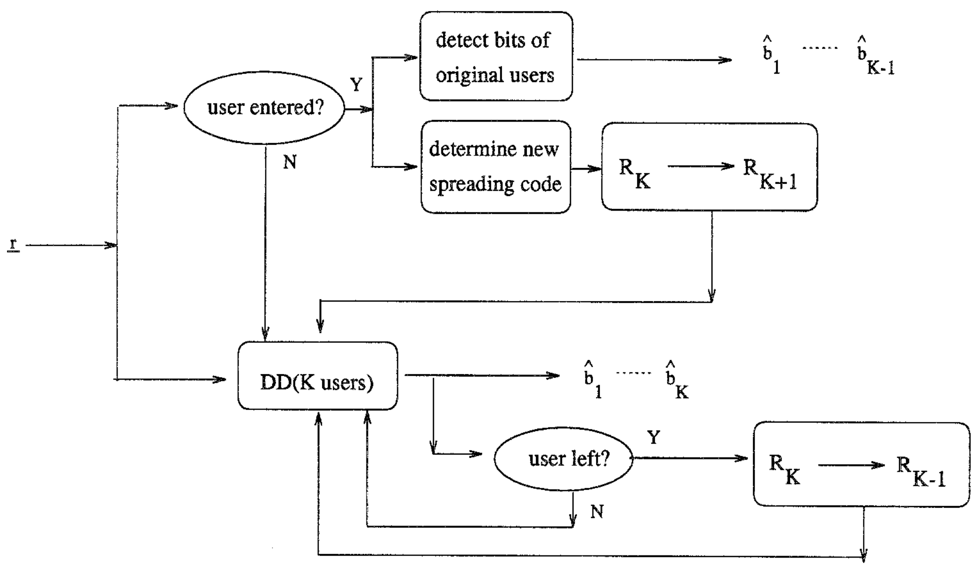

Fig. 2. The adaptive decorrelator receiver system.

to the process of incorporating the new user knowledge into the decorrelator and similarly $R_K \rightarrow R_{K-1}$ denotes the method by which the decorrelator structure is reduced with the removal of information about a user which has ceased communication. This paper is organized as follows: the communication environment and assumptions are described in Section 2. In Section 3, the required projection transformation is reviewed. Several likelihood-based tests for the determination of the spreading code of the new user are presented in Section 4 for the synchronous transmission channel. In Section 5, a scheme for accommodating asynchronous communication is described. The probability of detection error for the augmented decorrelator is discussed in Section 6. The performance of the generalized adaptive decorrelator schemes is addressed by considering the probability of determining the incorrect spreading code for the new user. Bounds on this probability of code-selection error are determined for one of the tests in Section 7. The accommodation of a user ceasing transmission is presented in Section 8. Simulation of the probability of code-selection error is presented in Section 9 for the various detectors developed. A method for bit detection of the original users during training is presented in Section 10. Final conclusions are drawn in Section 11.

2. Preliminaries

We consider binary Phase-Shift Keyed CDMA as implemented via direct-sequence spread-spectrum modulation. We shall assume coherent reception with K active users. The received signal with respect to the last user's symbol interval, after chip matched filtering and sampling at the chip rate with respect to the last user's symbol interval can be described as follows

$$\underline{r}_i = \underline{n}_i + \sum_{k=1}^{K} A_k \left[b^{(k)}(i-1)\underline{m}_k^L + b^{(k)}(i)\underline{m}_k^R \right] \tag{1}$$

$$\text{where } \underline{m}_k^L[j] = \underline{m}_k[N - \tau_k + j] I_{[j<\tau_k]}, \tag{2}$$

$$\text{and } \underline{m}_k^R[j] = \underline{m}_k[j - \tau_k] I_{[j \geq \tau_k]}. \tag{3}$$

The received amplitude is denoted by A_k, $b^{(k)}(i)$ is the transmitted bit of user k at time i. Members of the signal constellation are denoted as $\underline{m}_i$; these binary-valued codes are normalized. The delay of each user relative to user K is τ_k and thus $\tau_K = 0$. We shall assume that each delay is an integer multiple of the chip duration T_c and that N denotes the number of chips in the spreading sequence. The function $I_{[.]}$ is the indicator function; it is unity when its argument is true and zero otherwise. The additive Gaussian noise process is $\underline{n}_i$, which is assumed to be white.

We let $\underline{\tau} = [\tau_1, \cdots, \tau_{K-1}]$. Equation (1) can be re-written in matrix notation as

$$\underline{r_i} = A_K b_K(i) \underline{m}_K + S^L(\underline{\tau}) A^{K-1} \underline{b}_{K-1}(i-1) + S^R(\underline{\tau}) A^{K-1} \underline{b}_{K-1}(i) + \underline{n}, \tag{4}$$

where $S^L(\underline{\tau})$ is an $N \times (K-1)$ matrix whose columns are composed of the $\underline{m}_k^L$ and similarly for $S^R(\underline{\tau})$; and A^{K-1} is a diagonal $(K-1) \times (K-1)$ matrix whose non-zero entries are the received amplitudes of *all* of the active users *save* user K.

For synchronous communication we can drop the time reference i and (4) reduces to

$$\underline{r} = S_K A^K \underline{b} + \underline{n}, \tag{5}$$

where S_K is an $N \times K$ matrix whose columns are the normalized spreading codes of each of the active users, and $\underline{b}$ is the $K \times 1$ vector of bit values for the K users. The cross-correlation matrix is denoted as $R_K = S_K^T S_K$.

It is assumed that the reader is familiar with the operation of the *conventional decorrelator* [7]. It is of value to emphasize that the conventional decorrelator requires knowledge of all of the active users' spreading codes. With imperfect knowledge of all active users' sequences, the conventional decorrelator is near-far limited, as was observed in Fig. 1.

The *constrained adaptive decorrelator* [9] enables one to employ a decorrelating type structure to perform detection of a set of active users while adaptively determining properties about a new active user. The end result is a decorrelating detector for all active users. This constrained adaptive decorrelator is realized by constraining members of the signaling set to have a common cross-correlation value. We shall employ a methodology similar to what was used for the constrained adaptive decorrelator work. It is assumed that there are $K - 1$ active users for whom a decorrelating detector is in use. Initially we shall assume synchronous communication. In addition, at most one new user can enter the network at a time and it is assumed that the receiver has knowledge of this entrance. The new user is required to draw its signature sequence from some pre-defined code set. We next present the generalized adaptive decorrelator.

3. The Projection Operator

A methodology similar to the previous adaptive decorrelator work [9] is prescribed. Augmenting the decorrelating detector to detect the $K - 1$ original users and the new Kth user will be done in two phases. The first phase will involve isolating the new user's signal from the received signal. We propose to use a projection operator to create a residual signal. The second phase will use this residual signal to determine the unknown signature sequence.

We seek an operator which will remove the signal content of the original active users from the residual signal; failing to do so will result in a residual signal that is dependent on the unknown amplitudes of the existing users. We shall be considering the least squares estimate of the new user's spreading code given that the spreading codes of the original users are known. For *synchronous* communication (5), this is simply the projection of the received vector onto the orthogonal complement of the span of the spreading codes of the original users. This span is denoted by $R(S_{K-1})$ to indicate the range space of the columns of the matrix, S_{K-1}. Note that the orthogonal complement is simply the null space of the columns of the matrix; we denote this subspace as $N(S_{K-1})$. This results in the following linear transformation:

$$B = I - S_{K-1} R_{K-1}^{-1} S_{K-1}^{T}. \tag{6}$$

Projection operators have several interesting geometric properties: they are *idempotent* i.e. $B^2 = B$; they are self pseudo-inverses, $B^{+} = B$, where the superscript $+$ indicates the Moore-Penrose pseudo-inverse (see e.g. [3]). In addition, this particular projection, B, is symmetric.

If the cross-correlations between each of the original active users' and the new user's signals are relatively small, the residual signal formed by $\underline{x} = B\underline{r}$ should contain sufficient signal content of the new user for detection purposes. It should be pointed out that we are able to isolate the new user's signal from the existing users' signals without any knowledge of the received amplitudes of *any* of the users.

4. Determining the New Spreading Code: Synchronous Communication

We shall be considering Maximum Likelihood (ML) type detection for determining $\underline{m}_K$ from the residual signal $\underline{x}$ described above. Modifications shall be made in order to accommodate imperfect knowledge of the complete communication scenario. Below we describe methods for determining the new user's spreading code with the use of a training sequence.

