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Corrigendum:
On a Bound for Deformations in Terms
of Strains in an Elastic Cylinder

Vol. 60, 17-28 (1975)

This corrigendum is to correct an assumption made in the paper as well as the part of the proof based on this assumption. As will be seen below, the main theorem and the basic technique of the proof remain valid with the proper assumption.

In the Statement of the Main Theorem [page 21], $d_b(z)$ was defined as

$$d_b(z) = \sqrt{\frac{\lambda}{\mu \rho \omega_0^2(z)}}, \quad (\text{A.1})$$

where λ and μ are Lamé constants, ρ is mass density (λ, μ, ρ being constant) and $\omega_0(z)$ is the smallest characteristic frequency of free vibration in linear elasticity of a section of the cylinder of length z .

The assumption was then made that $d_b(z) < Mh$ for all $z > H$, where h and H are the inner and outer diameters of the cross-section of the cylinder for all $z > H$, where M depends on the ratio H/h and on the boundary curve of the cross-section. However, in free vibration it is possible that a mode of vibration may exist with wave length approximately equal to z as well. Thus, a proper assumption for $d_b(z)$ is that

$$d_b(z) < M \max(H, z) \quad (\text{A.2})$$

for any subcylinder of length z .

In the paper, the original assumption was only used to obtain (5.44) from (5.30). We shall now proceed to derive (5.44) from (5.30) with the proper assumption (A.2). The problem is to estimate $\left| \iiint_{\mathcal{Q}_{r, r+a}} q_{i3} v_i d\sigma dx_3 \right|$ and $\left| \iiint_{\mathcal{Q}_{r+a, r+2a}} q_{i3} v_i d\sigma dx_3 \right|$, with $a > 4H$. Let us choose a number H_1 , such that $H_1 \leq H$ and $a = NH_1$, with N an integer. $\mathcal{Q}_{r, r+a}$ will now be divided into N disjoint subcylinders of length H_1 and we denote the subcylinders by $\{\mathcal{S}_k\}_{k=1}^N$.

Thus

$$\iiint_{\mathcal{Q}_{r, r+a}} q_{i3} v_i d\sigma dx_3 = \sum_{k=1}^N \iiint_{\mathcal{S}_k} q_{i3} v_i d\sigma dx_3. \quad (\text{A.3})$$

The vector v_i is defined by (5.18) with arbitrary coefficients α_{ij} and d_i . In each $\mathcal{S}_k$, we set $\alpha_{ij} = \alpha_{ij}^{(k)}$ and $d_i = d_i^{(k)}$ such that the resulting $v_i^{(k)}$ is orthogonal to every rigid body motion in a subcylinder of length H_1 .

In the following, the summation convention will not apply to the superscript k . By Rayleigh's Theorem,

$$\iiint_{\mathcal{S}_k} v_i^{(k)} v_i^{(k)} d\sigma dx_3 \leq \frac{2}{\rho \omega^2(H_1)} \iiint_{\mathcal{S}_k} W(\bar{e}_{ij}) d\sigma dx_3, \quad (\text{A.4})$$

where $\bar{e}_{ij}$ is the linear strain corresponding to $v_i^{(k)}$. The tensor $\bar{e}_{ij}$ is independent of k , because in each case, $v_i^{(k)}$ differs from $\bar{u}_i$ by a rigid body motion and thus

$$\bar{e}_{ij} = \frac{1}{2} (\bar{u}_{i,j} + \bar{u}_{j,i}). \quad (\text{A.5})$$

Now

$$2 \left| \iiint_{\mathcal{S}_k} q_{i3} v_i^{(k)} d\sigma dx_3 \right| \leq \alpha \iiint_{\mathcal{S}_k} q_{i3} q_{i3} d\sigma dx_3 + 1/\alpha \iiint_{\mathcal{S}_k} v_i^{(k)} v_i^{(k)} d\sigma dx_3, \quad (\text{A.6})$$

for any $\alpha > 0$ by the standard arithmetic-geometric mean inequality.

Combining (A.4), (A.6) and (A.3), we obtain

$$2 \left| \iiint_{\mathcal{Q}_{r,r+a}} q_{i3} v_i d\sigma dx_3 \right| \leq \alpha \iiint_{\mathcal{Q}_{r,r+a}} q_{i3} q_{i3} d\sigma dx_3 + \frac{2}{\rho \alpha \omega_0^2(H_1)} \iiint_{\mathcal{Q}_{r,r+a}} W(\bar{e}_{ij}) d\sigma dx_3. \quad (\text{A.7})$$

By a similar argument,

$$\begin{aligned} 2 \left| \iiint_{\mathcal{Q}_{r+a,r+2a}} q_{i3} v_i d\sigma dx_3 \right| &\leq \alpha \iiint_{\mathcal{Q}_{r+a,r+2a}} q_{i3} q_{i3} d\sigma dx_3 \\ &+ \frac{2}{\rho \alpha \omega_0^2(H_1)} \iiint_{\mathcal{Q}_{r+a,r+2a}} W(\bar{e}_{ij}) d\sigma dx_3, \end{aligned} \quad (\text{A.8})$$

and adding (A.7) to (A.8), we obtain

$$\begin{aligned} &\left| \iiint_{\mathcal{Q}_{r,r+a}} q_{i3} v_i d\sigma dx_3 \right| + \left| \iiint_{\mathcal{Q}_{r+a,r+2a}} q_{i3} v_i d\sigma dx_3 \right| \\ &\leq \frac{1}{2} \left[\alpha \iiint_{\mathcal{Q}_{r,r+2a}} q_{i3} q_{i3} d\sigma dx_3 + \frac{2}{\rho \alpha \omega_0^2(H_1)} \iiint_{\mathcal{Q}_{r,r+2a}} W(\bar{e}_{ij}) d\sigma dx_3 \right]. \end{aligned} \quad (\text{A.9})$$

The relations (5.37), (5.38), (5.39) are unaffected by the change in assumption and therefore, by combining these relations and (5.3) with (A.9) we see that

$$\begin{aligned} &\left| \iiint_{\mathcal{Q}_{r,r+a}} q_{i3} v_i d\sigma dx_3 \right| + \left| \iiint_{\mathcal{Q}_{r+a,r+2a}} q_{i3} v_i d\sigma dx_3 \right| \\ &\leq \frac{1}{2} \left[\alpha + \frac{2K_1}{\rho \alpha \omega_0^2(H_1)} \right] \iiint_{\mathcal{Q}_{r,r+2a}} q_{ik} q_{ik} d\sigma dx_3, \end{aligned} \quad (\text{A.10})$$

where K_1 depends only on the physical elastic constants.

We now choose $\alpha = \sqrt{\frac{2K_1}{\rho\omega_0^2(H_1)}}$, and therefore

$$\begin{aligned} & \left| \iiint_{\mathcal{Q}_{r,r+a}} q_{i3} v_i d\sigma dx_3 \right| + \left| \iiint_{\mathcal{Q}_{r+a,r+2a}} q_{i3} v_i d\sigma dx_3 \right| \\ & \leq \sqrt{\frac{2K_1}{\rho\omega_0^2(H_1)}} \iiint_{\mathcal{Q}_{r,r+2a}} q_{ik} q_{ik} d\sigma dx_3. \end{aligned} \quad (\text{A.11})$$

Since $H_1 \leq H$, we have from (A.2) that

$$[\sqrt{\rho}\omega_0(H_1)]^{-1} = O(H) = O(h). \quad (\text{A.12})$$

Combining (A.12), (A.11), (5.3), and (5.30), we obtain (5.44) and the rest of the proof is unchanged.

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(Received August 19, 1976)