Chapter 8

Sensitivity and Uncertainty of Criticality

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Abstract Excess reactivity and control rod worth are generally considered important reactor physics parameters for experimentally examining the neutron characteristics of criticality in a core, and for maintaining safe operation of the reactor core in terms of neutron multiplication in the core. For excess reactivity and control rod worth at KUCA, as well as at the Fast Critical Assembly in the Japan Atomic Energy Agency, special attention is given to analyzing the uncertainty induced by nuclear data libraries based on experimental data of criticality in representative cores (EE1 and E3 cores). Also, the effect of decreasing uncertainty on the accuracy of criticality is discussed in this study. At KUCA, experimental results are accumulated by measurements of excess reactivity and control rod worth. To evaluate the accuracy of experiments for benchmarks, the uncertainty originated from modeling of the core configuration should be discussed in addition to uncertainty induced by nuclear data, since the uncertainty from modeling has a potential to cover the eigenvalue bias more than uncertainty by nuclear data. Here, to investigate the uncertainty of criticality depending on the neutron spectrum of cores, it is very useful to analyze the reactivity of a large number of measurements in typical hard (EE1) and soft (E3) spectrum cores at KUCA.

Keywords Sensitivity · Uncertainty · Criticality · Solid-moderated and solid-reflected core

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8.1 Experimental Settings

8.1.1 Core Configuration

The experiments of reactivity measurement [1, 2] were carried out in the A cores (EE1 and E3 cores in Figs. 8.1a, b, respectively) that have polyethylene moderator and reflector rods, and different fuel assemblies: “F” and “f” (Figs. 8.2a, b, respectively). In EE1 core (Fig. 8.1a), fuel assembly “F” (1/8”P60EUEU) in Fig. 8.2a is composed of 60 unit cells, and upper and lower polyethylene blocks about 25” and 20” long, respectively, in an aluminum (Al) sheath (2.1” × 2.1” × 60”). For the fuel assembly, a unit cell in the fuel region is composed of two highly enriched uranium (HEU) fuel plates 1/8” (2 × 1/16”) thick and a polyethylene moderator plate 1/8” thick. In E3 core (Fig. 8.1b), another fuel assembly “f” (3/8”P36EU) in Fig. 8.2b is composed of 36 unit cells with an HEU fuel plate 1/16” thick, polyethylene plates 3/8” thick, and upper and lower polyethylene blocks about 23” and 21” long, respectively, in the Al sheath as in fuel assembly “F”. The neutron spectrum in EE1 core is compared with that in E3 core as shown in Fig. 8.3, demonstrating representatively hard (EE1 core) and soft (E3 core) neutron spectra in the KUCA A-core.

8.1.2 Reactivity Measurements

For measuring the excess reactivity, the critical state was adjusted by maintaining a certain position of C3 rod in EE1 core and C1 rod in E3 core, and by withdrawing fully the other control (C1 and C2 in EE1; C2 and C3 in E3) and safety (S4, S5, and S6) rods from the core, respectively. Furthermore, excess reactivities in EE1 and E3 cores were measured by the positive period method, when C3 and C1 rods, respectively, were, under the critical state, fully withdrawn from the core.

For measuring the control rod worth, in cases of C1 and C2 rods in EE1 core, criticality was maintained by C3 rod, and the control rod worth of C1 or C2 rod was acquired by the rod drop method, after full insertion of C1 or C2 rod into the core. For C3 rod in EE1 core, criticality was adjusted by C1 rod, and the control rod worth of C3 rod was obtained by the rod drop method, after the full insertion of C3 rod into the core. Almost the same procedures were followed for the E3 core, with the use of the rod drop method, and the control rod worth of C1, C2, and C3 rods was obtained experimentally.

To estimate the experimental uncertainty of excess reactivity and control rod worth, the dimensions of the HEU plate comprising of core components are considered manufacturing tolerances among the significant factors taken into account, as shown in Table 8.1. In addition to the dimensions of the HEU plate, as previously demonstrated at the Fast Critical Assembly in the Japan Atomic Energy Agency [3], other important uncertainty factors include mechanical reproducibility of control rod position, measurement errors of doubling time by the positive period method, core

Fig. 8.1 Top view of KUCA A-core (Refs. [1, 2])

(a) Fuel "F" shown in Fig. 8.1(a)

(b) Fuel "f" shown in Fig. 8.1(b)

Fig. 8.2 Description of fuel rods in A-core shown in Fig. 8.1 (Refs. [1, 2])

Fig. 8.3 Neutron spectra of EE1 and E3 cores shown in Figs. 8.1a, b, respectively (Refs. [1, 2])

Table 8.1 Experimental uncertainties of HEU fuel plate (Refs. [1, 2])

Cause of uncertainty	Error (%)
Composition	3.0
Length of both sides (one side: 0.2%)	0.3
Thickness	3.1

Table 8.2 Measured results [pcm] of excess reactivity and control rod worth with standard deviation obtained from experimental data in EE1 and E3 cores (Refs. [1, 2])

Reactivity	EE1 (error: %)	E3 (error: %)
Excess	210 ± 6 (3.1)	264 ± 2 (0.8)
C1 rod	838 ± 11 (1.4)	551 ± 6 (1.2)
C2 rod	144 ± 3 (2.4)	429 ± 4 (1.0)
C3 rod	521 ± 8 (1.7)	329 ± 4 (1.4)

EE1: β_{eff} : 831 pcm and Λ : 3.027E-05 s by MCNP6.1 with JENDL-4.0
E3: β_{eff} : 805 pcm and Λ : 4.771E-05 s by MCNP6.1 with JENDL-4.0

temperature, delayed neutron parameters induced by numerical analyses, fuel composition uncertainty caused by heterogeneity distribution and deformation inside the Al sheath. Among these factors, the mechanical reproducibility of control rod position, in this study, was considered of great impact caused by tolerance and instability of control rod inside the Al sheath in a horizontal direction than the experimental uncertainty of excess reactivity and control rod worth, because the other uncertainties are relatively minor. Furthermore, heterogeneous arrangement of fuel plates in an axial direction was attributable to random selection of fuel plates for making fuel cells with a record number of fuel plates. Therefore, the experimental uncertainty of excess reactivity and control rod worth was finally determined to be about 3% at most by averaging the experimental data, and estimating its standard deviation, as observed in operations of the previous decade in the KUCA A-core, as shown in Table 8.2.

8.2 Criticality

8.2.1 Numerical Simulations

8.2.1.1 Stochastic Calculations

Excess reactivity was numerically deduced by the MCNP6.1 code [4] with the JENDL-4.0 [5] and the ENDF/B-VII.0 [6] libraries through the difference between the critical and super-critical states in the core; control rod worth was numerically obtained by the difference between critical and subcritical states. For the evaluation

of excess reactivity and control rod worth, in the critical core, the effective delayed neutron fraction (β_{eff}) was acquired by MCNP6.1 (2,000 active cycles of 50,000 histories; 5 pcm statistical error) with JENDL-4.0. The values of β_{eff} at critical state of EE1 and E3 cores were 831 and 805 pcm, respectively; those of the neutron generation time (Λ) were 3.027E-05 and 4.771E-05 s, respectively. Atomic number densities of core components comprise fuel elements, moderator (reflector), control rod, and Al sheath, for analyzing experimental values of excess reactivity and control rod worth.