4.1. Training Signal Dependent Algorithm

We will assume, without loss of generality, that the training sequence sent will consist entirely of $+1$'s, which implies that

$$\underline{x} = A_K B b_K \underline{m}_K + B\underline{n} \quad \text{where } b_K = 1. \tag{7}$$

We note that B is not full rank; in fact the projection achieves full rank only when it is the identity. We shall reduce the dimension of the problem by considering the restriction of all vectors and matrices to $N(S_{K-1})$. The eigenvectors corresponding to the non-zero eigenvalues of B form a basis for the subspace, $N(S_{K-1})$. Let V be a matrix whose columns are the eigenvectors of the non-zero eigenvalues of B. We can consider the residual signal with respect to this basis

$$\underline{x}^{\aleph} = V^T \underline{x} \tag{8}$$

$$\underline{m}_i^{\aleph} = V^T \underline{m}_i \tag{9}$$

$$B^{\aleph} = V^T V V^T V = I_{N-K+1}, \tag{10}$$

where the superscript $\aleph$ denotes the restriction to $N(S_{K-1})$. As is to be expected, $B^{\aleph}$ is simply the identity matrix of dimension $N-K+1$.

We return to the detection problem at hand. ML detection will be accomplished using a fixed sample size, where the relevant statistic will be determined in a sequential manner as more observations of the residual signal are collected. This statistic will be equivalent to the likelihood ratio. Essentially, we perform C-ary hypothesis testing [13], where C is the cardinality of the set of possible codes from which the new active user has drawn its code. We will consider the projections of the possible codes onto $N(S_{K-1})$. Although the set of codes is known, the received amplitude, A_K, is unknown, and thus we condition on A_K yielding the following probability density for $\underline{x}^{\aleph}$ conditioned on hypothesis, H_i.

$$p_{\underline{m}_i}(\underline{x}^{\aleph} | A_K, b_K = 1) = \frac{1}{(2\pi\sigma^2)^{\frac{N-K+1}{2}}} \exp\left\{ -\frac{1}{2\sigma^2} \|(\underline{x}^{\aleph} - A_K \underline{m}_i^{\aleph})\|^2 \right\}. \tag{11}$$

This yields the following *locally optimum* [13] ML decision rule:

$$\hat{\underline{m}}_t^{\text{LOTSD}} = \arg\max_{B\underline{m}_i} \frac{1}{M} \sum_{l=1}^{M} \underline{x}_l^T B \underline{m}_i, \tag{12}$$

where $\hat{\underline{m}}_t$ is the ML estimate of the new user's spreading code and M is the number of samples examined. We note that this test is globally optimum with respect to the variance of the underlying Gaussian noise $\underline{n}$.

4.2. Blind Algorithms

We next introduce decision rules for determining the new user's spreading code which do not require a training signal to be sent. The probability density of the residual signal conditioned on hypothesis H_i and the unknown amplitude, A_K, is now a Gaussian mixture as $b_K = \pm 1$ with probability $1/2$. This density is given below:

$$p_{\underline{m}_i}(\underline{x}^{\aleph}|A_K) = \frac{1}{2}\frac{1}{(2\pi\sigma^2)^{\frac{N-K+1}{2}}}\left(\exp\left\{-\frac{1}{2\sigma^2}\|(\underline{x}^{\aleph} - A_K\underline{m}_i^{\aleph})\|^2\right\}\right.$$
$$\left. + \exp\left\{-\frac{1}{2\sigma^2}\|(\underline{x}^{\aleph} + A_K\underline{m}_i^{\aleph})\|^2\right\}\right). \tag{13}$$

The resulting locally optimal rule is

$$\hat{\underline{m}}_t^{\mathrm{BLLO}} = \arg\max_{B\underline{m}_i} \frac{1}{M}\sum_{l=1}^{M}\frac{\left(\underline{x}_l^T B\underline{m}_i\right)^2}{\sigma^2} - \underline{m}_i^T B\underline{m}_i. \tag{14}$$

Unlike the training signal dependent tests, we have a test relying on knowledge of variance of the noise. Seeking a blind locally optimum test as a function of the SNR (*vs.* the amplitude) resulted in a meaningless test. Therefore, we fashion two blind tests:

$$\hat{\underline{m}}_t^{\mathrm{BLSQ}} = \arg\max_{B\underline{m}_i} \frac{1}{M}\sum_{l=1}^{M}\left(\underline{x}_l^T B\underline{m}_i\right)^2 \tag{15}$$

$$\hat{\underline{m}}_t^{\mathrm{BLMOD}} = \arg\max_{B\underline{m}_i} \frac{1}{M}\sum_{l=1}^{M}\frac{\left(\underline{x}_l^T B\underline{m}_i\right)^2}{\underline{m}_i^T B\underline{m}_i}. \tag{16}$$

We assume that the BLLO test will have superior performance to the BLSQ test, as the latter test does not consider the underlying noise variance or the norms of the transformed signature sequences. However, it is unclear *a priori* how the BLMOD test will perform *vis à vis* the first two blind tests.

This concludes our treatment of synchronous transmission; in the sequel, asynchronous communication is addressed.

5. Asynchronous Communication

In this section, we will further generalize the adaptive decorrelator to accommodate asynchronous transmission by each of the active users. We shall assume that the receiver has knowledge of the delays (τ_k) of each of the active users relative to the new active user, i.e. $\tau_K = 0$. We assume that the delays are integer multiples of the chip duration T_c. It is noted that, traditionally, signal acquisition requires knowledge of the signature sequence of the user to be acquired. However, preliminary work has been presented in [2], in which acquisition is done by serial correlation with an estimate of the new user's signal, which is created via removal of the estimated MAI from the received vector. A similar technique will be employed here with the residual signal. We shall correlate sums of adjacent received samples and estimate the delay within a chip from these statistics. We note that the schemes proposed in [2] suffer from the near-far problem because of imperfect estimates of the active users' amplitudes. The adaptive decorrelator technique presented herein is near-far resistant.

We shall consider the received vector synchronized to user K; this implies that each sample of the received vector contains signal information from a single symbol interval of user K, and from two symbol intervals of each of the original users.

If we re-consider (4), it is clear that we can think of the asynchronous system of K active users as a synchronous system with $2K - 1$ users. The formulation about to be presented is akin to that described in [19], which discusses the formulation of a one-shot asynchronous decorrelator. We let

$$S^*(\underline{\tau}) = [S^L(\underline{\tau}) | S^R(\underline{\tau})] \tag{17}$$

$$A^* = \begin{bmatrix} A^{K-1} & 0 \\ 0 & A^{K-1} \end{bmatrix} \tag{18}$$

$$\underline{b}^* = \begin{bmatrix} \underline{b}_{K-1}(i-1) \\ \underline{b}_{K-1}(i) \end{bmatrix}. \tag{19}$$

And so we can rewrite (4) as,

$$\underline{r} = S^*(\underline{\tau}) A^* \underline{b}^* + A_K b_K \underline{m}_K + \underline{n}. \tag{20}$$

We shall assume that $S^*(\underline{\tau})$ has full column rank for all possible sets of delays $\underline{\tau}$, thus we can form the non-singular cross-correlation matrix $R^*(\underline{\tau}) = S^{*T}(\underline{\tau}) S^*(\underline{\tau})$. It is clear that the desired projection operator for the asynchronous communication case is

$$B^* = I - S^*(\underline{\tau}) R^*(\underline{\tau})^{-1} S^{*T}(\underline{\tau}). \tag{21}$$

We note that the presentation above represents one possible mechanism for accommodating asynchronous transmission using decorrelator techniques. The next possible extension is to consider an n-shot receiver which uses the same technique described here, but for n symbols of the received signal; i.e. create $(n+1)(K-1)+n$ virtual users [19]. Another possibility is to use the same techniques proposed in this section but in conjunction with the asynchronous infinite impulse response (IIR) decorrelator [8] which operates on the entire received signal sequence. However, it will be observed that the one-shot asynchronous adaptive decorrelator, with its relatively lower complexity (with respect to an n-shot detector or an IIR filter), offers good performance.

The performance of the asynchronous training signal dependent detectors are provided in Section 9. Next we consider the probability of detection error of the augmented decorrelator.

6. Probability of Detection Error for the Augmented Decorrelator

We examine the probability of the ML algorithm making an incorrect decision (i.e. choosing the wrong code) to investigate the performance of the projection adaptive decorrelator algorithm in the sequel. However, there are a few relevant points to mention in regards to the probability of detection error, which is distinct from the probability of code-selection error studied in the subsequent section.

