

## ORIGINAL ARTICLE

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## Application of modal analysis by transfer function to nondestructive testing of wood I: determination of localized defects in wood by the shape of the flexural vibration wave

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**Abstract** This study was intended to detect nondestructively some defects such as knots and grain deviations in wood using modal analysis. The shapes of flexural vibration waves at the first mode generated by the tapping of wooden beams were determined using the transfer function. The wave shapes obtained were compared with the theoretical wave shape for a uniform material; and the possibility of detecting defects in wood was examined. The results are summarized as follows: (1) The shapes of flexural vibration waves at the first mode of wooden beams free of defects coincided almost completely with the theoretical wave shape. (2) The shapes of flexural vibration waves of wooden beams containing defects such as knots clearly differed from the theoretical wave shape, especially near the defect. (3) Based on these results, it should be possible to detect the presence of defects and to determine their location in wood.

**Key words** Modal analysis · Transfer function · Non-destructive test · Flexural vibration · Defect

### Introduction

If the elasticity and strength of wood for structural uses can be estimated nondestructively with high accuracy by applying a small deformation or vibration to a wood sample, the confidence of wood for structural uses will increase; moreover, various woods can be more appropriately utilized

according to their elasticity and strength. Therefore, it is useful for the forest industry that a wood's elasticity and strength be easily estimated with high accuracy by non-destructive testing.

A large number of nondestructive methods for testing wood properties such as elasticity and strength have been studied,<sup>1</sup> and many results obtained by the vibrational method (i.e., stress waves generated by tapping and ultrasonic longitudinal waves) have been reported. Most studies have showed the relation between the resonance frequency of longitudinal or flexural vibration and the average elastic modulus of the wood.<sup>2</sup> However, even if the elasticity and the strength of wood used for building or construction as whole can be estimated in terms of the resonance frequency, the elasticity and the strength of partial defects such as decays or knots cannot be estimated. Thus, in previous studies nondestructive vibration tests were used for uniform wood<sup>3</sup> and the average elasticity and strength of whole wood containing defects were studied,<sup>4</sup> but there has been no investigation of the elasticity or strength of partial defects such as knots in the wood. Defects such as knots are commonly present in wood, so it is difficult to avoid the influence of such defects on wood's elasticity or strength, and the elasticity or strength of the defects is also important. From these viewpoints, a method for detecting defects easily and estimating the elasticity or strength of the defective parts with high accuracy is necessary.

Modal analysis<sup>5</sup> was attempted for the above purpose in a series of studies. Miyamura and Nakajima<sup>6</sup> used this method to measure the unified degree of laminated woods but not to detect defects or to estimate the elasticity of the defective parts. In a series of studies, the vibrational properties of wood containing defects, especially the vibrational properties of defective parts (flexural vibration wave shape), were compared with a uniform wooden beam (defect free) to determine the possibility of detecting defects and estimating their elasticity or strength. In the present report, as the first step of these studies, the detection of defects such as knots and the estimation of the location of the defects were attempted based on the difference in the wave shape from that of a uniform wooden beam.

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## Materials and methods

### Materials

The wood beams were cut from a sugi (*Cryptomeria japonica*) log from Miyamacho, Japan; the tree was 76 years old with a diameter of 130 cm at breast height. Twelve wood beams with a dimension of 100 (L) × 4 (R) × 4 (T) cm were cut from air-dried sapwood and heartwood. Three kinds of wood beams – a uniform straight grain beam, a beam containing a knot, and a beam containing grain deviation near a knot – were prepared. These beams were stored in an air-conditioned room at 20°C and 65% relative humidity (RH) for at least 4 weeks. The wood density after conditioning ranged from 0.32 to 0.38 g/cm<sup>3</sup>, and the moisture content measured by a radiofrequency resistance-type moisture meter (CAS Electronic Co.) ranged from 7.9% to 8.5%.

