Corrigenda to “On the Flow Between a Rotating Disk and a Porous Disk”

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In my paper in volume 73 (hereafter referred to as [E]) an existence theorem was stated for the ordinary differential equation problem which results when von Karman's substitution is made in the Navier-Stokes equations for the flow between two (infinite) disks, in which one rotates with constant angular velocity, the other is at rest and there is constant blowing at the stationary disk. An essential ingredient in the proof was given by Lemma 2, which asserted that the solution f of

$$f'''' + gf''' + hh'' = 0,$$

$$f(0) = f'(0) = f'(d) = 0, \quad f(d) = \alpha,$$

where g is a given function in $C^1[0, d]$ satisfying the boundary conditions and h is a decreasing function in $C^1[0, d]$ such that $h(0) = \Omega$, $h(d) = 0$, is nondecreasing. Unfortunately, the proof given is not correct in the stated generality. The argument used works only if we know also that $f < \alpha$ in $[0, d)$.

A corrected version of the proof of this lemma and the corresponding modification of the main theorem of [E] are best understood if the problem is put in dimensionless form. After this is done, the problem considered becomes

$$F'''' + FF''' + HH' = 0,$$

$$H'' + FH' - F'H = 0,$$

$$H(0) = \Omega^*, \quad F(1) = \alpha^*,$$

$$H(1) = F(0) = F'(0) = F'(1) = 0,$$

where

$$\Omega^* = \frac{2\Omega d^2}{\nu}, \quad \alpha^* = \frac{2\alpha d}{\nu},$$

are the dimensionless angular velocity and blowing parameter, respectively.

Lemma 2. Suppose that

$$(1) \quad (\Omega^*)^2 \leq 2\alpha^* e^{-\alpha^*}.$$

Suppose also that G is an increasing positive function on $[0, 1]$ such that $G(1) = \alpha^*$, and that H is a decreasing function on $[0, 1]$ such that $H(0) = \Omega^*$ and $H(1) = 0$. Then for the solution F of

$$F'''' + GF''' + HH' = 0,$$

$$F(0) = F'(0) = F'(1) = 0, \quad F(1) = \alpha^*,$$

we have $F' \geq 0$.

Proof. The proof given in [E] need only be modified in the case in which there is an x_0 in $(0, 1)$ such that $F''' < 0$ on $[0, x_0]$ and $F''' > 0$ on $(x_0, 1]$. As before $F''(0) > 0$, and necessarily, $F''(x_0) < 0$ so that there must be (exactly) one inflection point to the left of x_0 .

We must show that $F(1) = \alpha^*$, $F'(d) = 0$ is impossible if there is only one inflection point in $(x_0, 1)$. This follows immediately if we know that $F < \alpha^*$ on $[0, x_0]$. To this end we will show that (a) $0 > F''' > -\frac{1}{2}(\Omega^*)^2 e^{\alpha^*}$ on $[0, x_0]$ and (b) $0 < F''' < \frac{1}{2}(\Omega^*)^2$ on $(x_0, 1]$. (We use (1) only in proving (a).) It then follows that $|F'| < \frac{1}{2}(\Omega^*)^2 e^{\alpha^*}$, since $F'(0) = F'(1) = 0$ and the Green's function for $y'' = 0$, $y(0) = y(1) = 0$ is not greater than 1 on $[0, 1]$. Then, since $F(0) = 0$, it follows that $|F| < \frac{1}{2}(\Omega^*)^2 e^{\alpha^*} < \alpha^*$.

Let $e(x) = \exp\left(\int_0^x G\right)$. Then, for $0 \leq x < x_0$,

$$\begin{aligned} e(x) F'''(x) &= \frac{1}{2} \int_x^{x_0} (H^2)' e \\ &= -\frac{1}{2} \int_x^{x_0} H^2 e' + \frac{1}{2} [H^2(x_0) e(x_0) - H^2(x) e(x)], \end{aligned}$$

and

$$\begin{aligned} 0 > F'''(x) &> -\frac{1}{2} H^2(x) e(x_0)/e(x) \\ &\geq -\frac{1}{2} (\Omega^*)^2 \exp\left(\int_x^{x_0} G\right). \end{aligned}$$

Since $0 < G < \alpha^*$, (a) follows, and (b) is shown in exactly the same way.

A nondimensional version of the main theorem of [E] is still true if we add (1) to the hypotheses. In the notation used there this requires that $\Omega^2 \leq 2\alpha d^{-3} e^{-2\alpha d}$. Several conclusions were drawn in [E] by taking the limit $d \rightarrow \infty$. These conclusions cannot be drawn from this reduced theorem.

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