Once adaptation is complete, the decorrelator can be augmented to decorrelate the complete set of active users. It would be possible to compute the cross-correlation matrix and then compute its inverse; however, it is possible to use matrix algorithms to update the matrix inverse. By combining the algorithm for determining the inverse of a partitioned matrix (see e.g. [3]) with the matrix inversion lemma (see e.g. [4]) we have the following algorithm for determining R_K^{-1} from R_{K-1}^{-1}, S_{K-1} and the new user's spreading code, $\underline{m}_K$:

$$R_K^{-1} = \left[\begin{array}{c|c} R_{K-1}^{-1} - \frac{R_{K-1}^{-1} \underline{\varrho}_K \underline{\varrho}_K^T R_{K-1}^{-1}}{c_+} & R_{K-1}^{-1} \underline{\varrho}_K c_- \\ \hline c_- \underline{\varrho}_K^T R_{K-1}^{-1} & \frac{1}{c_-} \end{array} \right] \tag{22}$$

$$\text{where}\quad c_- = 1 - \underline{\varrho}_K^T R_{K-1}^{-1} \underline{\varrho}_K, \tag{23}$$

$$\text{and}\quad c_+ = 1 + \underline{\varrho}_K^T R_{K-1}^{-1} \underline{\varrho}_K, \tag{24}$$

$$\text{and}\quad \underline{\varrho}_K = S_{K-1}^T \underline{m}_K. \tag{25}$$

Once the augmented cross-correlation matrix inverse has been determined, the probability of detection error is simply that of the conventional decorrelating detector; the probability of detection error for the decorrelating detector for user k will be,[1] $Q\left(\frac{A_k}{\sigma\sqrt{(R_K^{-1})_{kk}}}\right)$ where A_k is the kth user's received amplitude, σ^2 is the ambient noise variance, and R_K^{-1} is the inverse of the cross-correlation matrix of the spreading codes of all of the K active users. This value is conditioned on the true spreading code being chosen by the adaptive algorithm in use.

7. Probability of Choosing the Incorrect Code

We shall investigate the computation of the probability,P_k^c; this is the probability of making a correct decision conditioned on $\underline{m}_k$ being the correct code. Without loss of generality, the new user's amplitude, A_K will be absorbed into the noise variance of the Gaussian noise, so that effectively $A_K = 1$ and the noise variance will be σ^2/A_K^2. We then note that the average probability of choosing the incorrect code, P^e, is,

$$P^e = \frac{1}{C}\sum_{k=1}^{C} P_k^e \quad \text{and that}\quad P_k^e = 1 - P_k^c. \tag{26}$$

Although we assume equal prior probabilities for the possible spreading codes, the different cross-correlation values imply that the probability of error conditioned on a particular code is not the same for all codes. Hence we shall focus our efforts on determining the probability of code-selection error conditioned on a particular code being the correct code. The methodology described below can be used for each k to determine the average error probability described in (26).

If we define $T(\underline{m}_i)$ as the appropriate test statistic (dependent on which ML test was used) for signature sequence $\underline{m}_i$, the probability of a correct decision can be described by,

$$\begin{aligned} 1 - P_t^e = P_t^c &= P[T(\underline{m}_i) < T(\underline{m}_t) \quad \forall i \neq t] \\ &= \int P[T(\underline{m}_i) < z \quad \forall i \neq t \,|\, T(\underline{m}_t) = z] f_{T(\underline{m}_t)}(z)\, \mathrm{d}z, \end{aligned} \tag{27}$$

where $f_{T(\underline{m}_t)}(\cdot)$ is the probability density associated with the decision statistic $T(\underline{m}_t)$. Given that the decision variables are correlated for all tests devised in this work, this expression does not simplify into a closed form expression. However, due to the fact that the test statistics for the locally optimum training signal dependent (LOTSD) algorithm are jointly Gaussian, we can bound the expression for the probability of a correct decision. For the test described in (12), we have the following decision variables, $T(\underline{m}_i)$ corresponding to codes $\underline{m}_i$, that is,

$$T(\underline{m}_i) = \frac{1}{M}\sum_{l=1}^{M} \underline{x}_l^T B \underline{m}_i. \tag{28}$$

[1] $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\{-\frac{v^2}{2}\}\, \mathrm{d}v.$

These variables are jointly Gaussian, and in particular,

$$T(\underline{m}_i) \sim \mathcal{N}(\underline{m}_t^T B \underline{m}_i, \frac{\sigma^2}{M} \underline{m}_i^T B \underline{m}_i), \tag{29}$$

$$\text{and } \operatorname{Cov}\{T(\underline{m}_i) T(\underline{m}_j)\} = \frac{\sigma^2}{M} \underline{m}_i^T B \underline{m}_j. \tag{30}$$

where $\underline{m}_t$ is the new user's spreading code and $\mathcal{N}(\mu, \sigma^2)$ denotes a Gaussian density with mean μ and variance σ^2. Note that each decision statistic has a possibly unique mean and variance.

We next discuss methods for bounding the expression for the probability of a correct decision for the LOTSD algorithm.

7.1. Upper Bound on the Probability of Code-Selection Error for the LOTSD Test

We shall apply the multiple-hypothesis Chernoff bound to our problem. In [5], Kazakos developed the multiple-hypothesis Chernoff bound and its relationship to analogous large deviations results. This bound is as follows:

$$\frac{1}{L} \log P[\underline{y}_L \in D] \leq -\inf_{\underline{y} \in D} I(\underline{y}). \tag{31}$$

This bound holds for random variables $\underline{y}_L$ which are the sums of L identically and independently distributed random variables i.e. we let $\underline{y}_L = \sum_{i=1}^{L} \underline{x}_i$, where $\underline{y}_L \in \mathbb{R}^N$. Here D is any convex set in the event class of the sample space and $I(\underline{y})$ is the rate function associated with the probability of error. $I(\underline{y})$ is determined by the equation below,

$$I(\underline{y}) = \sup_{\underline{\theta}} \underline{y}^T \underline{\theta} - \log \Xi_{\underline{x}}(\underline{\theta}) \tag{32}$$

$$\text{where } \Xi_{\underline{x}}(\underline{\theta}) = \mathbf{E}_{\underline{x}}\{\exp^{\underline{x}^T \underline{\theta}}\}. \tag{33}$$

Note that $\Xi_{\underline{x}}(\underline{\theta})$ is simply the moment-generating function for the distribution of $\underline{x}$ with respect to the variable $\underline{\theta}$. For a multivariate Gaussian random variable $\underline{x}$ with mean $\underline{m}$, and covariance matrix Σ, it is easily shown that the log of the moment-generating function is

$$\log \Xi_{\underline{x}}(\underline{\theta}) = \underline{\theta}^T \underline{m} + \frac{1}{2} \underline{\theta}^T \Sigma \underline{\theta}. \tag{34}$$

From (32) above we find that the rate function for a Gaussian random vector $\underline{x}$ is

$$I(\underline{y}) = \frac{1}{2} (\underline{y} - \underline{m})^T \Sigma^{-1} (\underline{y} - \underline{m}). \tag{35}$$

We now specialize the bound to our particular multiple-hypothesis testing problem. Using the same properties of our projection matrix B as were used to determine the locally optimum maximum likelihood decision rule in (12), we determine the rate function for the projection of the received signal vector, $\underline{x}$, i.e. we shall be considering $\underline{y}_L = \sum_{i=1}^{L} B \underline{r}_i$,

$$I(\underline{y}) = \frac{1}{2\sigma^2} (\underline{y} - \underline{m}_t)^T B (\underline{y} - \underline{m}_t) \tag{36}$$

$$= \frac{1}{2\sigma^2} \|B\underline{y} - B\underline{m}_t\|^2. \tag{37}$$

We are required to consider our probability over a convex set D. This is due to the derivation of the bound which involved the interchange of minimum and maximum operations over a function on the set of interest [5]. We begin by addressing the issue of bounding the conditional probability of code-selection error $P_k^e = P[(\underline{m}_k - \underline{m}_j)^T B\underline{y} < 0 \ \textit{for some} \ j \neq k|H_k]$. Let us define the following subset of $\mathbb{R}^N$,

$$G_k = \left\{\underline{y} \in \mathbb{R}^N : (\underline{m}_k - \underline{m}_j)^T B\underline{y} \geq 0 \ \forall k \neq j\right\}. \tag{38}$$

Thus G_k represents the set of projected samples that yield a decision in favor of hypothesis H_k. Clearly the G_k are convex sets for all k since they are the intersection of a finite number of half-spaces in $\mathbb{R}^n$. What we would like to investigate is $P[\underline{y} \in \bar{G}_k|H_k]$ where $\bar{G}_k$ is the complement of G_k. This set, however, is not convex[2] and so we look at

$$P_k^e = \sum_{j \neq k} P[\underline{y} \in G_j|H_k]. \tag{39}$$

Applying (31) we have

$$\frac{1}{L} \log \ P[\underline{y} \in G_j|H_k] \leq - \inf_{\underline{y} \in G_j} I(\underline{y}) \tag{40}$$

$$= - \inf_{\underline{y} \in G_j} \|B\underline{y} - B\underline{m}_k\|^2. \tag{41}$$

If $\underline{m}_k \in G_j$ then clearly the infimum of the rate function is zero. This would be a highly undesirable scenario because it would imply that in the absence of noise, the desired signal vector $\underline{m}_k$ would not yield a maximum of the appropriate decision statistic. Therefore, we assume that this is not the case. Then, since $\underline{m}_k \notin G_j$, it is apparent that the infimum is achieved in (41) on the boundary of G_j. Thus determining our rate function is equivalent to minimizing the distance between a point and the boundary of a convex set. Techniques for solving such a linear programming problem are discussed in [6].

