Note Correcting a Remark in a Paper of Karl Bosch

M. Rosenblatt

Department of Mathematics, University of California, Box 109, San Diego, La Jolla, Cal. 92093, USA

In a paper "Notwendige und hinreichende Bedingungen dafür, daß eine Funktion einer homogenen Markoffschen Kette Markoffsch ist" appearing in vol. **31**, 199–202 (1975) of this journal, Karl Bosch states that Theorem 2 of a paper of C.J. Burke and myself "A Markovian function of a Markov chain" (p. 1113 Ann. Math. Statist. **29** (1958)) is false. A careful reading of the proof of Theorem 2 shows that Theorem 2 is correct as the proof involves a simple verification of the Markov property. Bosch in his argument looks at an example of a Markov chain we considered with transition matrix

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$$

and any initial distribution w_1, w_2, w_3, w_4, w_5 . The states 1, 2, 3 are collapsed into a single state S . The apparent error in Bosch's argument is the assumption that the collapsed process, if it is a Markov chain, will be homogeneous. In the example the collapsed Markov chain which is Markovian for any initial distribution, will be homogeneous if and only if $w_1 = w_3$.

Received September 15, 1975