Errata

Nonlinear Equivalence Transformations of Brownian Motion

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I am very grateful to Steven Orey for pointing out to me that [1] needs a correction to the proof of Lemma 5C.

On p. 256, in the definition of B_m , “ $y > z$ ” should be replaced by “ $y \geq x \geq z$ ”. (Without this change B_m is empty in general. Actually we will use only $y = x$ or $z = x$.)

At the top of p. 257, instead of citing Theorem 1 (which is irrelevant at this point) we need the following fact, where λ = Lebesgue measure. (This is not claimed as essentially new; it may follow even more directly from results in [2].)

Theorem. *Let K be a compact set of real numbers and f any real-valued function on K . Then*

$$\lambda\{x \in K: \lim_{h \downarrow 0} \{|f(x+h) - f(x)|/h: x+h \in K\} = +\infty\} = 0.$$

Proof. If K is an interval, the result is in Saks [2, p. 270, (4.4)]. Let $K \subset [a, b]$. Let

$$k(t) = \inf \{x: \lambda([a, x] \cap K) \geq t\}.$$

Then the function k is defined on the interval $[0, \lambda(K)]$, with $k(u) - k(t) \geq u - t$ for $0 \leq t \leq u \leq \lambda(K)$: k is a λ -preserving map into (onto almost all of) K . Being monotone, k is continuous except on an at most countable set.

For λ -almost all x in the set defined in the Theorem, there is some t with $k(t) = x$ and with k continuous at t . Then

$$\begin{aligned} & \lim_{h \downarrow 0} |f(k(t+h)) - f(k(t))|/h \\ & \geq \lim_{h \downarrow 0} |f(k(t+h)) - f(k(t))|/(k(t+h) - k(t)) \\ & \geq \lim_{u \downarrow 0} |f(x+u) - f(x)|/u = +\infty, \end{aligned}$$

so we apply the Theorem for the interval case and the function $f \circ k$, Q.E.D.

References

1. Dudley, R. M.: Nonlinear equivalence transformations of Brownian motion. Z. Wahrscheinlichkeitstheorie verw. Gebiete **20**, 249–258 (1971)
2. Saks, St.: Theory of the Integral (Transl. L. C. Young), 2d ed. (Monografie Matematyczne, Tom VII). Warsaw-New York: Hafner 1938

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