### Transfer function

When a force is applied to a beam, its response  $x(t)$  can be described by a convolution integral with the force  $f(t)$  and dynamic properties  $h(t)$  as shown in Eq. (1) as a function of time.<sup>7</sup>

$$x(t) = \int_{-\infty}^{\infty} h(\tau)f(t - \tau)d\tau \tag{1}$$

When this relation is converted to a function of frequency using the Fourier transform, it can be described by a simpler equation with the force  $F(f)$ , the response  $X(f)$ , and the dynamic properties  $H(f)$  as follows:

$$X(f) = H(f) \cdot F(f) \tag{2}$$

Accordingly, for analysis of the vibration, the response  $X(f)$  was generally measured. However, the response depends on the force, so to grasp the dynamic properties correctly and to analyze the response obtained under the various conditions the dynamic property  $H(f)$  (i.e., compliance transfer function) should be measured directly. For obtaining  $H(f)$ , the force  $F(f)$  and response  $X(f)$  should be measured simultaneously and can be calculated using the following equation.

$$H(f) = \frac{X(f)}{F(f)} \tag{3}$$

In the present report, using the modal analysis in the single degree of freedom based on the compliance transfer function, the imaginary value at the first mode (maximum amplitude) on each measuring point of the longitudinal direction of the beam was measured. Then the flexural vibration wave shape was determined and analyzed using the maximum amplitudes obtained.

### Measurement of flexural vibration wave shapes

Figure 1 illustrates the experimental apparatus. The beam was hung horizontally at two points located 22.4 cm from

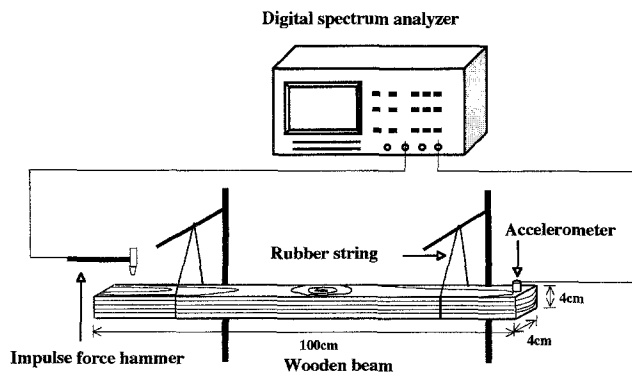


Fig. 1. Experimental apparatus

each end of the beam (the node point of the first mode), as shown in Fig. 1. The measuring points, every 1 cm (5 cm in some experiments) on the same surface of the beam, were tapped by an impulse force hammer, and the responses of the generated flexural vibration were measured by an accelerometer placed at a definite location (99 cm) on the same surface. The wave of the impulse from the impulse force hammer and the response measured by the accelerometer were read by digital spectrum analyzer, and the resonance frequency at the first mode of the beam and the imaginary value of the transfer function (i.e., the maximum amplitude at the first mode of each measuring point) were obtained by the digital spectrum analyzer. The flexural vibration wave shape of the beam was determined by these amplitudes. The flexural wave shape was then normalized by the amplitudes as described in the following section and compared with the theoretical flexural wave shape for a uniform material. All of the measurements were carried out in an air-conditioned room at 20°C and 65% RH.

### Theoretical flexural vibration wave shape and modal normalization

The wave shape of the flexural vibration<sup>8</sup> with the free ends can be described using the following equation.

$$Y(x) = \frac{\sinh k_1 l + \sin k_1 l}{\cos k_1 l - \cosh k_1 l} (\sinh k_1 x + \sin k_1 x) + \cosh k_1 x + \cos k_1 x \tag{4}$$

$$k_1 l = 4.730, k_2 l = 7.853, k_3 l = 10.996, k_4 l = 14.137, k_5 l = 17.279$$

where  $i$  is the number of modes; and  $l$  is the length of the beam.