The Chernoff upper bound is compared to simulation data in Section 9. As a caveat, we mention that the Chernoff bound and other large deviations bounds are tight asymptotically as the number of samples examined increases towards infinity. The bits used for the training sequence make no contribution to the information rate of the communications transmission. L refers to the number of training bits (samples) considered in the Chernoff bound. It will be observed that while the Chernoff bound determined in this section is adequate for performing systems engineering to determine the parameters necessary for good performance, this bound is in fact not very tight; the number of samples used is conservative, as this is desirable for a practical communications system.

We note that we are unable to use the multiple-hypothesis Chernoff bound to obtain a lower bound on the probability of code-selection error by upper bounding the probability of correctness. This is due to the fact that $\underline{m}_k \in G_k$ and so the infimum of the rate function is 0. This yields the trivial upper bound on the probability of correct code-selection of 1. Thus we must consider an alternative method for lower bounding the probability of code-selection error. This method is described in the sequel.

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[2] The set $\bar{G}_k$ is actually the union of a finite number of convex sets as is seen in (39).

7.2. Lower Bound on the Probability of Code-Selection Error for the LOTSD Test

It can be seen by modifying (shown in [12]) a lemma by Slepian [16] that the following result holds: if D and Q are positive definite symmetric matrices of dimension $n \times n$ such that $D_{ii} = Q_{ii} \;\; \forall\, i$ and $D_{ij} \geq Q_{ij} \;\; \forall\, i, j$, and these matrices are the covariance matrices of two sets of Gaussian random variables, where all random variables in question are zero-mean, then

$$P_n(D) \geq P_n(Q), \tag{42}$$

where $P_n(D)$ and $P_n(Q)$ are *orthant* probabilities for the same bounded region. Let $P_n(D)$ denote the following orthant probability, where D is the covariance matrix of the Gaussian random variables, $\underline{x}$, then

$$P_n(D) = P[x_1 < a_1, x_2 < a_2, \cdots, x_n < a_n] \tag{43}$$

$$= \int_{-\infty}^{a_1} dx_1 \int_{-\infty}^{a_2} dx_2 \cdots \int_{-\infty}^{a_n} dx_n f_n(x_1, x_2, \cdots, x_n : D), \tag{44}$$

where $f_n(x_1, x_2, \cdots, x_n : D)$ is the joint density of the random variables x_i with covariance matrix D.

The probability of correct code selection (see (27)) that we wish to determine is actually the expected value of an orthant probability, and so we can use the previously described generalization of Slepian's lemma to obtain a bound on the probability of correct code selection.

In order to apply Slepian's result, we first normalize the covariance matrix of the decision variables conditioned on the true spreading code and then determine the maximum cross-correlation value. We shall upper bound the probability of correct-code selection by determining the probability of correct code-selection for a corresponding equi-correlated system. This common cross-correlation value for the new system will be the maximum cross-correlation value of the normalized original system.

In general, the maximum cross-correlation value will be positive. In the event that the maximum cross-correlation value is negative, it is most convenient (for numerical evaluation) to consider the corresponding orthogonal system. In [17], Stuart has shown how a set of n jointly Gaussian random variables with equal, positive cross-correlation can be generated by a set of $n + 1$ independent Gaussian random variables with equal variance. That is, let $x_1, x_2, \cdots, x_n$ be our set of equi-correlated random variables and let $y_0, y_1, y_2, \cdots, y_n$ be $n+1$ independent random variables. We use the following construction,

$$x_i = y_i - b y_0. \tag{45}$$

We assume the following holds:

$$\mathbf{E}\{y_0\} = 0 \;\; and \;\; \mathbf{E}\{y_i\} = \mathbf{E}\{x_i\} \;\; \forall\, i \neq 0 \tag{46}$$

$$\mathbf{Var}\{y_i\} = \alpha^2. \tag{47}$$

The desired values of b and α^2 are determined by the actual statistics of the x_i. We can rewrite the orthant probability, $P_n(D)$ in terms of the independent random variables, y_i.

$$P_n(D) = \int_{-\infty}^{\infty} P[y_1 < a_1 + bz, y_2 < a_2 + bz, \cdots, y_n < a_n + bz] f_{y_0}(z)\, dz \tag{48}$$

$$= \int_{-\infty}^{\infty} \prod_{i=1}^{n} P[y_i < a_i + bz] f_{y_0}(z) dz, \tag{49}$$

where f_{y_0} is the density of the random variable y_0. Equation (49) follows from (48) due to the fact that the y_i are independent and thus the orthant probability reduces to a product of probabilities for single random variables. These probabilities are simply the cumulative distribution function of the Gaussian random variables y_i and can be easily determined.

This lower bound is compared to simulation data in Section 9. Due to the fact that there are both positive and negative cross-correlation values present in the covariance matrix of the original decision variables, we expect this lower bound to be loose. This seems quite intuitive when we consider Slepian's lemma which shows that the orthant probability grows with the cross-correlation value.

We now consider the procedure for modifying the decorrelator system when a user terminates communication.

8. A User Exits Communication

The dominant issue in the creation of the adaptive decorrelator is the accommodation of a new transmitting user into the reception of the multi-user signal; however, one must still consider a procedure for dealing with a user discontinuing transmission.

We begin by assuming that it is known that a user has ceased communication, and which user that is. At this point, the matched filter corresponding to that user is removed from operation and the matrix R_K^{-1} is replaced by R_{K-1}^{-1}. As in the augmentation of the cross-correlation matrix when a user enters into communication, appropriate matrix algorithms are to be used for computing R_{K-1}^{-1} from R_K^{-1}. It is clear that the exodus of multiple users can easily be accommodated. The crux of the matter is then determining when a user has left and the identity of that user. It is easily shown that the output of the K-user original decorrelator for user i is

$$\underline{s}_i = A_i b_i + \tilde{n}_i, \tag{50}$$

where $\tilde{n}_i$ is a Gaussian random variable with zero mean and variance $\sigma^2[R_K^{-1}]_{ii}$. As in the previous detection problems investigated herein, it is possible to form a hypothesis testing problem to determine the presence or absence of the ith user's signal:

$$H_0 \; : \; s_i = \tilde{n}_i \tag{51}$$

$$H_1 \; : \; s_i = A_i b_i + \tilde{n}_i. \tag{52}$$

We note that b_i is a binary random variable which takes on the values ± 1 with equal probability. We form the decision rule conditioned on the values of A_i and σ^2. This approach is considered as one can determine a minimum SNR necessary for proper system operation and thus design a decision rule for that value of SNR. The resulting likelihood ratio is

$$\mathcal{L}(s_i) = \frac{1}{2}\left[\exp - \left\{\frac{2s_iA_i + A_i^2}{2\sigma^2[R_K^{-1}]_{ii}}\right\} + \exp - \left\{\frac{-2s_iA_i + A_i^2}{2\sigma^2[R_K^{-1}]_{ii}}\right\}\right]. \tag{53}$$

For equal priors and uniform costs, we choose H_1 if

$$\cosh\left(\frac{s_iA_i}{\sigma^2[R_K^{-1}]_{ii}}\right) > 2\exp\left(\frac{A_i^2}{2\sigma^2[R_K^{-1}]_{ii}}\right) \tag{54}$$

$$\text{i.e. if } |s_i| > \frac{\sigma^2[R_K^{-1}]_{ii}}{A_i}\cosh^{-1}\left[2\exp\left(\frac{A_i^2}{2\sigma^2[R_K^{-1}]_{ii}}\right)\right]. \tag{55}$$

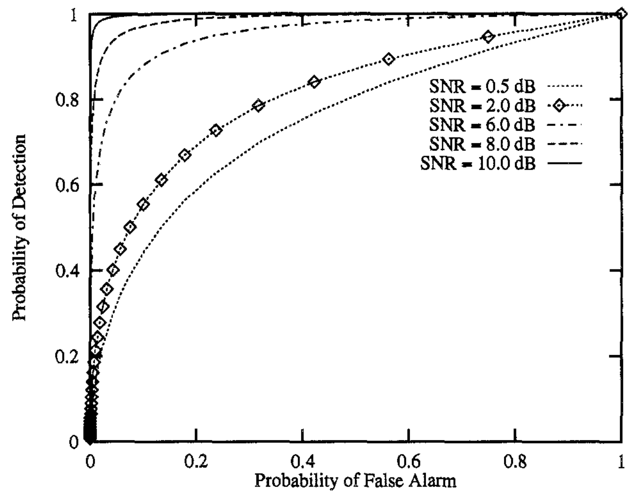

Fig. 3. Receiver Operator Characteristics for the binary hypothesis test determining that a user has ceased communication.