8.2.1.2 Deterministic Calculations

Numerical analyses by deterministic calculations were performed by combining the SRAC2006 [7] and the MARBLE [8] code systems: collision probability calculations (PIJ [7]) and eigenvalue calculations (CITATION [9]) of SRAC2006, sensitivity coefficient calculations (SAGEP [10]) and uncertainty calculations (UNCERTAINTY [8, 11]) of MARBLE, coupled with the JENDL-4.0 nuclear data library. Experimental analyses of uncertainty were conducted with the use of covariance data of cross sections contained in JENDL-4.0, including uncertainty of excess reactivity and control rod worth induced by the nuclear data, and the effects of decreasing uncertainty on the accuracy of excess reactivity and control rod worth. Here, in a series of deterministic calculations, the CITATION code was notably executed for obtaining sensitivity coefficients by the SAGEP code based on the diffusion calculations.

8.2.2 Sensitivity and Uncertainty

8.2.2.1 Numerical Reactivity

The experimental values of excess reactivity (positive value) $\Delta\rho_{\text{Excess}}^{\text{Exp}}$ and control rod worth (negative value) ($-\Delta\rho_{\text{Rod}}^{\text{Exp}}$) were deduced by effective multiplication factors $k_{\text{Clean}}^{\text{Exp}}$ and $k_{\text{Rod}}^{\text{Exp}}$ in super-critical (clean core) and subcritical states (rod insertion core) obtained by the positive period method and the rod drop method, respectively, as follows:

$$\Delta\rho_{\text{Excess}}^{\text{Exp}} = 1 - \frac{1}{k_{\text{Clean}}^{\text{Exp}}}, \quad (8.1)$$

$$-\Delta\rho_{\text{Rod}}^{\text{Exp}} = 1 - \frac{1}{k_{\text{Rod}}^{\text{Exp}}}. \quad (8.2)$$

In MCNP analyses, numerical value $\Delta\rho_{\text{Excess}}^{\text{MCNP}}$ or ($-\Delta\rho_{\text{Rod}}^{\text{MCNP}}$) was deduced by the difference between two effective multiplication factors $k_{\text{Clean}}^{\text{MCNP}}$ or $k_{\text{Rod}}^{\text{MCNP}}$ and $k_{\text{Critical}}^{\text{MCNP}}$ in the super-critical or subcritical and critical cores, respectively, as follows:

$$\Delta\rho_{\text{Excess}}^{\text{MCNP}} = \frac{1}{k_{\text{Critical}}^{\text{MCNP}}} - \frac{1}{k_{\text{Clean}}^{\text{MCNP}}}, \quad (8.3)$$

$$-\Delta\rho_{\text{Rod}}^{\text{MCNP}} = \frac{1}{k_{\text{Critical}}^{\text{MCNP}}} - \frac{1}{k_{\text{Rod}}^{\text{MCNP}}}. \quad (8.4)$$

In MCNP calculations, $k_{\text{Critical}}^{\text{MCNP}}$ needs to be defined as the value of the effective multiplication factor in the critical core, since the numerical value is not always a unit.

On the basis of the experimental methodology shown in Eqs. (8.1) and (8.2), a numerical approach of excess reactivity $\Delta\rho_{\text{Excess}}^{\text{CITATION}}$ and control rod worth ($-\Delta\rho_{\text{Rod}}^{\text{CITATION}}$) by deterministic calculations (CITATION) is generally expressed, respectively, as follows, as were experimental values:

$$\Delta\rho_{\text{Excess}}^{\text{CITATION}} = 1 - \frac{1}{k_{\text{Clean}}^{\text{CITATION}}}, \quad (8.5)$$

$$-\Delta\rho_{\text{Rod}}^{\text{CITATION}} = 1 - \frac{1}{k_{\text{Rod}}^{\text{CITATION}}}, \quad (8.6)$$

where $k_{\text{Clean}}^{\text{CITATION}}$ and $k_{\text{Rod}}^{\text{CITATION}}$ indicate the effective multiplication factors in the super-critical and subcritical cores, respectively.

Of the two numerical values by CITATION and MCNP, as mentioned in Sect. 8.2.2.1, the CITATION calculations needed to conduct a series of sensitivity and uncertainty analyses by SAGEP and UNCERTAINTY, respectively. Meanwhile, the MCNP calculations were requisite to assess the precision of eigenvalue calculations, such as eigenvalue bias [12]. That is why two numerical values were introduced by the stochastic (Eqs. (8.3) and (8.4)) and the deterministic (Eqs. (8.5) and (8.6)) approaches, and compared differently with the experimental values shown in Eqs. (8.1) and (8.2).

8.2.2.2 Sensitivity Coefficient

Sensitivity coefficient S of the integral reactor physics parameter (effective multiplication factor) R is defined by the ratio of the rate of change in R and a certain parameter x as follows:

$$S = \frac{dR}{R} \bigg/ \frac{dx}{x}. \quad (8.7)$$

The effective multiplication factor k_{eff} is expressed by a balance equation of neutrons as follows:

$$\mathbf{A}\phi = \frac{1}{k_{\text{eff}}}\mathbf{F}\phi, \quad (8.8)$$

where **A** and **F** indicate operators of transport and fission terms, respectively, and ϕ the forward neutron flux. Multiplying Eq. (8.8) by adjoint neutron flux ϕ^* and integrating over whole volume and energy, the following equation is obtained:

$$\frac{1}{k_{\text{eff}}} = \frac{\langle \phi^* \mathbf{A} \phi \rangle}{\langle \phi^* \mathbf{F} \phi \rangle}, \quad (8.9)$$

where brackets $\langle \rangle$ indicate integration over the whole volume and energy.

