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Propositions (2.4), (2.5) must be replaced by:

PROPOSITION. *Let us assume that G is semi-simple with finite center, and that U is a quasi-simple K -finite representation of G . Let π^λ be a representation of g on A^K such that:*

$$\pi_X^\lambda = dU_X + \sum_{n>0} \frac{\lambda^n}{n!} t_X^n, \quad t_X^n \in \mathcal{L}(A),$$

the series converging in $\mathcal{L}(A)$ when $|\lambda| < R \neq 0$; moreover, $\pi_K^\lambda = dU_K$, if $K \in k$. Then:

(1) π^λ can be extended to a representation (denoted by π^λ) of g on A^∞ . The mapping $\lambda \rightarrow \pi_X^\lambda$, $X \in g$, is analytic from $B(O, R)$ into $\mathcal{L}_s(A^\infty)$ (the space of continuous linear mappings from the Fréchet space A^∞ into itself, endowed with the topology of simple convergence). The following relations hold:

$$t_{[X,Y]}^\pi = \sum_{i=0}^n C_n^i [t_X^i, t_Y^{n-i}] \text{ on } A^\infty, \text{ with } t^0 = dU \quad \text{and}$$

$$t_X^n(A^K) \subset A^K, \quad X \in g.$$

(2) The mapping $\lambda \rightarrow \pi_u^\lambda$, $u \in \mathcal{U}$, is analytic from $B(O, R)$ into $\mathcal{L}_s(A^\infty)$. If t^n denotes the canonical extension of t^n to $\mathcal{U}$ defined by Lemma (1.9), one has:

$$\pi_u^\lambda = dU_u + \sum_{n>0} \frac{\lambda^n}{n!} t_u^n, \quad u \in \mathcal{U}, \text{ on } A^\infty.$$

Proof. Starting from $dU_K \pi_X^\lambda = \pi_{[K,X]}^\lambda + \pi_X^\lambda dU_K$, $K \in k$, $X \in g$ on A^K , we obtain by induction

$$dU_{K^n} \pi_X^\lambda = \sum_{i=0}^n C_n^i \pi_{(\text{ad } K)^i(X)}^\lambda dU_{K^{n-i}}, \quad \text{on } A^K.$$

Let us write $\pi_X^\lambda = dU_X + F_X^\lambda$, $F_X^\lambda \in \mathcal{L}(A)$. We have

$$(*) \quad dU_{K^n} \pi_X^\lambda = \sum_{i=0}^n C_n^i (\text{ad } dU_K)^i (dU_X) dU_{K^{n-i}} + \sum_{i=0}^n C_n^i F_{(\text{ad } K)^i(X)}^\lambda dU_{K^{n-i}}.$$

U being quasi-simple, and G semi-simple:

$$\lim_p \varphi_p = \varphi \text{ in } A^\infty \text{ if and only if } \lim_p dU_{K^n}(\varphi_p) = dU_{K^n}(\varphi), \quad \forall K \in k, n \geq 0.$$

It results from (*) that π_X^λ is a continuous mapping from A^K into A^∞ (for the Fréchet topology) and, since A^K is dense, it can be extended to a representation of g on A^∞ .

By the same argument, formula (*) now holds on A^∞ . Using the analyticity of $\lambda \rightarrow F_X^\lambda$, we obtain that $\lambda \rightarrow \pi_X^\lambda$ is a continuous mapping from $B(O, R)$ into $\mathcal{L}_s(A^\infty)$. Therefore, so does $\lambda \rightarrow F_X^\lambda$, and since this mapping is analytic from $B(O, R)$ into $\mathcal{L}(A)$, Morera's theorem proves that it is an analytic mapping from $B(O, R)$ into $\mathcal{L}_s(A^\infty)$. Clearly $\pi_X^\lambda = dU_X + F_X^\lambda$ is also analytic.

From the formula $\pi_{[X, Y]}^\lambda = \pi_X^\lambda \pi_Y^\lambda - \pi_Y^\lambda \pi_X^\lambda$, we deduce that

$$t_{[X, Y]}^n = \sum_{i=0}^n C_n^i [t_X^i, t_Y^{n-i}].$$

Since $\pi_K^\lambda = dU_K$, $K \in k$ we have $t_K^n = 0$, $n > 0$, and therefore

$$t_{[K, X]}^n = [dU_K, t_X^n], \quad K \in k, \quad X \in g.$$

Denote by V_δ , $\delta \in \hat{k}$, the analytic component of type δ in A^K , and define θ from $g \otimes V_\delta$ into A^∞ by $\theta(X \otimes v) = t_X^n(v)$. The preceding formula shows that θ is a morphism of k -modules, and therefore $t_X^n(A^K) \subset A^K$.

When $u \in \mathcal{U}$, there exists $X_1 \dots X_p \in g$ such that $u = X_1 \dots X_p$. Writing $\pi_u^\lambda = \pi_{X_1}^\lambda \dots \pi_{X_p}^\lambda$, we obtain the analyticity and the development. Q.E.D.

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