To compare the experimentally obtained wave shape with the theoretical wave shape, the wave shape of the beams were normalized. To make clear the difference between a wave shape of a beam containing a knot and that of the theoretical wave shape, the amplitudes between the two node points of the beam were used for normalization, as shown in Fig. 2. The area enclosed by the original wave shape among two node points and the x-axis (the integral of

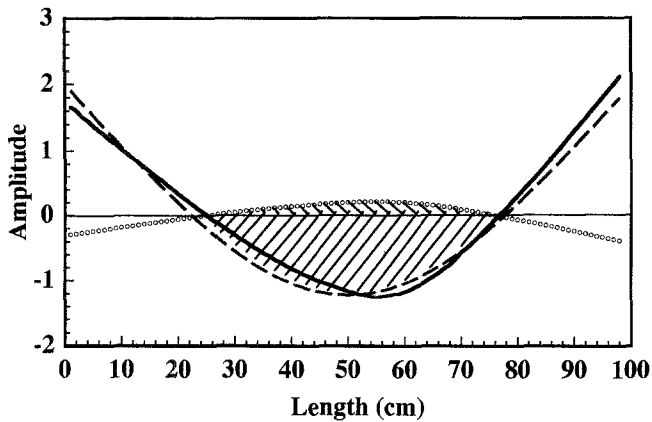


Fig. 2. Normalization of flexural vibration wave shape at the first mode. *Open symbols*, original wave shape; *solid line*, normalized wave shape; *broken line*, theoretical wave shape

the amplitudes between two node points to the length direction of the beam) (shown by the right oblique lines) and the area enclosed by the theoretical wave shape between two node points and the x-axis (shown by the left oblique lines) were computed. The ratio of the latter to the former was multiplied by the original wave shape, thereby obtaining the normalized wave shape of the beam. From this point on, the normalized wave shapes determined by this method are used for comparison with the theoretical wave shape.

## Results and discussion

Determination of the flexural vibration wave shape using the transfer function

For determining the wave shape of the flexural vibration at the first mode, the maximum amplitude of each measuring point on the longitudinal direction of the beam must be measured simultaneously. An apparatus with multiple channels and a number of sensors is necessary for such analysis. Such a system is expensive, and difficulty is often encountered because of the mass of the sensors used in the measurement. Therefore, measurements using a less expensive apparatus with two channels and a single sensor is desirable. In the present study, the method using the wave shape determined by the transfer function, obtained from the displacement/impulse force ratio was attempted. A beam with straight grain and no defects was tapped twice at different forces at the identical measuring point using the impulse force hammer. The impulse from the impulse force hammer and the generated flexural vibration were measured by the accelerometer set near one end of the beam and were fed into the digital spectrum analyzer. The results calculated by the transfer function are shown in Fig. 3. Despite the different tapping forces, the maximum amplitude at the first mode was 0.264, at the second mode it was 0.124, and at the third mode it was 1.53, with the amplitudes of the two taps being identical at each mode. Therefore, it was possible to obtain identical maximum wave shape