The locally optimal rule with respect to the unknown amplitude is found to be: choose H_1 if

$$s_i^2 > \sigma^4[R_K^{-1}]_{ii}^2 + \sigma^2[R_K^{-1}]_{ii} \tag{56}$$

Thus both the optimal and locally optimal rules are of the form $|s_i| \gtrless \tau$, where τ is some threshold. Given such a threshold we can calculate the probability of a missed detection P_M and the probability of false alarm, P_F, for a rule of this form

$$P_M = 1 - Q\left(\frac{\tau + A_i}{\sigma\sqrt{[R_K^{-1}]_{ii}}}\right) - Q\left(\frac{\tau - A_i}{\sigma\sqrt{[R_K^{-1}]_{ii}}}\right) \tag{57}$$

$$P_F = 2Q\left(\frac{\tau}{\sigma\sqrt{[R_K^{-1}]_{ii}}}\right). \tag{58}$$

The resulting receiver operating characteristics are provided in Fig. 3. We note that the SNR values shown are the *output* SNRs and are thus independent of the number of active users. It should be noted that the output SNR may differ from the input SNR; the output SNR will be a function of the number of actives users and more specifically the value of $[R_K^{-1}]_{ii}$ for user i. From this characteristic, it can be observed that this simple detection scheme performs well. It should be noted that more involved change detection methods are possible as well. Sequential detection schemes (e.g. the classical work found in [20]) which would involve collecting samples until an appropriate threshold has been exceeded or extensions of such work such as quickest detection schemes (e.g. [1] and references therein) could prove useful for this problem.

We shall next investigate the performance of each of the projection adaptive decorrelators presented herein via simulations. In addition, the experimental results for the LOTSD test will be compared to the upper and lower bounds previously described in Section 7.

9. Performance

Performance of the projection adaptive decorrelating detector was evaluated as a function of three parameters: the number of active users, the signal to noise ratio between the new user's power and that of the ambient Gaussian noise, and the number of received symbols considered to make the code determination. Codes of length $N = 31$ were used where each code was a shifted version of one of five possible m-sequences [14]. M-sequences are linear shift register sequences of length N where N must be of the form $2^m - 1$ for a positive integer m. In particular, the first m-sequence was generated using the shift-register polynomial described by the number 45. The subsequent m-sequences are generated by decimation of the original sequence. The following decimation rates were used: $3, 5, 7$, and 15. The rest of the sequences were generated by shifting this set of m-sequences by factors of 5. The cross-correlation matrix of the original set of 5 m-sequences is given below,

$$R = \begin{bmatrix} 1 & \frac{7}{31} & -\frac{9}{31} & -\frac{9}{31} & -\frac{9}{31} \\ \frac{7}{31} & 1 & \frac{7}{31} & \frac{3}{31} & -\frac{9}{31} \\ -\frac{9}{31} & \frac{7}{31} & 1 & \frac{7}{31} & \frac{7}{31} \\ -\frac{9}{31} & \frac{3}{31} & \frac{7}{31} & 1 & \frac{7}{31} \\ -\frac{9}{31} & -\frac{9}{31} & \frac{7}{31} & \frac{7}{31} & 1 \end{bmatrix}. \tag{59}$$

It is important to emphasize that the performance curves presented in this section are not universal. A very specific set of spreading codes is employed. Not all combinations of spreading codes were used to determine the conditional probability of error for, say, $K = 5$ users. Therefore different combinations would lead to different results. However, the experiments conducted do give an idea of the type of performance possible from the projection adaptive decorrelator.

We begin with the results for synchronous communication. From the figures it is readily apparent that the generalized adaptive decorrelator has good performance for reasonable SNR and for very few symbols. We first examine the utility of the bounds derived in Section 7 with simulation data for the locally optimum training signal dependent test (LOTSD) seen in (12). The trends observed hold for all three experiments conducted. The upper bound is fairly tight while the lower bound is very loose. These results were expected as the lower bound derived loses the 'dynamics' between the decision statistics through the construction of an equi-correlated system. In Fig. 4 there were 10 active users and the training sequence was of length 3 and the SNR was varied. Next the effects of increasing the length of the training sequence was investigated as seen in Fig. 5 where 10 active users are considered in a channel with an SNR of 8 dB. It is clear that we can achieve arbitrarily good performance by simply increasing the length of the training sequence. Finally, the probability of code-selection error is studied as a function of the number of active users. In Fig. 6 the SNR was 8 dB and the training sequence was of length 5. It is noticed that the projection decorrelator's performance degrades as more users become active. This is to be expected as we are reducing the effective dimension of the nullspace of the original active users as more users enter the system. Thus the residual signal will become weaker and weaker with each new user. However, as long as there are at most $N/2$ users in active communication for length N spreading codes, the algorithms deliver good performance. In addition, one can compensate by considering more samples of the signal in the decision-making process.

The next set of simulations studied the efficacy of the various blind algorithms derived in comparison to the LOTSD test. The three blind tests considered were: the locally optimum

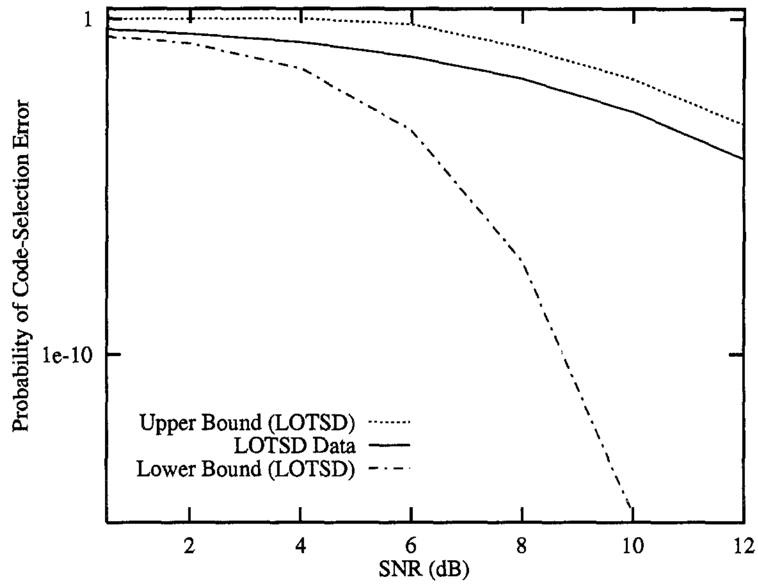

Fig. 4. Simulation data and bounds for the LOTSD Adaptive Decorrelator as the SNR is varied in synchronous communication.

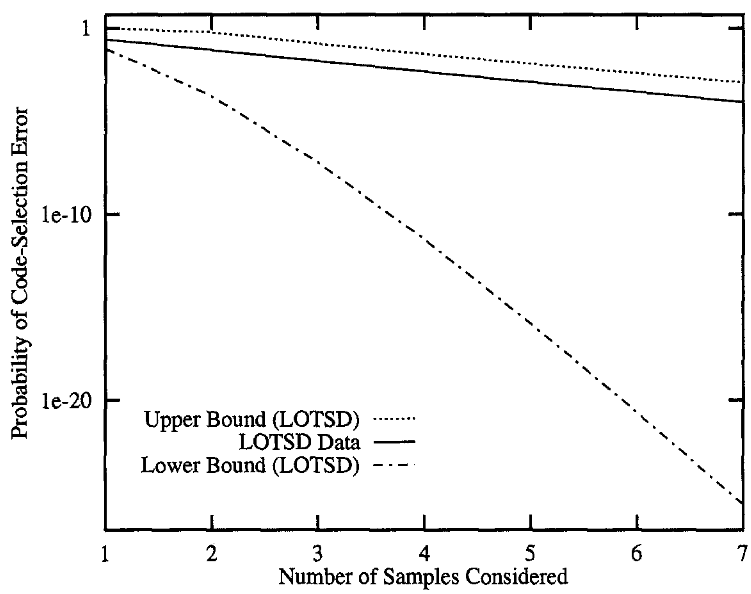

Fig. 5. Simulation data and bounds for the LOTSD Adaptive Decorrelator as the number of samples collected is varied in synchronous communication.

blind test (BLLO in (14)) and the two ad hoc tests (BLSQ in (15) and BLMOD in (16)). In Fig. 7, the probability of code-selection is considered as a function of the SNR with 10 active users and 3 samples of the received signal used to determine the new user's code. We notice that there is little difference between the performances of the BLLO and BLSQ tests, while

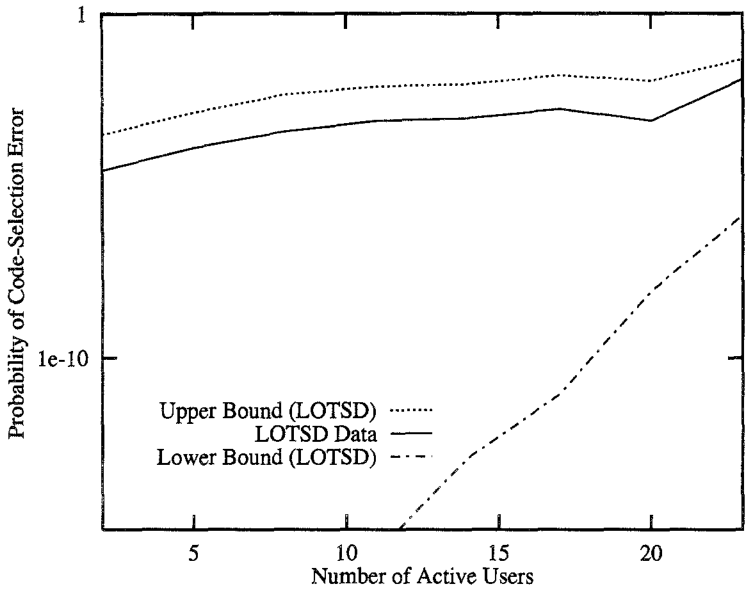

Fig. 6. Simulation data and bounds for the LOTSD Adaptive Decorrelator as the number of active users is varied in synchronous communication.