With the use of an operator **B**, Eq. (8.8) is expressed as follows:

$$\left(\mathbf{A} - \frac{1}{k_{\text{eff}}} \mathbf{F} \right) \phi = \mathbf{B} \phi = 0. \quad (8.10)$$

Here, assuming that parameter x , operator **B**, and neutron flux ϕ are changed into $x + \delta x$, **B** + $\delta \mathbf{B}$, and $\phi + \delta \phi$, respectively, in a critical state, the following equations are obtained:

$$(\mathbf{B} + \delta \mathbf{B})(\phi + \delta \phi) = 0. \quad (8.11)$$

Neglecting second-order perturbation terms, Eq. (8.10) is expressed as follows:

$$\mathbf{B} \delta \phi + \delta \mathbf{B} \phi = 0. \quad (8.12)$$

Introducing the generalized adjoint flux Γ^* , the following equation is obtained with the use of adjoint operator $\mathbf{B}^*$ and a certain adjoint source term q^* , defined as reactivity in these analyses:

$$\mathbf{B}^* \Gamma^* = q^*. \quad (8.13)$$

Considering the theoretical background [13–16] and using Eqs. (8.10) through (8.13), the sensitivity coefficient in Eq. (8.7) is finally expressed by applying the first-order perturbation approximation [17], as follows:

$$S = \frac{\langle \phi^* \left(-\frac{\partial \mathbf{B}}{\partial x} \right) \phi \rangle}{\langle \phi^* \mathbf{F} \phi \rangle} k_{\text{eff}}. \quad (8.14)$$

Finally, applicability of sensitivity analyses to k_{eff} was investigated for a thermal spectrum core, such as the KUCA core, with the use of SAGEP that had been originally developed for conducting the sensitivity analyses of fast reactors.

8.2.2.3 Uncertainty

In analyzing the cross-section uncertainty of nuclear data [18], the uncertainty of reactor physics parameter ν is expressed as follows:

$$v = \mathbf{G}_{tar} \mathbf{M} (\mathbf{G}_{tar})^t = \sum_i \sum_j s_i c_{i,j} s_j \equiv \sum_i \sum_j v_{i,j} \quad (1 \leq i, j \leq p) \quad , \quad (8.15)$$

where $\mathbf{G}_{tar}$ ($1 \times p$) indicates the sensitivity vector of reactor physics parameters, $\mathbf{M}$ ($p \times p$) the covariance matrix of nuclear reaction parameters, s_i the sensitivity coefficient, $c_{i,j}$ the covariance, $v_{i,j}$ the factor of uncertainty and p the number of nuclear reactions including the nuclides. Thus, the contribution of uncertainty u_i in each nuclear reaction can be defined as follows:

$$u_i \equiv \sum_j v_{i,j}. \quad (8.16)$$

Generally, since sensitivity coefficient s_i and covariance $c_{i,j}$ are dominant in the energy group, the factor of uncertainty is finally expressed with the use of the maximum number of energy group G as follows:

$$v_{i,j} = \sum_g \sum_{g'} s_g^i c_{g,g'}^{i,j} s_{g'}^j \quad (1 \leq g, g' \leq G), \quad (8.17)$$

where g and g' indicate the energy groups.

8.2.3 Results and Discussion

8.2.3.1 Eigenvalue Calculations

In MCNP simulations, excess reactivity and control rod worth were obtained by two eigenvalue calculations in critical and super-critical, and, critical and subcritical states, respectively: the difference between the inverse values of eigenvalue calculations in the two states. Here, MCNP eigenvalue calculations were made with 2,000 active cycles of 50,000 histories, resulting in a standard deviation within 10 pcm. The numerical results of excess reactivity and control rod worth were obtained by MCNP6.1 with JENDL-4.0 in EE1 and E3 cores, as shown in Table 8.3a, b, respectively. Moreover, estimation of C/E (calculation/experiment) values revealed an accuracy of around 5% error with the use of the experimental and numerical results of excess reactivity and control rod worth, as shown in Eqs. (8.1) and (8.2), respectively, excluding small values of excess reactivity and C2 control rod worth in EE1 core. From the calculated results in Table 8.3a, b, the difference between numerical analyses by MCNP6.1 with JENDL-4.0 and ENDF/B-VII.0 was found within an error of 3% of the C/E value.

The ability of MCNP6.1 calculations was confirmed at a critical state in terms of the eigenvalue bias by the MCNP approach, as shown in Table 8.4. In EE1 core, eigenvalue bias by MCNP6.1 with JENDL-4.0 demonstrated a relatively small value

Table 8.3 Comparison between measured and calculated (MCNP6.1) results of excess reactivity and control rod worth in EE1 and E3 cores (Ref. [1])

(a) EE1			
Reactivity	Exp. [pcm]	Cal. [pcm] (C/E value)	
		JENDL-4.0	ENDF/B-VII.0
Excess	210 ± 6	198 ± 18 (0.94 ± 0.09)	196 ± 18 (0.93 ± 0.09)
C1 rod	838 ± 11	857 ± 18 (1.02 ± 0.02)	843 ± 18 (1.01 ± 0.02)
C2 rod	144 ± 3	134 ± 18 (0.93 ± 0.13)	135 ± 18 (0.94 ± 0.13)
C3 rod	521 ± 8	500 ± 18 (0.96 ± 0.04)	491 ± 18 (0.94 ± 0.04)
(b) E3			
Reactivity	Exp. [pcm]	Cal. [pcm] (C/E value)	
		JENDL-4.0	ENDF/B-VII.0
Excess	264 ± 2	246 ± 16 (0.93 ± 0.06)	241 ± 16 (0.91 ± 0.06)
C1 rod	551 ± 6	526 ± 16 (0.95 ± 0.03)	536 ± 16 (0.97 ± 0.03)
C2 rod	429 ± 4	400 ± 16 (0.93 ± 0.04)	388 ± 16 (0.90 ± 0.04)
C3 rod	329 ± 4	314 ± 16 (0.95 ± 0.05)	315 ± 16 (0.96 ± 0.05)

Table 8.4 Numerical results by MCNP6.1 eigenvalue calculations in critical state (Ref. [1])

EE1		E3	
JENDL-4.0	ENDF/B-VII.0	JENDL-4.0	ENDF/B-VII.0
1.00246 ± 0.00009	1.00461 ± 0.00009	1.00443 ± 0.00008	1.00578 ± 0.00008

of about 250 pcm, and with ENDF/B-VII.0 a value about 460 pcm (Table 8.4). Similarly, in E3 core, the value with JENDL-4.0 was small, about 440 pcm, compared with that with ENDF/B-VII.0, about 580 pcm. From the analyses of eigenvalue bias, a significant index of the accuracy of experimental analyses was acquired by MCNP calculations in EE1 and E3 cores at KUCA; also, as in previous studies [12, 19], JENDL-4.0 was considered reliable as a reference nuclear data library in a series of sensitivity and uncertainty analyses of excess reactivity and control rod worth in the KUCA A core.

The numerical values of k_{eff} in excess reactivity and control rod worth shown in Eqs. (8.5) and (8.6), respectively, were deduced, for a clean core (withdrawal of all control and safety rods) and a subcritical core (control rod insertion) in super-critical and subcritical states, respectively, through the results of diffusion-based eigenvalue calculations (CITATION) in the 107-energy-group and x - y - z dimensions (3-D) with JENDL-4.0. Also, on the basis of accuracy of experimental analyses by CITATION (Table 8.5), sensitivity analyses of k_{eff} in excess reactivity and control rod worth were notably conducted by diffusion-based calculations (SAGEP).

