ERRATA CORRIGE

On the Accelerating Property of an Algorithm for Function Minimization without Calculating Derivatives

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Abstract. It is shown that the algorithm of Ref. E1, when converging on a uniformly convex function and when technical condition (13) of Ref. E1 is satisfied, has an n -iteration Q -superlinear rate of convergence and a behaviour which is a precursor of every-iteration Q -superlinearity. This result overrides and corrects main result Theorem 3.1 of Ref. E1.

Key Words. Conjugate-direction methods, unconstrained minimization without calculating derivatives, superlinearly convergent algorithms, mathematical programming.

Discussion. We shall use the numbering, references, assumptions, and notations of Ref. E1.

In the proof of Theorem 3.1 of Ref. E1, page 544, fourth line from the bottom, it is stated that, since (40) is not true, every subsequence $\{p\}$ of $\{j\}$ satisfies

$$\|g_{n+1}^p\|/\|g_{n+1}^{p+1(p)}\| \rightarrow \infty.$$

Unfortunately, this is not correct: the extended real number infinity is only a cluster point and not a limit point; the negation of (40) is that, for every subsequence $\{p\}$ of $\{j\}$,

$$\liminf_{\substack{p \rightarrow \infty \\ p \in \{p\}}} \|g_{n+1}^{p+1(p)}\|/\|g_{n+1}^p\| = 0.$$

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Therefore, the statement of Theorem 3.1 of Ref. E1, page 535, is not true: no every-iteration Q -superlinearity is obtained. However, a comparative study, with the results available to the authors (essentially Remark 5.3, Ref. 10 of Ref. E1), revealed that this main result should be replaced by the following theorem.

New Theorem 3.1. Consider the sequence of points $\{x_r^i\}$ produced by the algorithm. Assume that $f(\cdot)$ satisfies Assumptions (A.1)–(A.2) and that Assumption (A.3) is satisfied. Under these conditions, if³

$$\|g(x_{n+1}^i)\| = O(\|x_{n+1}^{i+1} - x_{n+1}^i\|), \quad (\text{E1})$$

then (a) for every subsequence $\{p\}$ of $\{j\}$,

$$\liminf_{\substack{p \rightarrow \infty \\ p \in \{p\}}} \|x_{n+1}^{p+1(p)}\| / \|x_{n+1}^p\| = 0, \quad (\text{E2})$$

where $1(p)$ means an increment in the subsequence $\{p\}$; in particular,

$$\liminf_{j \rightarrow \infty} \|x_{n+1}^{j+1}\| / \|x_{n+1}^j\| = 0, \quad (\text{E3})$$

and thus, if the sequence $\{\|x_{n+1}^{j+1}\| / \|x_{n+1}^j\|\}_{j=0}^{\infty}$ converges, then we obtain an every-iteration Q -superlinear rate of convergence; (b) moreover,

$$\lim_{j \rightarrow \infty} \|x_{n+1}^{j+n}\| / \|x_{n+1}^j\| = 0, \quad (\text{E4})$$

i.e., we obtain an n -iteration Q -superlinear rate of convergence. $\square$

Indication of Proof. (a) (E2) is a consequence of the above-mentioned correction; (E3) is an obvious consequence of (E2).

(b) (E4) is obtained as follows. Because of (20)–(21), it is sufficient to prove that

$$\|g_{n+1}^{j+n}\| / \|g_{n+1}^j\| = o(1). \quad (\text{E5})$$

Suppose that (E5) is not true. This implies that there exists a subsequence $\{p\}$ of $\{j\}$ such that, for $p \in \{p\}$,

$$\|g_{n+1}^p\| = O(\|g_{n+1}^{p+n}\|). \quad (\text{E6})$$

This is seen to lead to a contradiction as follows. Using similar arguments as in proof of Theorem 3.1, Step 1 of Ref. E1, and (E6), we obtain that

$$\begin{aligned} |\langle s_{k+1}^{p+n}, H^* s_{i+1}^{p+n} \rangle| &= o(1) \quad \text{for } p \in \{p\}, \\ k &= 1, \dots, n-1, \quad i = 0, 1, \dots, k-1. \end{aligned} \quad (\text{E7})$$

³ Equation (E1) is the same as condition (13) of Ref. E1.

Using (E7) and similar arguments as in the proof of Theorem 3.1, Step 2 of Ref. E1, we obtain, by studying the subsequence $\{x_r^{p+n}\}_{r=0, \dots, n+1}^{p \in \{p\}}$, that

$$\|g_{n+1}^{p+n}\|/\|g_1^{p+n}\| = o(1) \quad \text{for } p \in \{p\}. \quad (\text{E8})$$

Hence, by (22)–(23) and (E8), for $p \in \{p\}$,

$$\|g_{n+1}^{p+n}\|/\|g_{n+1}^p\| = O(\|g_{n+1}^{p+n}\|/\|g_1^{p+n}\|) = o(1).$$

Hence, (E6) leads to a contradiction: indeed, for $p \in \{p\}$,

$$1 = (\|g_{n+1}^{p+n}\|/\|g_{n+1}^p\|)(\|g_{n+1}^p\|/\|g_{n+1}^{p+n}\|) = o(1)O(1) = o(1).$$

Therefore, the assumption that (E5) does not hold is false, and we obtain (E4). $\square$

It follows, therefore, that (i) the observation that our algorithm produces directions $\{\{s_{j+i}\}_{i=0}^n\}_{j=0}^\infty$ which become asymptotically H^* -conjugate (Theorem 4.1) and (ii) the ideas of McCormick and Ritter on conjugate-direction methods (Ref. 6 of Ref. E1) can be exploited to show that our algorithm, when it converges and condition (13) is satisfied, has an n -iteration Q -superlinear rate of convergence [new Theorem 3.1(b)] and has a behavior which is a precursor of every-iteration Q -superlinearity [new Theorem 3.1(a)]. With these facts, Comment 3.1 still holds: *if all goes well* [condition (13) holds, for which it is sufficient that the nn matrices of search directions (15) satisfy (16)], then *it goes even better*: we get an accelerating property.

Reference

- E1. TOINT, P. L., and CALLIER, F. M., *On the Accelerating Property of an Algorithm for Function Minimization without Calculating Derivatives*, Journal of Optimization Theory and Applications, Vol. 23, pp. 531–547, 1977.