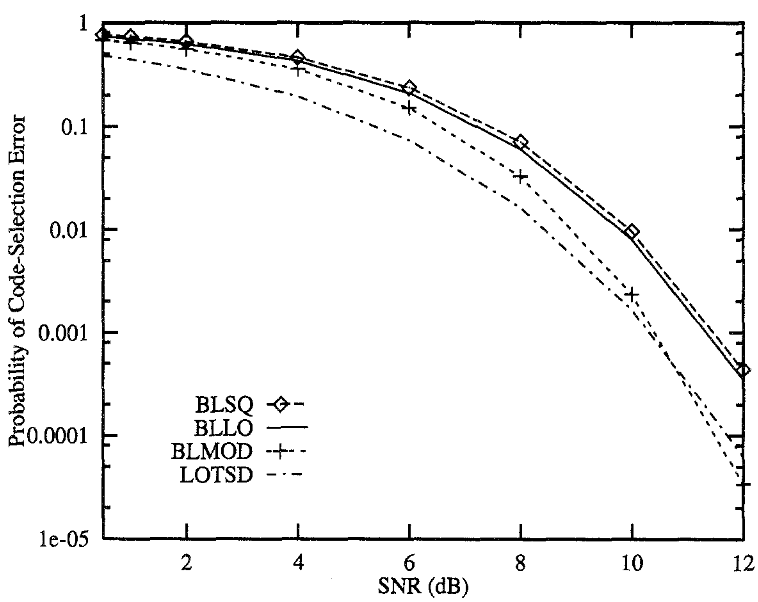

Fig. 7. Performance of Adaptive Decorrelators as the SNR is varied in synchronous communication.

the BLMOD test offers superior performance. These relationships continue to hold when we examine performance as a function of the number of received signal samples employed to make the code determination (Fig. 8). In this experiment the SNR was 8 dB and 10 active users were present. The final figure shows the effects of increasing the number of active users

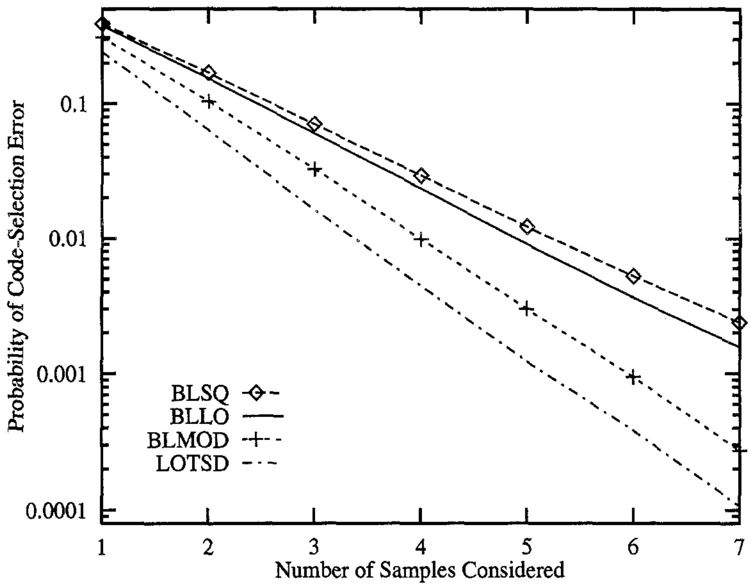

Fig. 8. Performance of Adaptive Decorrelators as the number of samples collected is varied in synchronous communication.

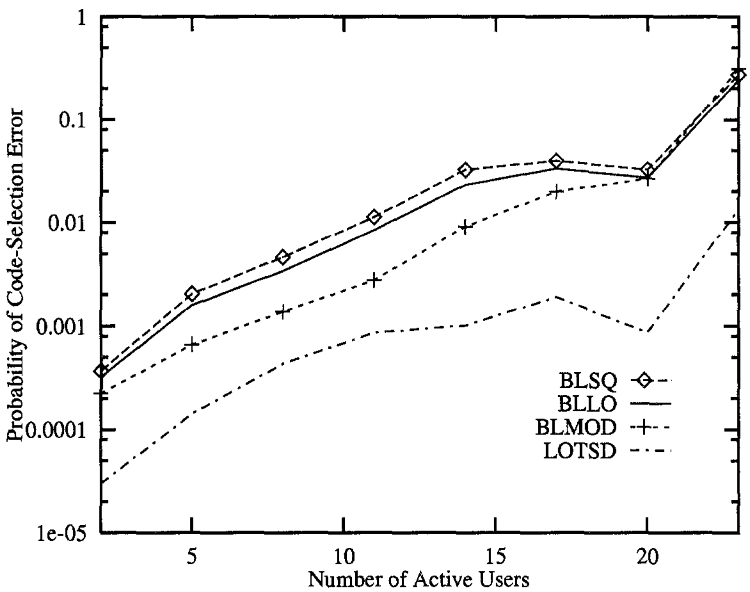

Fig. 9. Performance of Adaptive Decorrelators as the number of active users is varied in synchronous communication.

on the probability of code-selection error. In Fig. 9, the SNR was 8 dB and 5 samples were employed.

The next two figures show experiments for the use of the projection technique for asynchronous communication. We shall consider the locally optimum test described in (12). The trends observed between the various tests used in the synchronous case are observed to hold in the asynchronous case as well. Once again the $N = 31$ signature sequences were employed. Figure 10 shows the probability of incorrectly determining the delay of the new user employing the projected signal as a function of the number of active users. This was done via a correlation test (CORR) between sums of adjacent samples. The delay of user k was $\tau_k = 2(k-1)$ chips. Two sets of $3, 5$ and 7 samples were correlated against all possible shifts of each other. As can be seen from the figure, the algorithm achieves a reasonable probability of incorrect acquisition within a chip. Figure 11 shows the probability of code-selection error as a function of the number of active users for the scenario where each active user has a different delay with respect to the new user's symbol interval. The delay for user k is $2(k-1)$ chips; $4, 6$ and 8 symbol intervals of data were collected. It is observed that good performance can be achieved in the asynchronous case if simply more samples of the received signal are employed in the tests. In addition, it is clear that one can buffer the received samples (or some relevant statistic like the average for the LOTSD test) while determining the delay of the new user. After the delay determination has been made, the buffered samples can be used immediately to determine the new user's code without incurring further delay.

The final section of receiver development and analysis focuses on how the information streams of the original users are to be demodulated during the training process.

10. Detection of the Original $K-1$ Users

While our projection adaptive decorrelator is training to determine the spreading code of the new active user, it would be desirable to be able to continue to detect the information streams of the original users. It would be possible to apply the original decorrelating detector for $K-1$ users to the K active users. However, given that the spreading codes are not orthogonal, there will be a portion of the new user's signal present in each estimate of the original users' information streams and thus, such a detector would be near-far limited by the received signal of the new user. Thus an alternative method must be found.

We assume a communication environment where maximum likelihood detection of the new user's code is possible, therefore we have access to the set of possible codes from which the new user has chosen its code. Given this information, it is possible to project the received signal onto the orthogonal complement of the range space of the set of possible codes, S_P and then use the decorrelating detector for the original $K-1$ active users. We shall assume that the elements of the set of codes formed by the union of the set of possible codes,S_P and the set of codes of the original users, S_{K-1} are linearly independent. To begin, we shall consider the scenario of synchronous communication.