Table 8.5 Comparison between measured and calculated (CITATION with JENDL-4.0) results of k_{eff} (excess reactivity: super-critical state; control rod worth: subcritical state) in EE1 and E3 cores (Ref. [1])

Reactivity	EE1		E3	
	Experiment	Calculation	Experiment	Calculation
Excess	1.00210	1.00030	1.00265	1.00067
C1 rod	0.99169	0.99525	0.99452	0.99513
C2 rod	0.99856	0.99903	0.99573	0.99673
C3 rod	0.99482	0.99447	0.99672	0.99762

8.2.3.2 Sensitivity Coefficients

Sensitivity coefficients Eq. (8.7) of k_{eff} in excess reactivity and control rod worth were analyzed by the SAGEP code for assessing cross-section data of inelastic scattering, elastic scattering and capture reactions of ^{27}Al , boron isotopes ($^{10}, ^{11}\text{B}$), carbon (^{12}C), hydrogen (^1H), oxygen (^{16}O), and uranium isotopes ($^{234}, ^{235}, ^{236}, ^{238}\text{U}$) comprising the core components.

For excess reactivities in EE1 and E3 cores, the sensitivity coefficients of elastic scattering reactions were relatively highly positive, mostly 1 MeV, in ^{27}Al , ^{12}C , and ^1H , as shown in Fig. 8.4a, and b, respectively. Sensitivity coefficients were dominant over the high-energy (MeV) region of the inelastic scattering reactions of ^{27}Al in k_{eff} (excess reactivities) at EE1 and E3 cores shown in Fig. 8.5a, b, respectively. In thermal neutron region shown in Fig. 8.6a, b, the capture cross sections of ^{27}Al , ^1H , and ^{235}U were highly sensitive at EE1 and E3 cores, respectively. Also, the sensitivity coefficients of ^{27}Al , ^1H , and ^{235}U were remarkably higher in E3 core than in EE1 core ranging between 0.01 and 100 eV shown in Fig. 8.6b, because E3 core is a relatively soft-spectrum core shown in Fig. 8.3. In a series of sensitivity analyses shown in Figs. 8.4 through 8.6, effects of Al on sensitivities were observed in entire reactions and energy regions, and attributable to containing Al itself comprising of U-Al alloy (HEU) fuel plates and Al sheath of fuel assembly.

Further study of the sensitivity coefficients was made of k_{eff} (worth of C1 control rod) at EE1 and E3 cores, since the worth of C1 control rod was mostly larger in both EE1 and E3 cores, compared with other reactivities shown in Table 8.3. Also, the worth of C1 control rod was selected to investigate directly the effect of the boron isotope component of the control rod. As shown in Fig. 8.7a, b, the sensitivity coefficients of ^{27}Al , ^{10}B , ^1H , and ^{235}U were negative in the capture reactions; among these, the capture cross sections of ^{27}Al and ^{235}U were highly sensitive in the thermal neutron region, as well as for capture reactions in the excess reactivity shown in Fig. 8.6a, b. Moreover, the sensitivity coefficient of ^{27}Al was remarkably large in the E3 core with a thermal neutron spectrum, as shown in Fig. 8.7b, as was that of ^{235}U . Finally, in C1 control rod worth, sensitivity coefficient of ^{10}B was found relatively of negligible significance due to an insertion of C1 control rod, although that of ^{27}Al comprising of core components (U-Al alloy fuel plates and Al sheath) was large.

Fig. 8.4 Sensitivity coefficients for elastic scattering reactions in excess reactivity (Ref. [1])

8.2.3.3 Uncertainty

The uncertainty analyses by the UNCERTAINTY code of the MARBLE system were conducted with the use of JENDL-4.0 covariance data (107-energy-group). Since the covariance data of ^{27}Al , ^{12}C , and ^1H isotopes consisted mainly of core components that are not contained in JENDL-4.0, uncertainty analyses were executed for various reactions of $^{10,11}\text{B}$, ^{16}O , and $^{235,238}\text{U}$ isotopes comprising of control and fuel rods in the KUCA A-core, including capture, elastic scattering, inelastic scattering, fission, and $(n, 2n)$ reactions covered in the SAGEP code. As shown in Table 8.6, the uncertainty of excess reactivity in EE1 core induced by covariance data was large, total uncertainty 135.8 pcm, compared with an experimental error of 6 pcm (Table 8.2). The value of total uncertainty was acquired through the square root of the sum of squares for reaction-wise contributions. Among the isotopes, uncertainty was

Fig. 8.5 Sensitivity coefficients for inelastic scattering reactions in excess reactivity (Ref. [1])

dominant over the sum of all contributions, including the capture and fission reactions of ^{235}U ; a large contribution [19] was attributable to the sensitivity coefficients of ^{235}U capture and fission reactions. For the worth of C1 control rod in E3 core, the total uncertainty was 164.1 pcm, although the reaction-wise contribution was slight in the boron isotopes, as shown in Table 8.7, demonstrating the same tendency of excess reactivity as in EE1 core shown in Table 8.6. Finally, the uncertainty of k_{eff} in excess reactivity and control rod worth (C1, C2 and C3 rods) was summarized around 150 pcm in the EE1 and E3 cores.

Furthermore, the effect of decreasing uncertainty induced by nuclear data on calculated k_{eff} was investigated by the cross-section adjustment method [20, 21], with the use of uncertainty of k_{eff} values. The result of this investigation demonstrated a great improvement from around 150 to 3 pcm induced by nuclear data of JENDL-4.0 in all excess reactivity and control rod worth in EE1 and E3 cores. Moreover, by

Fig. 8.6 Sensitivity coefficients for capture reactions in excess reactivity (Ref. [1])

applying the cross-section adjustment method to the uncertainty analyses, the C/E value of all excess reactivity and control rod worth in both EE1 and E3 cores reached around a unit.

Fig. 8.7 Sensitivity coefficients for capture reactions in C1 rod worth (Ref. [1])

Table 8.6 Reaction-wise contribution [pcm] to excess reactivity induced by covariance data of JENDL-4.0 in EE1 core (Ref. [1])

Isotopes	Reactions					
	Capture	Elastic	Inelastic	Fission	(n, 2n)	Total
¹⁰ B	0.0	0.0	–	–	–	0.0
¹¹ B	0.0	0.0	–	–	–	0.0
¹⁶ O	0.0	0.0	0.0	–	0.0	0.0
²³⁵ U	111.7	5.3	12.9	44.7	0.7	121.1
²³⁸ U	2.8	0.2	0.9	0.1	0.1	2.9
					Total	135.8

Table 8.7 Reaction-wise contribution [pcm] to the worth of C1 control rod induced by covariance data of JENDL-4.0 in E3 core (Ref. [1])

Isotopes	Reactions					
	Capture	Elastic	Inelastic	Fission	(<i>n</i> , 2 <i>n</i>)	Total
¹⁰ B	9.0	0.0	–	–	–	9.0
¹¹ B	0.0	4.4	–	–	–	4.4
¹⁶ O	0.0	1.2	0.0	–	0.0	1.2
²³⁵ U	113.0	1.1	2.8	81.8	0.2	139.5
²³⁸ U	0.7	0.1	0.2	0.0	0.0	0.8
					Total	164.1