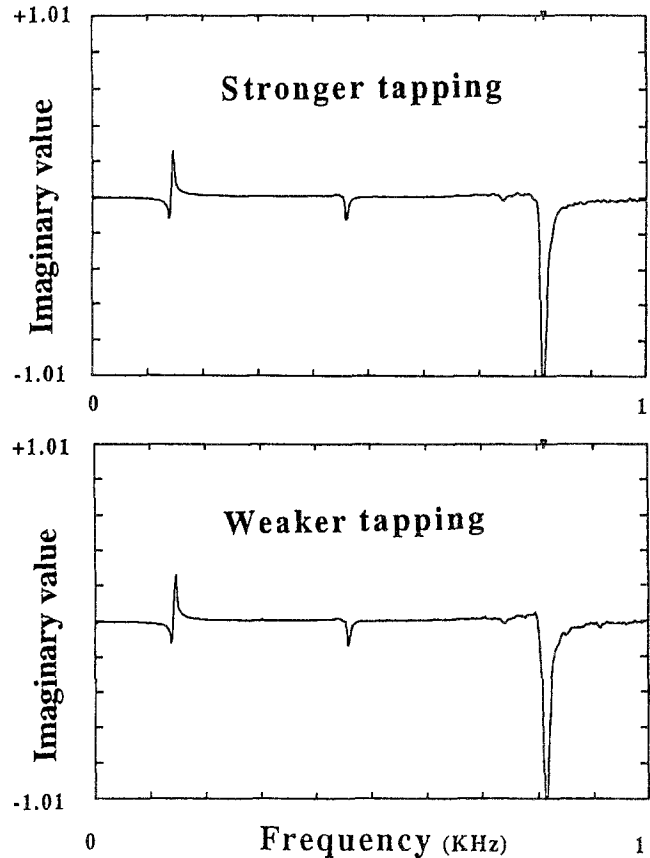


Fig. 3. Influence of tapping force on transfer function analysis

amplitudes by tapping each measuring point of the beam individually. This apparatus system is relatively inexpensive and does not involve the difficulty entailed by the mass of multiple sensors because it uses only a single sensor. Accordingly, the wave shape of the flexural vibration can be obtained easily and accurately with the present method.

As mentioned above, the maximum amplitude of each measuring point can be determined by tapping each measuring point of the beam and using the transfer function, thereby allowing the wave shape to be obtained from these maximum amplitudes. Consequently, it is possible to detect wood defects such as knots and to estimate the elasticity or the strength of the wood based on the wave shape.

Flexural vibration wave shape of uniform beam

Using the method described above, a beam with straight grain and without defects was tapped at each measuring point every 1 cm on the flat grain surface, and the maximum amplitudes of each measuring point were determined at the first mode; the wave shape was obtained by these maximum amplitudes. The wave shape of the beam was then normalized and compared with the theoretical wave shape of the homogeneous material. As seen by the results shown in Fig. 4, the normalized wave shape of the uniform beam completely coincided with the theoretical wave shape. Because this beam did not contain defects and its grain was straight,

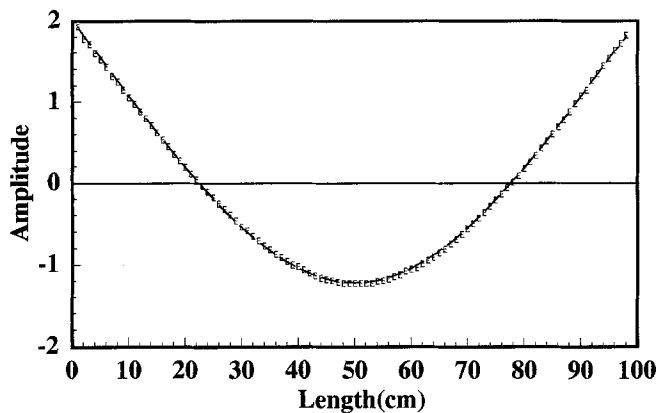


Fig. 4. Flexural vibration wave shape at the first mode of a clear wooden beam. *Open symbols*, normalized wave shape of a clear wooden beam; *broken line*, theoretical wave shape

the elasticity of the beam was considered uniform. The results shown in Fig. 4 also suggest that the elasticity of the beam is almost uniform.

Effect of partial mass and elasticity variation on the flexural vibration wave shape

The elasticity of the area containing a mechanical defect such as a knot or decay is generally lower in the direction of the tree axis than in a uniform part of the same beam; and the density of such an area is also different from that of the uniform part. Therefore, when detecting the defect and estimating the elasticity of the defective part using the wave shape, the effect of the partial elasticity and density variation on the wave shape must be examined. In the present report, this issue was investigated by attaching a weight with glue to simulate density variation and by drilling a hole to simulate elasticity variation (more exactly, the bending stiffness variation). That is, after obtaining the wave shape of a uniform beam with straight grains, a 5-g iron bar with glue was attached to the beam in the width direction at a position 63cm from one end, and the wave shape of the beam was then remeasured. Moreover, after removing the iron bar, a 2cm diameter hole was drilled at the same position on the flat grain surface, and the wave shape was measured for a third time. The mass removed by drilling the hole was almost equal to the mass of the iron bar attached to the beam. Figure 5 shows these three wave shapes. As shown in Fig. 5A, the node points of the wave shape of the beam to which the iron bar was attached shifted to the center of the beam compared with those of the uniform beam. At the point (63cm) of attachment to the iron bar, the amplitude was somewhat higher than that of the uniform beam, and the curvature of this point increased. This result shows the shift in the node points to the location of the weight increase due to the increase in the mass and the attendant increase in the curvature of the location. In contrast, the node points of the wave shape of the beam with a hole drilled at the same position (63cm) scarcely shifted, although the curvatures of the wave shape apparently increased, as shown in Fig. 5B.

