The Borda Game: A correction

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In a recent paper, Peleg (forthcoming) points out an error in Proposition 2 of Gardner (1977). This note corrects the statement of this proposition.

Let $c(f)$ denote the proportion of all voters that a coalition must exceed in size if it is to be winning, given there are m alternatives, n voters and the social choice function is f . For the Borda social choice function, f_B , Proposition 2 of Gardner (1977) states:

$$c(f_B) = 2(m - 1)/(3m - 2), \text{ for } n \text{ large enough.} \tag{1}$$

Gardner (1977) already pointed out that for $m = 3$ and $n = 5$, formula (1) understates the correct answer. According to (1) in this case, a 3-player coalition is winning, when it actually takes four agents to win. To see this, consider voting on the alternatives x, y , and z . If three voters wish to elect x , they cannot do better than to adopt the strategy 3-tuple (xyz, xzy, xzy) . However, this only guarantees a tie between x and z when the counter coalition plays the strategy duple (zyx, zyx) . Indeed, even for large but finite n , the formula sometimes understates by one the required number of members in a winning coalition. For $m = 3$, the following table shows the cases that require this correction:

n	Least integer greater than $c(f_B) \cdot n$	Minimum number required to win
5	3	4
12	7	8
19	11	12
26	15	16
33	19	20

and so on.

* I would like to thank B. Peleg for bringing to my attention the need for this correction.

Peleg (forthcoming) shows, in general, that whenever there is an n for which $(3m - 2)$ is an even divisor of $2(m - 1)n + 1$, this underestimate of formula (1) occurs. Thus, when $m = 4$, the underestimate cannot occur; but it does reappear when $m = 5$ and $n = 8, 21, 34, \dots$

Formula (1) is thus strictly true only if there is a continuum of voters. This, then, is the appropriate sense of 'large enough' in the statement of the proposition.

REFERENCES

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