8.3 Benchmarks

8.3.1 Experimental Analyses

8.3.1.1 Reactivity Measurements

Before quantifying the uncertainty in criticality, the validity of the standard modeling of the core configuration was verified by the comparison of the excess reactivity and the control rod worth between the calculation and the experiment, without consideration of variation in the position and material property. The calculation of the reactivity was performed through two eigenvalue calculations with MCNP6.1 and KENO-VI module of SCALE6.2 code system [22] together with ENDF/B-VII.1. The calculated reactivity was obtained as follows:

$$\rho_{\text{excess}}^{\text{cal}} = \frac{1}{k_{\text{eff}}^{\text{critical}}} - \frac{1}{k_{\text{eff}}^{\text{clean}}}, \quad (8.18)$$

$$\rho_{\text{rod}}^{\text{cal}} = \frac{1}{k_{\text{eff}}^{\text{rod}}} - \frac{1}{k_{\text{eff}}^{\text{critical}}}, \quad (8.19)$$

where $\rho_{\text{excess}}^{\text{cal}}$ and $\rho_{\text{rod}}^{\text{cal}}$ are the calculated excess reactivity and control rod worth, respectively, $k_{\text{eff}}^{\text{critical}}$ the k_{eff} value at the critical state, $k_{\text{eff}}^{\text{clean}}$ the k_{eff} value at the withdrawal of all control rods and $k_{\text{eff}}^{\text{rod}}$ the k_{eff} value at the insertion of control rod C1, C2, or C3 in the critical state.

8.3.1.2 Numerical Simulations

Through the development of the methodology of the adjoint flux [23, 24], sensitivity and uncertainty analyses were easily conducted with the use of the Monte Carlo method applying to rigorous modeling without homogenization [25]. Sensitivity and

uncertainty analyses of k_{eff} induced by nuclear data were performed by TSUNAMI-3D module of SCALE6.2 together with ENDF/B-VII.1 and its 56-group covariance library (56groupcov7.1) for standard modeling of EE1 and E3 core configurations. Here, the adjoint flux was obtained by the CLUTCH method [26].

Variation in the manufacturing tolerance of HEU plates is provided by statistically processing the measured results (enrichment and thickness) of all plates at KUCA and with specification sheet in the fabrication for length of each side, as shown in Table 8.1. On the basis of the variation in HEU plates, the uncertainty analyses for the enrichment and tolerance of length of both sides were conducted with KPERT option in MCNP6.1 together with ENDF/B-VII.1. Since the variation is too small to explicitly calculate the impact (difference in k_{eff} values), a pseudo variation was considered 10%, instead of the actual variation shown in Table 8.1, in the perturbation for the enrichment and length of sides of the HEU plates. Here, while varying the length of the sides of HEU plates, their mass was maintained by retaining the original number of atoms. The pseudo uncertainty of k_{eff} attributed to varying the enrichment and length was converted to the actual one (Table 8.1) on the basis of the standard method provided in the guideline by OECD/NEA [27] as follows:

$$\Delta k_{\text{eff}} = \sqrt{\sum_{i=1}^N \frac{1}{n_i} \left(\frac{\Delta k_{\text{eff},i}}{\Delta q_i} \cdot \frac{\Delta q_i}{\Delta x_i} \right)^2}, \quad (8.20)$$

where, Δk_{eff} is the difference between k_{eff} values before and after variation Δq_i (the intensity of the pseudo variation), Δx_i the real uncertainty shown in Table 8.1, n_i the number of HEU plates involved in the variation, finally, i and N the region of perturbation and the total number of regions, respectively. In the calculation for the variation in the enrichment and the length of HEU plates, the core was divided into fuel rod regions in the x - y plane (25 divisions in EE1 core; 21 divisions in E3 core) and into three regions in z axis ($N = 75$ and 63 in EE1 and E3 cores, respectively). The uncertainty of k_{eff} by varying the thickness of HEU plates was evaluated by setting all HEU plates 3.1% decreasing in thickness according to Ref. [14].

The uncertainty attributed to the reproducibility of control rod position was approximately evaluated by eigenvalue calculations (Eq. (8.20)) with MCNP6.1, and by varying control rod position (x - y directions) in the casing for 5.15 mm radially (Δq_i in Eq. (8.20)) and by 8 segments circumferentially. Then, the actual uncertainty of control rod position in the x - y plane is limited to a radius of 1.15 mm (Δx_i in Eq. (8.20)). In the case of one (two) control rod(s) inserted, N in Eq. (8.20) becomes 8 (64). The uncertainty was finally deduced by subtracting the k_{eff} value obtained at the reference position from that obtained by varying the position.

8.3.2 Uncertainty

8.3.2.1 Eigenvalue Bias

The eigenvalue calculations with the use of MCNP6.1 and SCALE code system were performed for a total of $1\text{E} + 08$ histories ($1\text{E} + 03$ active cycles of $1\text{E} + 05$ each); the statistical error was less than 10 pcm. Through the comparison of the results of k_{eff} between MCNP6.1 and SCALE6.2/KENO-VI, the difference was found about 100 pcm in EE1 core and about 200 pcm in E3 core, as shown in Tables 8.8 and 8.9, respectively. Moreover, eigenvalue bias was dependent on the neutron spectrum of the core: about 350 pcm and about 600 pcm in EE1 and E3 cores, respectively. Interestingly, the uncertainty induced by nuclear data showed about 950 pcm in the k_{eff} evaluation by the SCALE code. Also, the uncertainty was found almost same value regardless of the control rod position, indicating that the neutronic characteristics are the same even at insertion and withdrawal of control rods.

To validate the modeling of core configuration and eigenvalue calculations, the calculated results of excess reactivity and control rod worth compared with the measured ones in EE1 and E3 cores, as shown in Tables 8.10 and 8.11, respectively. Although eigenvalue bias was smaller than the uncertainty induced by nuclear

Table 8.8 Comparison of k_{eff} values between experiments and calculations (MCNP6.1 and SCALE6.2) in EE1 core (Ref. [2])

Case	Core	Inserted control rod (position)	Effective multiplication factor k_{eff}		
			Experiment	MCNP6.1	SCALE6.2
I-1	EE1	–	1.00202 ± 0.00006	1.00534 ± 0.00009	1.00574 ± 0.00009 (947.7 ± 0.3 pcm)*
I-2	EE1	C1 (critical position)	1.00000	1.00348 ± 0.00009	1.00395 ± 0.00010 (948.3 ± 0.3 pcm)
I-3	EE1	C3 (critical position)	1.00000	1.00374 ± 0.00009	1.00383 ± 0.00010 (949.0 ± 0.3 pcm)
I-4	EE1	C3 (critical position) C1 (full inserted)	0.99199 ± 0.00001	0.99651 ± 0.00009	0.99533 ± 0.00010 (956.5 ± 0.3 pcm)
I-5	EE1	C3 (critical position) C2 (full inserted)	0.99861 ± 0.00002	1.00359 ± 0.00009	1.00242 ± 0.00010 (949.7 ± 0.3 pcm)
I-6	EE1	C1 (critical position) C3 (full inserted)	0.99501 ± 0.00008	1.00002 ± 0.00009	0.99899 ± 0.00010 (953.7 ± 0.3 pcm)