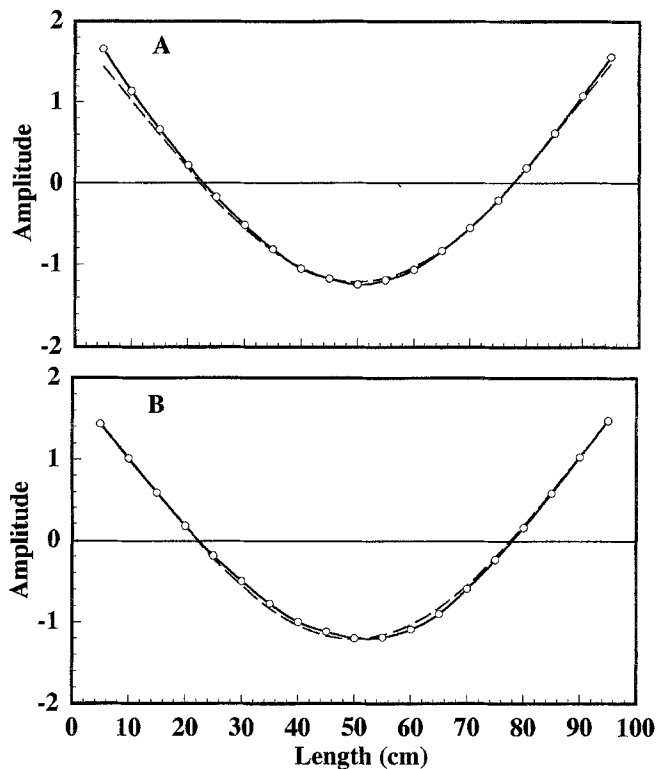


Fig. 5A,B. Influence of partial density and rigidity variation on the flexural vibration wave shape at the first mode. **A** Wave shape of the beam with attached weight. **B** Wave shape of the beam with a hole. *Open symbols*, wave shape of the wooden beam with weight in case A and with a hole in case B; *broken line*, theoretical wave shape

This result shows that in the beam with a knot 2cm in diameter, even if the density of the knot is twice that of the uniform part, the effect of the density variation on the wave shape was only slight. However, if the bending stiffness or elasticity of only 2cm of the partial defect against the 100cm length of the beam was decreased by half, the effect of the elasticity variation on the wave shape would be marked.

The preceding results suggest that it is possible to detect defects such as knots or decay based on the wave shape, especially in the case of knots, as both the increase in density and the decrease in elasticity increase the curvature of part of the knot, making the knot more easily detected.

Determining the presence of a knot in a beam in terms of flexural wave shape

To discuss the characteristics of the flexural vibration wave shape of a beam containing defects such as knots, two beams with the dimensions described under materials were prepared. One contained a 3cm diameter knot, with the center of the knot located 62cm from the base end of the beam. The other was taken from the adjacent part in a direction tangential to the above beam, and it contained grain deviations around the knot in the above beam.