The desired projection is

$$B_P = I - S_P R_P^{-1} S_P^T, \tag{60}$$

$$\text{where} \quad R_P = S_P^T S_P. \tag{61}$$

Thus the estimate of the original $K-1$ active users' information streams $\hat{\underline{b}}$ is

$$\hat{\underline{b}} = \operatorname{sgn}(R_{K-1}^{-1} S_{K-1}^T B_P \underline{r}) \tag{62}$$

$$= \operatorname{sgn}\Big[(I - R_{K-1}^{-1} S_{K-1}^T S_P R_P^{-1} S_P^T S_{K-1}) A \underline{b} + R_{K-1}^{-1} S_{K-1}^T (I - S_P R_P^{-1} S_P^T)\underline{n}\Big]. \tag{63}$$

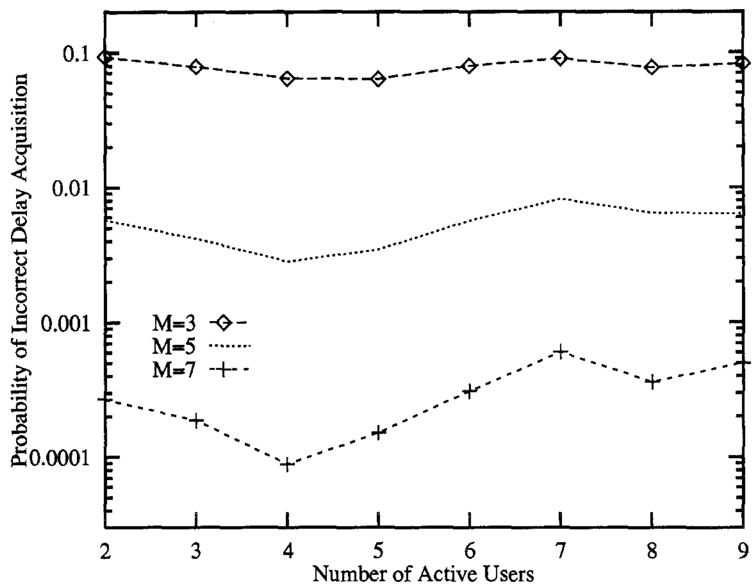

Fig. 10. Probability of incorrectly determining the delay.

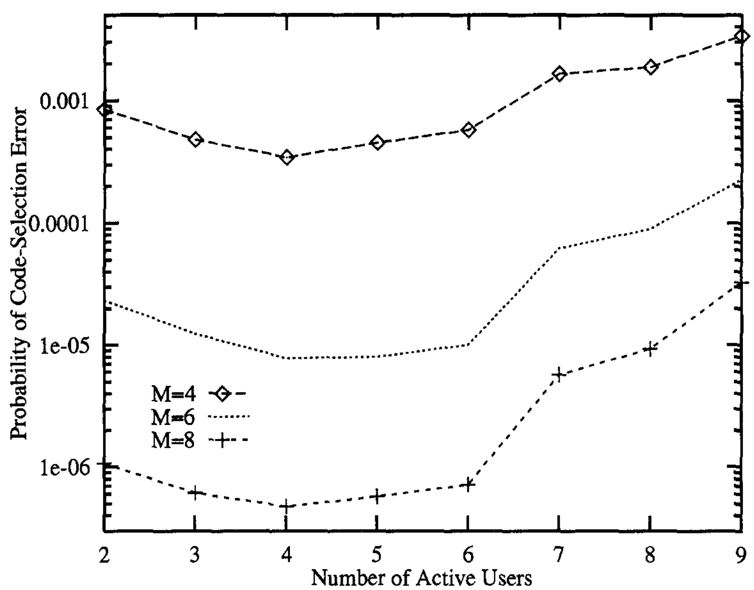

Fig. 11. Performance of the training signal dependent adaptive decorrelators in asynchronous communication.

Recalling that projections are idempotent, the probability of error for this detector for user k is

$$P_e(k) = Q\left(\frac{A_k(1 - [R_{K-1}^{-1} S_{K-1}^T S_P R_P^{-1} S_P^T S_{K-1}]_{kk}}{\sigma\sqrt{[R_{K-1}^{-1} - R_{K-1}^{-1} S_{K-1}^T S_P R_P^{-1} S_P^T S_{K-1} R_{K-1}^{-1}]_{kk}}}\right). \tag{64}$$

We shall refer to this detector as the *modified projection decorrelator* to distinguish it from the decorrelator developed by Lupas and Verdú [7], as well as from the projection adaptive decorrelator proposed in this work.

To compare the efficacy of this modified decorrelating detector to be used in the training phase with the conventional decorrelating detector in full reception phase, we look to the near-far resistance. The near-far resistance of the conventional decorrelating detector [7] is given by

$$\bar{\eta}_k = \frac{1}{[R^{-1}]_{kk}}, \tag{65}$$

where R is the cross-correlation matrix of the active users' spreading codes. Using an analysis similar to Lupas [7], we can show that the near-far resistance of user k of the modified projection decorrelating detector during the training phase is

$$\bar{\eta}_k^* = \frac{(1 - [R_{K-1}^{-1} S_{K-1}^T S_P R_P^{-1} S_P^T S_{K-1}]_{kk})^2}{[R_{K-1}^{-1} - R_{K-1}^{-1} S_{K-1}^T S_P R_P^{-1} S_P^T S_{K-1} R_{K-1}^{-1}]_{kk}}. \tag{66}$$

We next prove the following proposition.

PROPOSITION 1. *The modified decorrelating detector is near-far resistant, i.e.* $\bar{\eta}_k^* > 0$ *for all* $k \in [1, K-1]$.

Proof: From the expression for the near-far resistance of the modified detector for user k in (66) it is clear that the detector attains zero near-far resistance under two conditions

$$[R_{K-1}^{-1} - R_{K-1}^{-1} S_{K-1}^T S_P R_P^{-1} S_P^T S_{K-1} R_{K-1}^{-1}]_{kk} = \infty \tag{67}$$

$$or \;\; [R_{K-1}^{-1} S_{K-1}^T S_P R_P^{-1} S_P^T S_{K-1}]_{kk} = 1. \tag{68}$$

The equivalence in (67) is prevented from occurring due to the constraints on the spreading codes ($\underline{m}_i$ is drawn from $\{-\frac{1}{\sqrt{N}}, \frac{1}{\sqrt{N}}\}^N$) and the linear independence assumption. Thus we focus on the latter condition (68).

Let the $K \times 1$ vector $\underline{r}_i$ denote the ith column (or row transpose) of the cross correlation matrix R_{K-1}. Similarly, let $\underline{\zeta}_i$ denote the $K \times 1$ column (or row transpose) of R_{K-1}^{-1}. We decompose the spreading code $\underline{m}_i$ (where $i \in [1, K-1]$) into its projections into $R(S_P)$ and $N(S_P)$[3]. Thus,

$$\underline{m}_i = \underline{m}^P + \underline{m}^{P\perp} \tag{69}$$

$$\text{where} \;\; \underline{m}^P = S_P R_P^{-1} S_P^T \underline{m}_i \tag{70}$$

$$\text{and} \;\; \underline{m}^{P\perp} \perp R(S_P). \tag{71}$$

We also define the following subspaces: $V_1 = R(\underline{r}_i)$ and $V_2 = R(\underline{r}_1, \cdots, \underline{r}_j, \cdots, \underline{r}_K)$ for $j \neq i$. Since S_{K-1} is full column rank, R_{K-1} is full rank and thus the columns of R_{K-1} are linearly independent. This implies

$$V_1 \cap V_2 = \underline{0}. \tag{72}$$

[3] Recall that $R(S_P)$ and $N(S_P)$ denote the range space and null space of the columns of S_P, respectively.

We make two more observations before delving into the proof of the proposition

$$\underline{\zeta}_i^T \underline{r}_i = 1 = \underline{\zeta}_i^T [S_{K-1}^T(\underline{m}^P + \underline{m}^{P\perp})] \tag{73}$$

$$\underline{\zeta}_i \perp V_2. \tag{74}$$

These observations stem from the properties of the inverse of a matrix. Equipped with such observations and definitions

$$\bar{\eta}_k^* = 0 \tag{75}$$

$$\Leftrightarrow [R_{K-1}^{-1} S_{K-1}^T S_P R_P^{-1} S_P^T S_{K-1}]_{kk} = 1 \tag{76}$$

$$\Leftrightarrow \underline{\zeta}_i^T (S_{K-1}^T \underline{m}^P) = 1 \tag{77}$$

$$\Leftrightarrow S_{K-1}^T \underline{m}^{P\perp} \in V_2 \tag{78}$$

$$\Leftrightarrow S_{K-1}^T \underline{m}^{P\perp} \in V_2 \cap V_1 = \underline{0} \tag{79}$$

$$\Leftrightarrow \underline{m}_i^T \underline{m}^{P\perp} = 0 \tag{80}$$

$$\Leftrightarrow (\underline{m}^P + \underline{m}^{P\perp})^T \underline{m}^{P\perp} = \|\underline{m}^{P\perp}\|^2 = 0 \tag{81}$$

$$\Leftrightarrow \underline{m}^{P\perp} = \underline{0} \tag{82}$$

$$\Leftrightarrow \underline{m}_i \in R(S_P) \tag{83}$$

$$\Longrightarrow \quad \text{contradiction,} \tag{84}$$

where (79) follows from the fact that $\underline{m}^{P\perp}$ is the result of projecting $\underline{m}_i$ onto the null space of S_P and thus must fall into the span of $\underline{m}_i$. The final contradiction stems from the assumption that the signaling set (for existing and potential users) is linearly independent, thus $\underline{m}_i$ cannot be contained in $R(S_P)$. ■

The proof confirms what is known intuitively, as long as the existing users' signature sequences are not linear combinations of the possible codes, signal information for the existing users will be present after the received vector has been projected onto $N(S_P)$ and thus the conventional decorrelating detector operating on such a signal will continue to be near-far resistant. We also note the following

COROLLARY 2. *The modified projection detector achieves the near-far resistance for user k of the conventional decorrelating detector if $\underline{m}_k \in N(S_P)$, for some $k \in [1, K-1]$.*

Proof: This follows trivially from the expression for the near-far resistance of the modified decorrelator in (66) and the manipulations for Proposition 1. ■

Note that Corollary 2 states the conditions for the modified projection decorrelator to achieve optimal near-far resistance.