*Uncertainty by SCALEW6.2/TSUNAMI-3D

Table 8.9 Comparison of k_{eff} values between experiments and calculations (MCNP6.1 and SCALE6.2) in E3 core (Ref. [2])

Case	Core	Inserted control rod (position)	Effective multiplication factor k_{eff}		
			Experiment	MCNP6.1	SCALE6.2
II-1	E3	–	1.00253 ± 0.00002	1.00823 ± 0.00008	1.01045 ± 0.00009 (911.7 ± 0.3 pcm)*
II-2	E3	C1 (critical position)	1.00000	1.00578 ± 0.00008	1.00811 ± 0.00009 (912.6 ± 0.3 pcm)
II-3	E3	C2 (critical position)	1.00000	1.00614 ± 0.00008	1.00820 ± 0.00009 (913.4 ± 0.3 pcm)
II-4	E3	C2 (critical position) C1 (full inserted)	0.99477 ± 0.00006	1.00275 ± 0.00008	1.00312 ± 0.00009 (918.6 ± 0.3 pcm)
II-5	E3	C1 (critical position) C2 (full inserted)	0.99592 ± 0.00004	1.00212 ± 0.00008	1.00396 ± 0.00009 (918.0 ± 0.3 pcm)
II-6	E3	C1 (critical position) C3 (full inserted)	0.99687 ± 0.00004	1.00074 ± 0.00008	1.00495 ± 0.00010 (916.3 ± 0.3 pcm)

*Uncertainty by SCALEW6.2/TSUNAMI-3D

Table 8.10 Comparison between measured and calculated reactivity at critical experiments in EE1 core (Ref. [2])

Reactivity	Experiment [pcm]	MCNP6.1 [pcm]	SCALE6.2/KENO-VI [pcm]
Excess	202 ± 6	184 ± 13 (0.91 ± 0.07)	189 ± 19 (0.93 ± 0.10)
C1 rod	807 ± 11	876 ± 13 (1.09 ± 0.02)	862 ± 14 (1.07 ± 0.02)
C2 rod	139 ± 3	151 ± 13 (1.09 ± 0.09)	152 ± 14 (1.09 ± 0.10)
C3 rod	502 ± 8	525 ± 13 (1.05 ± 0.03)	494 ± 14 (0.98 ± 0.03)

(): C/E (calculation/experiment) value

data, the comparison revealed the relative difference of 10% in C/E (calculation/experiment) values, indicating that the calculations of EE1 and E3 cores were valid and pertinently modeled for experimental analyses.

Table 8.11 Comparison between measured and calculated reactivity at critical experiments in E3 core (Ref. [2])

Reactivity	Experiment[pcm]	MCNP6.1 [pcm]	SCALE-6.2/KENO-VI [pcm]
Excess	252 ± 2	242 ± 11 (0.96 ± 0.04)	229 ± 13 (0.91 ± 0.05)
C1 rod	526 ± 6	536 ± 11 (1.02 ± 0.02)	502 ± 13 (0.95 ± 0.03)
C2 rod	410 ± 4	377 ± 11 (0.92 ± 0.03)	410 ± 13 (1.00 ± 0.03)
C3 rod	314 ± 4	315 ± 11 (1.04 ± 0.04)	312 ± 13 (0.99 ± 0.04)

(): C/E (calculation/experiment) value

8.3.2.2 Core Components

The impact of ²³⁵U was observed about a significant 900 pcm in both EE1 and E3 cores, as shown in Tables 8.12 and 8.13, respectively. Especially, in the EE1 core, the uncertainty was mostly composed of the values of χ , ν and (n, γ) (capture cross section) reactions of ²³⁵U that are related to the infinite multiplication factor. In fact, sensitivity profiles of the χ value of ²³⁵U shown in Fig. 8.8 were large with a large number of standard deviations. A marked difference was observed in the $\bar{\nu}$ value of ²³⁵U shown in Fig. 8.9 between EE1 and E3 cores at thermal neutron and resonance regions in sensitivity profiles; also, core spectrum dependence on sensitivity was effectively canceled by constant standard deviation at these regions.

The uncertainty of capture reactions of polyethylene with the consideration of the thermal scattering law $S(\alpha, \beta)$ (termed ¹H in Tables 8.12 and 8.13) was effective in E3 core more than scattering (¹H (n, n)) reactions caused by the soft spectrum core with a large number of moderators as indicated by sensitivity profiles in Figs. 8.10 and 8.11.

Furthermore, the core spectrum was found to be dependent on the uncertainty of aluminum (Al) in the HEU plate (U-Al alloy) and aluminum in the core component (Al sheath) at KUCA indicated by “²⁷Al” and “*²⁷Al,” respectively, as shown in Figs. 8.12 and 8.13. In the EE1 core, elastic scattering and inelastic scattering (²⁷Al (n, n')) reactions in HEU plates affected the uncertainty because of the large sensitivity at the fast neutron region as shown in Figs. 8.14 and 8.15, respectively. Also, at the hard spectrum core, elastic scattering reactions of the Al sheath indicated higher value than that of ²⁷Al in HEU plates. Here, uncertainty that is not varied by control rod insertion should be emphasized, because the impact of nuclear reactions related to boron are very small and not shown in Tables 8.12 and 8.13. Accordingly, the calculated results of excess reactivity and control rod worth were considered in good agreement with the measured ones within a 10% difference because the uncertainty of boron is not induced by control rod insertions.