The flexural vibration wave shapes at the first mode of these beams were obtained and compared with the theoretical wave shape. In the case in which the beam contained a

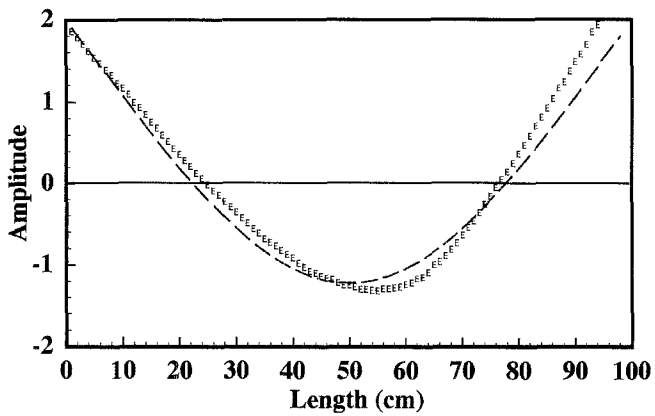


Fig. 6. Influence of a knot on the flexural vibration wave shape at the first mode. *Open symbols*, wave shape of wooden beam with a knot; *broken line*, theoretical wave shape

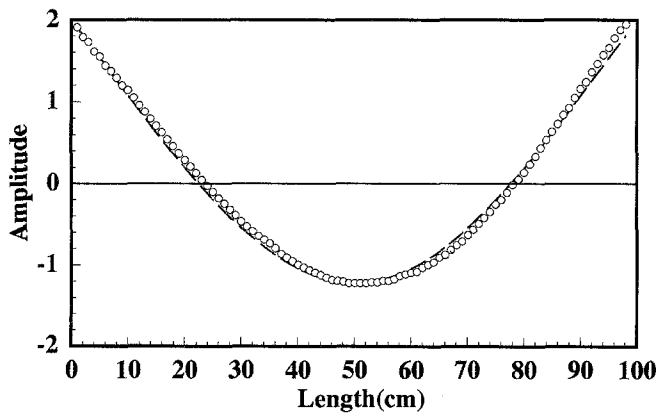


Fig. 7. Influence of localized grain deviations on the flexural vibration wave shape at the first mode. *Open symbols*, wave shape of a wooden beam containing localized grain deviations; *broken line*, theoretical wave shape

knot at 62 cm, the wave shape was obviously different from the theoretical wave shape. The maximum amplitude of the wave shape shifted to the right, and the amplitudes near the knot were higher than those of the theoretical wave shape at the same position, as shown in Fig. 6. A similar tendency was found in the beam containing the disordered grain, as shown in Fig. 7, though the tendency was much less than in the case shown in Fig. 6. In the case of the beam in which a hole was drilled (Fig. 5B), the amplitudes of the part in which a hole was drilled were also higher than that of the theoretical wave shape, and the curvature of that part was higher than that of the same part of the theoretical wave shape.

As mentioned above, it was shown that in the case of the beam containing a defect with lower elasticity (e.g., a knot, grain deviation, or hole) the amplitudes near the defect and the curvature of the defective part increased compared with the theoretical wave shape.

These results show that not only the presence of defects can be detected but also that the location and the elasticity of the defective parts can be estimated by comparing the beam's wave shape with the theoretical wave shape.

