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Correction:
Thermodynamics of Electromagnetic Materials
with Memory

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The principal conclusions in the above cited paper are incorrect, except for the case when the extra entropy flux vector $\mathbf{k}$, as defined in equation (2.7), vanishes identically. The error is caused by the fact that I failed to take into account the restrictions imposed by equations (2.1)₂ on the derivatives occurring in the entropy inequality (5.4).

The inequality (5.14) should be replaced by

$$\begin{aligned} \theta \gamma = & -(D_i + D_{E_i} \mathcal{F}) \dot{E}_i + (H_i - D_{B_i} \mathcal{F}) \dot{B}_i - (\eta + D_\theta \mathcal{F}) \dot{\theta} - \partial_{g_i} \mathcal{F} \dot{g}_i \\ & - \delta_{E_i} \mathcal{F} (\sim | \dot{E}_{(r)i}) - \delta_{B_j} \mathcal{F} (\sim | \dot{B}_{(r)j}) \\ & - \delta_\theta \mathcal{F} (\sim | \dot{\theta}_{(r)}) + \theta D_{E_j} \mathcal{K}_i E_{j,i} + \theta D_{B_j} \mathcal{K}_i B_{j,i} \\ & + \theta D_\theta \mathcal{K}_i g_i + \theta \partial_{g_j} \mathcal{K}_i \theta_{,ij} + \theta \delta_{E_j} \mathcal{K}_i (\sim | E_{(r)j,i}) + \theta \delta_{B_j} \mathcal{K}_i (\sim | B_{(r)j,i}) \\ & + \theta \delta_\theta \mathcal{K}_i (\sim | g_{(r)i}) - \frac{\partial_i g_i}{\theta} + \mathcal{J}_i E_i + v_s (e_{spq} H_{q,p} - \dot{D}_s - \mathcal{J}_s) \geq 0, \end{aligned} \quad (1)$$

where the vector v_s is an undetermined Lagrange multiplier and $f_{(r)}^t$ denotes the restriction of the history f^t to $(0, \infty)$. Consequently, equations (5.27)–(5.29) should be replaced by

$$D_i = -D_{E_i} \mathcal{F} - v_s D_{E_i} D_s, \quad (2)$$

$$H_i = D_{B_i} \mathcal{F} + v_s D_{B_i} D_s + \frac{1}{2} e_{irs} (\theta D_{E_s} \mathcal{K}_r + v_m e_{mrq} D_{E_s} H_q), \quad (3)$$

$$\eta = -D_\theta \mathcal{F} - v_s D_\theta D_s \quad (4)$$

and, in addition, $\mathcal{F}$ is not independent of g_i but satisfies the relation

$$\partial_{g_i} \mathcal{F} + v_s \partial_{g_i} D_s = 0. \quad (5)$$

In order to identify v_s , it is necessary to reformulate the problem using $\mathbf{E}$, $\mathbf{H}$, θ and $\mathbf{g}$ independent variables. After a long calculation it is found that

$$v_s = \frac{1}{2} \theta e_{sji} D_{H_j} \mathcal{K}_i (E_n^t, H_n^t, \theta^t; g_n)$$

where

$$\begin{aligned}\mathcal{K}_i = & A_i + e_{irs} \omega_r^{(1)} g_s + e_{irs} \omega_r^{(2)} E_s + e_{irs} \omega_r^{(3)} H_s + \Delta^{(1)} e_{irs} g_r E_s \\ & + \Delta^{(2)} e_{irs} g_r H_s + \Delta^{(3)} e_{irs} E_r H_s + e_{irp} E_r \int_0^\infty \Lambda^{(1)}(s) E_p^t(s) ds \\ & + e_{irp} H_r \int_0^\infty \Lambda^{(2)}(s) H_p^t(s) ds,\end{aligned}\quad (6)$$

and here A_i is a polar vector, $\omega_r^{(1)}, \omega_r^{(2)}, \omega_r^{(3)}$ are parallel axial vectors, $\Delta^{(1)}, \Delta^{(2)}, \Delta^{(3)}, \Lambda^{(1)}(s), \Lambda^{(2)}(s)$ are scalars and all of these quantities depend on θ^t only. Equation (6) replaces the erroneous relations (6.19) and (6.23). I have been unable to obtain a simple expression for $\mathcal{K}_i$ in terms of E_j^t, B_j^t, θ^t and g_j .

When the expression for v_s is substituted into equations (2), (3) and (5), these may be written in the form

$$(1 - \theta \Delta^{(3)}) D_i = -D_{E_i} F, \quad (7)$$

$$(1 - \theta \Delta^{(3)}) H_i = -\theta(\omega_i^{(2)} + \Delta^{(2)} g_i) - \theta \int_0^\infty \Lambda^{(1)}(s) E_i^t(s) ds + D_{B_i} F, \quad (8)$$

$$\partial_{g_i} F - \theta \Delta^{(2)} D_i = 0 \quad (9)$$

where $F = \mathcal{F} + v_s D_s$. These expressions may be further simplified by calculating D and B as functionals of E^t, H^t, θ^t, g and then applying a Legendre transformation to the resulting equations. When the resulting equations are applied to materials without memory, one finds that equations (8.1) should be replaced by

$$D_i = -\partial_{\bar{E}_i} \bar{F}(\bar{E}_j, B_j, \theta), \quad (10)$$

$$(1 - \theta \Delta^{(3)}) H_i = -\theta(\omega_i^{(2)} + \Delta^{(1)} g_i) + \partial_{B_i} \bar{F}(\bar{E}_j, B_j, \theta) \quad (11)$$

where $F = F(E_i, B_i, \theta, g_i) = \bar{F}(\bar{E}_i, B_i, \theta)$ and

$$\bar{E}_i = (1 - \theta \Delta^{(3)}) E_i - \theta \Delta^{(2)} g_i - \theta \omega_i^{(3)}. \quad (12)$$