Figure 12 compares the near-far resistance of the modified projection decorrelator with that of the conventional decorrelator detector operating as if there were no new user. Therefore we are comparing the modified projection decorrelator operating in the presence of the new user to the conventional decorrelator operating in more *idealized* circumstances. The near-far resistance of the Lupas decorrelator with knowledge of $K-1$ users in the presence of K users is easily shown to be zero. This value is also included in Fig. 12 as a point of reference. The code set employed was the one described in Section 9.

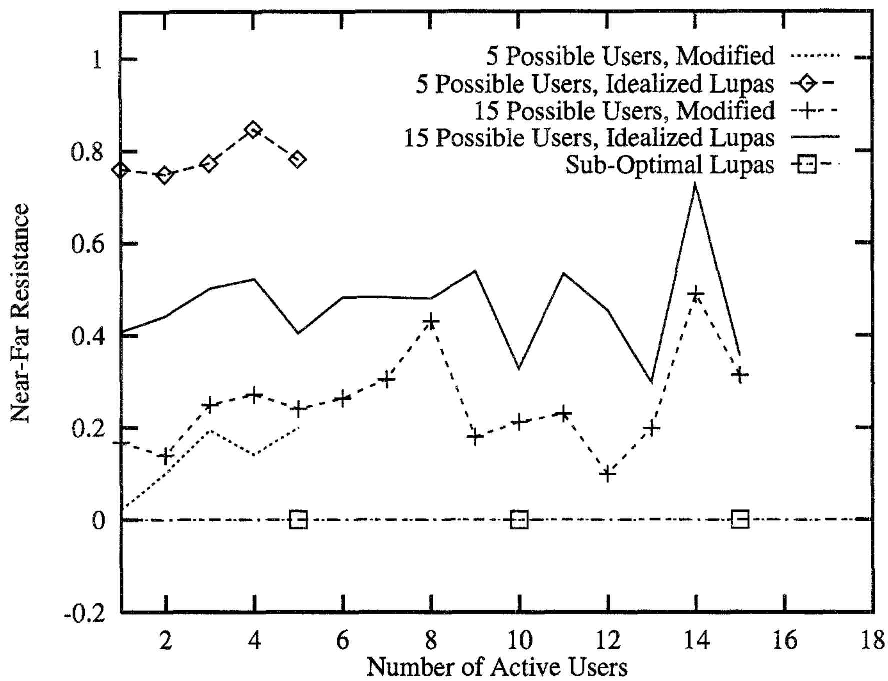

Fig. 12. Comparison of the near-far resistances of the Modified Projection Decorrelator and the Conventional Decorrelator for detection of the original users.

The phrase "n Possible Users" refers to the number (n) of total active users possible. Several observations can be made from Fig. 12: generally the near-far resistance of the modified projection decorrelator is less than that of the conventional decorrelator; however, neither decorrelator ever has zero near-far resistance. This is expected as the conventional decorrelator is operating in an idealized environment and it was just shown that the modified conventional decorrelating detector is near-far resistant. In addition, it is observed that the near-far resistances of the two decorrelators approach each other as the number of total possible users increases. This intuitively makes sense when we note that the limiting case of increasing the number of total possible active users is to have *all* users active and thus the projection B_P above is simply the identity since there are no possible codes available. Clearly in this scenario, the modified projection decorrelator is identically the conventional decorrelator. It is clear that this scheme can be modified to accommodate asynchronous communication by using the techniques developed in Section 5.

11. Conclusions

It can be seen that the generalized adaptive decorrelator schemes provide straightforward methods for augmenting an existing decorrelating detector to accommodate a new active user. Performance of the algorithms developed (with respect to the probability of code-selection error) was simulated and observed to be good for realistic communication scenarios. Relatively few received symbols of data are necessary to make an accurate determination of a new user's signature sequence. We observe that the methodology presented herein can be applied to augmenting other static detectors beyond the decorrelator.

It is noted that adaptive decorrelator presented herein assumed knowledge of the existence of the new active user. Research is in progress to determine the presence of the new active user using non-parametric detection methods [10, 11]. It is also desired to derive a tighter lower bound for the LOTSD test.

Future work will focus on the fading communication environment. Due to the inherent geometry of the adaptive decorrelating structure, it is presumed that beam-forming techniques will be necessary for this extension.

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References

1. M. Basseville and I. V. Nikiforov, *Detection of Abrupt Changes: Theory and Application.* Englewood Cliffs: Prentice Hall, 1993.
2. K. W. Halford and M. Brandt-Pearce, "User identification and multiuser detection of l out of k users in a CDMA system," in *Proceedings of the Twenty-eighth Annual Conference on Information Sciences and Systems, Princeton University, New Jersey*, pp. 115–120, 1994.
3. R. A. Horn and C. R. Johnson, *Matrix Analysis.* Cambridge: Cambridge University Press, 1985.
4. A. Householder, *The Theory of Matrices in Numerical Analysis.* New York: Blaisdell Press, 1964.
5. D. Kazakos, "Asymptotic error probability expressions for multihypothesis testing using multisensor data," *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 21, pp. 1101–1114, 1991.
6. D. G. Luenberger, *Optimization by Vector Space Methods.* New York: John Wiley & Sons, 1969.
7. R. Lupas and S. Verdú, "Linear multiuser detectors for synchronous code-division multiple-access channels," *IEEE Trans. Inform. Theory*, Vol. 35, pp. 123–136, 1989.
8. R. Lupas and S. Verdú, "Near-far resistance of multi-user detectors in asynchronous channels," *IEEE Trans. on Comm.*, Vol. 38, pp. 496–508, 1990.
9. U. Mitra and H. V. Poor, "Analysis of an adaptive decorrelating detector for synchronous CDMA channels," in *Proceedings of the Seventh IEE European Conference on Mobile Radio and Personal communications, Brighton, UK, December 1993*, pp. 155–160, IEE, 1993.
10. U. Mitra and H. V. Poor, "Detection of spread-spectrum signals in a multi-user environment," in *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing, Detroit*, May 1995.
11. U. Mitra and H. V. Poor, "Activity detection in a multi-user environment," *Wireless Personal Communications*, to appear in special issue on "Signal Separation and Interference Cancellation for Personal, Indoor and Mobile radio Communications", in press.
12. U. Mitra, *Adaptive Multi-User Detection.* PhD thesis, Princeton University, June 1994.
13. H. V. Poor, *An Introduction to Signal Detection and Estimation.* New York: Springer-Verlag, 2nd ed., 1994.
14. D. V. Sarwate and M. B. Pursley, "Crosscorrelation properties of pseudorandom and related sequences," *Proc. IEEE*, Vol. 68, pp. 593–619, 1980.
15. D. L. Schilling, R. L. Pickholtz and L. B. Milstein, "Spread spectrum goes commercial," *IEEE Spectrum*, pp. 40–45, 1990.
16. D. Slepian, "The one-sided barrier problem for Gaussian noise," *The Bell System Technical Journal*, Vol. 41, pp. 463–501, 1962.
17. A. Stuart, "Equally correlated variates and the multinormal integral," *Journal of the Royal Statistical Society*, Vol. 20, pp. 373–378, 1958.
18. S. Verdú, "Minimum probability of error for asynchronous Gaussian multiple-access channels," *IEEE Trans. Inform. Theory*, Vol. 32, pp. 85–96, 1986.
19. S. Verdú, "Recent progress in multi-user detection," in *1988 Conference Proceedings, International Conference on Advances in Communications and Control Systems and* Lecture Notes in Control and Information Sciences Series, Vol. 129, NY: Springer Verlag, 1988, IEEE, 1988.
20. A. Wald, *Sequential Analysis.* New York: Wiley, 1947.

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