Table 8.12 Breakdown of the uncertainty [pcm] of k_{eff} in nuclides and nuclear reactions in EE1 core (Ref. [2])

Component	Nuclides reaction versus reaction		Case					
			I-1	I-2	I-3	I-4	I-5	I-6
HEU	^{235}U	^{235}U	770.86	772.51	772.28	779.51	773.71	777.73
	χ	χ	± 0.27	± 0.27	± 0.27	± 0.28	± 0.27	± 0.27
	^{235}U	^{235}U	360.18	360.12	360.13	359.77	360.06	359.88
	$\bar{\nu}$	$\bar{\nu}$	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01
	^{235}U	^{235}U	216.32	216.33	216.36	216.34	216.34	216.34
	(n, γ)	(n, γ)	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01
	^{235}U	^{235}U	103.38	103.38	103.41	103.39	103.38	103.38
	(n, f)	(n, γ)	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01
Polyethylene	^{235}U	^{235}U	69.71	69.77	69.80	69.95	69.80	69.88
	(n, f)	(n, f)	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01
	^{27}Al	^{27}Al	147.30	144.61	146.57	154.01	143.87	149.45
	(n, n)	(n, n)	± 0.07	± 0.07	± 0.07	± 0.08	± 0.07	± 0.08
	^{27}Al	^{27}Al	142.17	142.97	143.74	146.16	144.26	144.64
	(n, n')	(n, n')	± 0.03	± 0.03	± 0.03	± 0.03	± 0.03	± 0.03
	^{27}Al	^{27}Al	43.41	43.45	43.48	43.63	43.50	43.56
	(n, γ)	(n, γ)	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01
Polyethylene	^1H	^1H	192.70	191.94	192.73	196.16	192.90	193.34
	(n, n)	(n, n)	± 0.02	± 0.02	± 0.02	± 0.02	± 0.02	± 0.02
	^1H	^1H	120.60	119.22	118.78	113.26	117.20	115.26
	(n, γ)	(n, γ)	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01
	C (n, n)	C (n, n)	61.06	60.62	60.81	61.24	60.90	60.97
Aluminum sheath	C (n, n')	C (n, n')	24.88	25.41	24.66	24.53	24.94	24.66
	C (n, n)	C (n, n)	-24.07	-24.37	-23.94	-23.88	-24.10	-23.95
	C (n, n)	C (n, n)	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01
Aluminum sheath	^{27}Al	Al (n, n)	93.22	92.56	95.85	96.56	95.91	96.67
	^{27}Al	Al (n, n')	± 0.03	± 0.03	± 0.03	± 0.03	± 0.03	± 0.03
Control rod	^{10}B	Al (n, n')	49.57	49.80	49.80	50.28	50.28	49.92
	(n, γ)	(n, γ)	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01	± 0.01
Control rod	^{10}B	^{10}B	9.5E-08	1.0E-03	9.5E-04	5.42E-03	1.59E-03	3.52E-03
	(n, γ)	(n, γ)						

*With consideration of thermal scattering law $S(\alpha, \beta)$

Table 8.13 Breakdown of the uncertainty [pcm] of k_{eff} in nuclides and nuclear reactions in E3 core (Ref. [2])

Component	Nuclides reaction versus reaction		Case					
			II-1	II-2	II-3	II-4	II-5	II-6
HEU	$^{235}\text{U } \chi$	$^{235}\text{U } \chi$	726.35 ± 0.25	729.64 ± 0.25	731.48 ± 0.25	735.58 ± 0.25	735.63 ± 0.25	735.89 ± 0.25
	$^{235}\text{U } \bar{\nu}$	$^{235}\text{U } \bar{\nu}$	379.52 ± 0.01	379.50 ± 0.01	379.49 ± 0.01	379.46 ± 0.01	379.47 ± 0.01	379.47 ± 0.01
	$^{235}\text{U } (n, \gamma)$	$^{235}\text{U } (n, \gamma)$	182.95 ± 0.01	182.85 ± 0.01	182.86 ± 0.01	182.70 ± 0.01	182.77 ± 0.01	182.75 ± 0.01
	$^{235}\text{U } (n, f)$	$^{235}\text{U } (n, f)$	126.42 ± 0.01	126.44 ± 0.01	126.44 ± 0.01	126.46 ± 0.01	126.45 ± 0.01	126.46 ± 0.01
	$^{235}\text{U } (n, f)$	$^{235}\text{U } (n, \gamma)$	109.66 ± 0.01	109.76 ± 0.01	109.74 ± 0.01	109.87 ± 0.01	109.82 ± 0.01	109.84 ± 0.01
	$^{27}\text{Al } (n, n)$	$^{27}\text{Al } (n, n)$	40.01 ± 0.01	39.38 ± 0.01	40.38 ± 0.01	40.25 ± 0.01	41.52 ± 0.01	41.07 ± 0.01
	$^{27}\text{Al } (n, n')$	$^{27}\text{Al } (n, n')$	37.25 ± 0.01	37.68 ± 0.01	38.11 ± 0.01	37.84 ± 0.01	37.66 ± 0.01	37.64 ± 0.01
Polyethylene	$^1\text{H } (n, \gamma)$	$^1\text{H } (n, \gamma)$	217.00 ± 0.01	215.12 ± 0.01	214.89 ± 0.01	211.03 ± 0.01	211.60 ± 0.01	212.23 ± 0.01
	$^1\text{H } (n, n)$	$^1\text{H } (n, n)$	188.58 ± 0.03	188.17 ± 0.03	187.72 ± 0.03	191.17 ± 0.03	190.96 ± 0.03	190.40 ± 0.03
	$\text{C } (n, n)$	$\text{C } (n, n)$	57.98 ± 0.01	57.70 ± 0.01	57.90 ± 0.01	58.40 ± 0.01	58.44 ± 0.01	58.42 ± 0.01
	$\text{C } (n, n')$	$\text{C } (n, n')$	31.96 ± 0.01	32.25 ± 0.01	32.29 ± 0.01	31.79 ± 0.01	32.25 ± 0.01	32.37 ± 0.01
	$\text{C } (n, n)$	$\text{C } (n, n')$	-28.83 ± 0.01	-28.94 ± 0.01	-29.01 ± 0.01	-28.70 ± 0.01	-28.98 ± 0.01	-28.94 ± 0.01
Aluminum sheath	$^{27}\text{Al } (n, n)$	$^{27}\text{Al } (n, n)$	68.36 ± 0.02	69.46 ± 0.02	68.11 ± 0.02	68.72 ± 0.02	70.06 ± 0.02	69.49 ± 0.02
	$^{27}\text{Al } (n, n')$	$^{27}\text{Al } (n, n')$	37.69 ± 0.01	37.50 ± 0.01	37.67 ± 0.01	37.61 ± 0.01	37.42 ± 0.01	37.60 ± 0.01
	$^{27}\text{Al } (n, \gamma)$	$^{27}\text{Al } (n, \gamma)$	30.81 ± 0.01	30.43 ± 0.01	30.39 ± 0.01	29.52 ± 0.01	29.66 ± 0.01	29.82 ± 0.01
Control rod	$^{10}\text{B } (n, \gamma)$	$^{10}\text{B } (n, \gamma)$	1.1E-07	1.1E-03	1.0E-03	3.5E-03	2.9E-03	2.5E-03

*With consideration of thermal scattering law $S(\alpha, \beta)$

8.3.2.3 Tolerance of HEU Plate

For the variation in manufacturing tolerance of HEU plates shown in Table 8.1, uncertainty was evaluated with MCNP6.1 together with ENDF/B-VII.1 with the same number of histories as was reactivity evaluation, as shown in Tables 8.14 and 8.15. Based on the evaluation of the uncertainty for HEU plates, the uncertainty of the

Fig. 8.8 Sensitivity profile and standard deviation covariance data for χ value of ^{235}U (Ref. [2])

Fig. 8.9 Sensitivity profile and standard deviation covariance data for $\bar{\nu}$ value of ^{235}U (Ref. [2])

enrichment (4–14 pcm), length of sides (5 pcm), and thickness (10 pcm) was small regardless of the position of the control rods. Nonetheless, the difference of neutron spectra demonstrated its dependence on the uncertainty between EE1 and E3 cores, emphasizing especially the variation of total number of fuel plates: 3000 and 756 plates in EE1 and E3 cores, respectively. Additionally, the uncertainty attributed to variation in manufacturing tolerance was smaller than that induced by nuclear data, indicating that uncertainty induced by nuclear data is reasonable for the accuracy of criticality at KUCA.