It follows from equations (10) and (11) that in a holohedral isotropic material

$$\begin{aligned}\bar{F} = & \bar{F}(I_1, I_2, I_3, \theta), \\ I_1 = & \bar{E}^2, \quad I_2 = B^2, \quad I_3 = (\bar{E} \cdot B)^2\end{aligned}\quad (13)$$

and

$$\begin{aligned}D = & -2 \frac{\partial \bar{F}}{\partial I_1} \bar{E} - 2 \frac{\partial \bar{F}}{\partial I_3} (\bar{E} \cdot B) B, \\ (1 - \theta \Delta^{(3)}) H = & 2 \frac{\partial \bar{F}}{\partial I_2} B + 2 \frac{\partial \bar{F}}{\partial I_3} (\bar{E} \cdot B) \bar{E},\end{aligned}\quad (14)$$

respectively, where $\bar{\mathbf{E}}$ is given by (12) with $\omega^{(2)}=0$. Consequently, equations (8.2) should be replaced by

$$\begin{aligned}(1-\theta\Delta^{(3)})\mathbf{D} = & -2\frac{\partial F}{\partial J_1}\mathbf{E} - 2\frac{\partial F}{\partial J_6}(\mathbf{E}\cdot\mathbf{B})\mathbf{B} \\ & + \frac{2\theta\Delta^{(2)}}{(1-\theta\Delta^{(3)})}\frac{\partial F}{\partial J_1}\mathbf{g} + \frac{2\theta\Delta^{(2)}}{(1-\theta\Delta^{(3)})}\frac{\partial F}{\partial J_6}(\mathbf{g}\cdot\mathbf{B})\mathbf{B}, \\ (1-\theta\Delta^{(3)})\mathbf{H} = & 2\frac{\partial F}{\partial I_2}\mathbf{B} + 2\frac{\partial F}{\partial I_6}(\mathbf{E}\cdot\mathbf{B})\mathbf{E} \\ & - \frac{2\theta\Delta^{(2)}}{(1-\theta\Delta^{(3)})}\frac{\partial F}{\partial J_6}(\mathbf{g}\cdot\mathbf{B})\mathbf{E} - \frac{2\theta\Delta^{(2)}}{(1-\theta\Delta^{(3)})}\frac{\partial F}{\partial J_6}(\mathbf{E}\cdot\mathbf{B})\mathbf{g} \\ & + \frac{2\theta^2\Delta^{(2)^2}}{(1-\theta\Delta^{(3)})^2}\frac{\partial F}{\partial J_6}(\mathbf{g}\cdot\mathbf{B})\mathbf{g}\end{aligned}\quad (15)$$

and $J_1=\mathbf{E}^2$, $J_2=\mathbf{B}^2$, $J_3=\mathbf{E}\cdot\mathbf{g}$, $J_4=\mathbf{g}^2$, $J_5=(\mathbf{B}\cdot\mathbf{g})^2$, $J_6=(\mathbf{E}\cdot\mathbf{B})^2$, $J_7=[(\mathbf{E}\cdot\mathbf{B})(\mathbf{B}\cdot\mathbf{g})]$.

In a hemihedral isotropic material without memory equations (8.4) should be replaced by

$$\begin{aligned}(1-\theta\Delta^{(3)})\mathbf{D} = & -2\frac{\partial F}{\partial J_1}\mathbf{E} - 2\frac{\partial F}{\partial J_8}\mathbf{B} + \frac{2\theta\Delta^{(2)}}{(1-\theta\Delta^{(3)})}\frac{\partial F}{\partial J_1}\mathbf{g}, \\ (1-\theta\Delta^{(3)})\mathbf{H} = & 2\frac{\partial F}{\partial J_2}\mathbf{B} + \frac{\partial F}{\partial J_8}\mathbf{E} - \frac{\theta\Delta^{(2)}}{(1-\theta\Delta^{(3)})}\frac{\partial F}{\partial J_8}\mathbf{g} - \theta\Delta^{(1)}\mathbf{g}\end{aligned}\quad (16)$$

where $F=F(J_1, J_2, J_3, J_4, J_8, J_9, \theta)$ and $J_8=\mathbf{E}\cdot\mathbf{B}$, $J_9=\mathbf{B}\cdot\mathbf{g}$.

It is now clear that, in general, the conclusions drawn from my original equations (8.3) and (8.4) are false. Since the cross product terms $\mathbf{E}\times\mathbf{B}$, $\mathbf{E}\times\mathbf{g}$ and $\mathbf{B}\times\mathbf{g}$ can be expressed as linear combinations of $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{g}$, provided $[\mathbf{E}, \mathbf{B}, \mathbf{g}] \neq 0$, there is no fundamental difference between my equations (15), (16) and the corresponding equations given by RIVLIN [1]. However, if the vectors $\mathbf{E}$, $\mathbf{B}$, $\mathbf{g}$ are coplanar, or if any one of them vanishes, it is no longer possible to reconcile his equations with mine.

References

1. RIVLIN, R. S., Some comments on a paper by M. F. McCarthy. Arch. Rational Mech. Anal. **46**, 61-65 (1972).

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