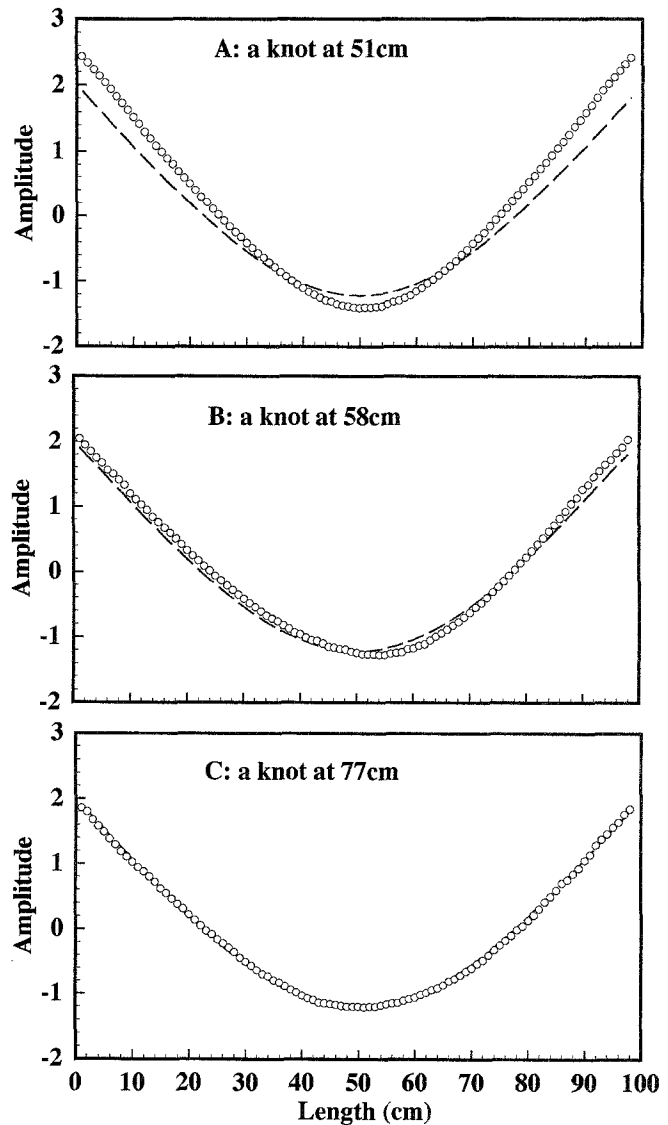


Fig. 8. Influence of knot location on the flexural vibration wave shape at the first mode. *Open symbols*, wave shape of wooden beam with a knot; *broken line*, theoretical wave shape

Estimating the location of the defect by means of the flexural vibration wave shape

As described in the preceding section, the location of the defect can be estimated by tapping the beam containing the defect and examining the wave shape. A beam containing a knot whose center was located near the center (51 cm) of the beam, a beam containing a knot whose center was located slightly apart from the center (58 cm) of the beam, and a beam containing a knot whose center was located near the node point (77 cm) were used for measurements. The wave shapes of these three beams were determined to investigate the reliability of estimating the defect's location.

In the case of the beam containing a knot (A: about 5 cm diameter, perforated) near the center of the beam, as shown in Fig. 8A, the wave shape of the beam was markedly different from the theoretical wave shape, and the node points shifted to the center of the beam. Therefore, the site where

the amplitude was most separated from the theoretical wave corresponded to the location of the knot. This result suggested that the location of the defect can be estimated with high accuracy. In the case of the beam containing a knot (B: about 3 cm diameter, perforated) slightly distant from the center of the beam, a similar result was obtained; that is, the amplitude of the knot's location was most separated from the theoretical wave, and the location of the knot can be accurately estimated. In the case of the beam containing a knot (C: about 6 cm diameter, about 1 cm depth) far from the center of the beam and near the node point, the wave shape was significantly different from the theoretical wave, and the maximum amplitude shifted to the right. Because the depth of the knot was shallow, there was only a small difference from the theoretical wave shape. However, the amplitudes were most separated from the theoretical wave shape in the range of 75–80 cm in the same location as that of the knot.

These results suggested that defects located near the central part of a beam can be easily detected, and the location of the defect can be estimated with high accuracy based on the flexural vibration wave shape obtained by the transfer function. Moreover, it is thought that the elasticity of every part of the beam can be estimated using the curvature of the wave shape. Consequently, the testing method described in the present study should be further investigated.

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## Conclusions

For the purpose of nondestructively detecting wood defects such as knots, the flexural vibration wave shape at the first mode generated by tapping the beam was determined using

the transfer function, and the wave shape was compared with the theoretical wave shape for homogeneous material. As a result, the flexural vibration wave shape at the first mode of the uniform beam almost completely coincided with the theoretical wave shape. In the case where the beam contained a defect such as a knot, the wave shape obviously differed from the theoretical wave shape around the defect. These results suggest that it is possible to detect the presence of defects and estimate their locations. Moreover, because it is thought that the defective part's elasticity can be estimated by analyzing the curvature of the wave shape, related studies will be further pursued.

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