Fig. 8.10 Sensitivity profile and standard deviation covariance data for H (n, n) reactions (polyethylene) (Ref. [2])

Fig. 8.11 Sensitivity profile and standard deviation in covariance data for H (n, γ) reactions (polyethylene) (Ref. [2])

8.3.2.4 Reproducibility of Control Rod Position

The uncertainty induced by the reproducibility of control rod positions, as shown in Tables 8.14 and 8.15, indicated about 3 pcm even at the withdrawal (no insertion) of control rods in Cases I-1 and II-1, showing an index of accuracy induced by the approximation in Eq. (8.20). By considering the index from the control rod insertion pattern, the uncertainty attributed to varying control rod position was evaluated about 8 pcm depending slightly on the insertion pattern. Here, the evaluated uncertainty was nearly the same as the experimental uncertainty in the reactivity shown in Tables 8.10

Fig. 8.12 Uncertainty of aluminum in HEU (U-Al alloy) plates, aluminum sheath, and control rod guide tube induced by nuclear data in EE1 core (Ref. [2])

Fig. 8.13 Uncertainty of aluminum in HEU (U-Al alloy) plates, aluminum sheath, and control rod guide tube induced by nuclear data in E3 core (Ref. [2])

and 8.11 (2 pcm through 11 pcm), demonstrating that the measured reactivity was slightly varied by the position of control rods.

8.4 Conclusion

Sensitivity and uncertainty analyses were conducted with the combined use of experimental results (excess reactivity and control rod worth) carried out at KUCA and numerical simulations by the MCNP6.1 calculations, the SRAC2006 and MARBLE code systems. The experimental value was compared with the calculated one by

Fig. 8.14 Sensitivity profile and standard deviation in covariance data for ^{27}Al (n, γ) reactions (HEU plate) (Ref. [2])

Fig. 8.15 Sensitivity profile and standard deviation in covariance data for ^{27}Al (n, n') reactions (HEU plate) (Ref. [2])

Table 8.14 Uncertainty [pcm] of k_{eff} attributed to manufacturing variation in HEU plates in Table 8.1 and reproducibility of control rods for EE1 core (Ref. [2])

Case	Enrichment of ^{235}U	Vertical length	Horizontal length	Thickness	Reproducibility of control rod
I-1	4.25 ± 0.09	4.19 ± 0.38	3.68 ± 0.43	9.44 ± 0.23	3.23 ± 3.55
I-2	4.21 ± 0.09	3.80 ± 0.40	4.10 ± 0.41	9.60 ± 0.23	2.22 ± 3.77
I-3	4.08 ± 0.09	4.51 ± 0.34	3.59 ± 0.40	9.68 ± 0.23	7.22 ± 3.57
I-4	4.21 ± 0.09	4.59 ± 0.39	4.40 ± 0.38	9.68 ± 0.23	5.54 ± 3.56
I-5	4.14 ± 0.09	3.69 ± 0.37	4.22 ± 0.41	10.08 ± 0.23	4.65 ± 3.56
I-6	4.13 ± 0.09	4.15 ± 0.38	4.23 ± 0.41	9.80 ± 0.23	4.28 ± 3.56

Table 8.15 Uncertainty [pcm] of k_{eff} attributed to manufacturing variation in HEU plates in Table 8.1 and reproducibility of control rods for E3 core (Ref. [2])

Case	Enrichment of ^{235}U	Vertical length	Horizontal length	Thickness	Reproducibility of control rod
II-1	13.20 ± 0.11	3.00 ± 0.25	3.09 ± 0.28	8.40 ± 0.41	2.47 ± 3.16
II-2	13.22 ± 0.11	3.15 ± 0.31	3.41 ± 0.35	7.71 ± 0.41	2.90 ± 3.17
II-3	13.27 ± 0.11	3.00 ± 0.29	3.19 ± 0.30	8.11 ± 0.41	3.56 ± 3.17
II-4	13.36 ± 0.11	3.10 ± 0.29	2.60 ± 0.31	8.04 ± 0.41	6.78 ± 3.14
II-5	13.50 ± 0.11	2.90 ± 0.32	3.03 ± 0.31	7.42 ± 0.41	3.99 ± 3.16
II-6	13.30 ± 0.11	3.08 ± 0.28	2.89 ± 0.32	7.93 ± 0.41	4.10 ± 3.15

the deterministic approach with the covariance data of JENDL-4.0. Sensitivity and uncertainty analyses demonstrated that the impact of ^{27}Al and ^{235}U was remarkably large in the KUCA A cores, respectively. Moreover, the numerical results revealed the quantitative evaluation (about 150 pcm) of uncertainty induced by the JENDL-4.0 data library in the A cores. Also, these results indicated that further investigation is needed of the numerical analyses of uncertainty of ^{27}Al composed mainly of core components in the A cores, with the use of ^{27}Al covariance data, in order to assess the effect of the uncertainty of ^{27}Al cross sections on reactivity.

To ensure the accuracy of criticality by experimental analyses at KUCA, the modeling of core configuration was examined through the comparison of excess reactivity and control rod worth between the calculation and the experiment in hard and soft spectrum cores. Furthermore, uncertainty was evaluated for manufacturing tolerance in HEU plates, for reproducibility of the control rod position, and for nuclear data. In the validation estimation of calculated k_{eff} values in the modeling of reference core materials and core configurations with MCNP6.1 and SCALE6.2/KENO-VI, the bias in calculated k_{eff} values showed that the difference in the spectrum was about 350 pcm in EE1 core and about 600 pcm in E3 core. Moreover, uncertainty of k_{eff} induced by nuclear data indicated about 950 pcm for k_{eff} evaluation with a slight variation in control rod position and core spectrum. In the breakdown of the uncertainty of k_{eff} induced by nuclear data, the impact of ^{235}U was significantly dominant in over 90% for both cores. The sensitivities of Al in HEU plates and in Al sheaths were marked in fast neutron and resonance regions, leading to large uncertainty about 100 pcm and 40 pcm in EE1 and E3 cores, respectively. Also, uncertainty was evaluated about 10 and 8 pcm in the tolerance of HEU plates and the reproducibility of control rod positions, respectively.

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