Chapter 4

Effective Delayed Neutron Fraction

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Abstract In kinetic analyses on ADS, although adjoint flux distribution is defined under the existence of an external neutron source, an issue of the proper determination of the weighting function still remains in the definition to obtain the kinetics parameters in the fixed-source calculations. Here, an alternative methodology is proposed with the combined use of the k-ratio method and the reaction rates obtained by the fixed-source calculations, when the subcriticality level or the spectrum of the external neutron source is varied. In ADS experiments, the measurement of β_{eff} is expected to provide complementary verification of the calculation and reliability of nuclear data. Then, the formulation of the Rossi- α method in the pulsed-neutron source has been already available for application to the subcriticality measurement in the pulsed-neutron source (PNS) experiments. Accordingly, the methodology is applied uniquely to deduce the β_{eff} value with the pulsed-neutron source (spallation neutrons), with the combined use of the results of experiments and calculations. Using parameters α and ρ_s , the values of $\beta_{\text{eff}}/\Lambda$ are deduced at near-critical configurations through experimental analyses. To estimate the numerical precision of Λ , the value of $\beta_{\text{eff}}/\Lambda$ is used as an index of Λ evaluation that is defined by a ratio of Λ values in the super-critical and subcritical states.

Keywords Effective delayed neutron fraction • k-ratio method • Rossi- α method • Pulsed-neutron source • Neutron generation time

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4.1 Dependency of External Neutron Source

4.1.1 Experimental Settings

4.1.1.1 Uranium-Fueled ADS Core

The critical EE1 core (reference core) was assembled at the KUCA-A core and was made up of 25 fuel rods surrounded by polyethylene reflectors as shown in Fig. A2.1. Each fuel rod (1/8"p60EUEU) was composed of highly enriched uranium (HEU; 2" $\times$ 2" and 1/16" thick) and a polyethylene moderator (PE; 2" $\times$ 2" and 1/4" thick) as shown in Fig. A2.2. The core was selected for considering the variation of β_{eff} along the subcriticality level. For the measurement of subcriticality, protons accelerated to 100 MeV were injected onto a disk-type tungsten (W) target in order to generate spallation neutrons. The accelerator was operated in pulsed mode and the repetition rate of the pulse was 20 Hz. The time width of the pulsed proton beam was 100 ns. The averaged proton current was 50 pA. The target was located at (15, H; Fig. A2.1) grid. The diameter and the thickness of the target were 50 mm and 12 mm, respectively. The subcriticality was measured by the extrapolated area ratio method [1] without considering the spatial effects. The neutron signals were obtained with the use of a BF₃ detector inserted diagonally at (10, U; Fig. A2.1) to the core for the measurements of the subcriticality. For the reference core in the critical experiment, excess reactivity and control rod worth (C1, C2, and C3) were measured by the positive period method and the rod drop method, respectively. Experimental analyses [2] were available to examine the precision of eigenvalue calculations by the Monte Carlo method and the accuracy of measured subcriticality by the extrapolated area ratio method. To achieve deep subcriticality, some of the fuel rods "F" (Fig. A2.1) were substituted for polyethylene reflectors and configured as shown in Fig. A2.3d, f, and the subcriticality level then ranged between about 1300 and 7500 pcm.

4.1.1.2 Thorium-Fueled ADS Core

In this experiment, different external neutron sources (spallation neutrons by the injection of 100 MeV protons onto the W target, and 14 MeV neutrons by the injection of deuteron beams onto the tritium target) were used for considering the variation of β_{eff} caused by the spectrum of external neutron source in the subcritical estimation. The subcritical core at $k_{\text{eff}} \simeq 0.85$ (Th-HEU-5PE core shown in Figs. A5.3 and A5.4) was composed of the polyethylene reflectors, fuel rods of thorium (Th; 2" $\times$ 2" and 1/8" thick), HEU, and PE moderators, as shown in Fig. A5.7. 14 MeV neutrons were produced by 0.4 mA deuteron beam, 10 Hz pulsed frequency, and 10 μ s pulsed width. 100 MeV proton beams were injected onto the W target at 50 mm spot size, 10 pA intensity, 20 Hz pulsed frequency, and 100 ns pulsed width. The subcriticality was measured by the same method as that in uranium-fueled ADS core. Here, the measured subcriticality could be affected by spatial effects especially in such a deep

subcritical core. Then, the neutron signals were obtained with the use of an optical fiber detector [3] at the core center (Figs. A5.3 and A5.4) in Th-HEU-5PE core. The optical fiber (1 mm diam. and 200 mm long) was coated with a powdered mixture of ^6LiF (95% enrichment) for detection of thermal neutrons based on $^6\text{Li}(n, t)^4\text{He}$ reactions and $\text{ZnS}(\text{Ag})$ for scintillation.

4.1.2 Numerical Simulations

4.1.2.1 Eigenvalue Calculations

To ensure the accuracy of eigenvalue calculations with the use of MCNPX-2.5.0 [4] and ENDF/B-VII.0 [5], experimental measurements of the excess reactivity and control rod worth of C1, C2, and C3 were carried out for comparing measured and calculated reactivities in the reference core. In the experiments, the critical state was attained by partial insertion of control rod C2 (full withdrawal of C1, C3, S4, S5, and S6), and excess reactivity was deduced from the coupling with control rod worth (C2 rod) and its integral calibration curve obtained by the positive period method. Control rod worth of S4, S5, and S6 was regarded as the same as those of C1, C2, and C3 obtained by the rod drop method, respectively, because of the symmetrical configuration of the rods, as shown in Fig. A2.1.

The MCNPX eigenvalue calculations were performed in a total of $1\text{E} + 08$ histories ($1\text{E} + 03$ active cycles of $1\text{E} + 05$ each); the statistical errors were less than 9 pcm. As shown in Table 4.1, the calculated excess reactivity and control rod worth of C1, C2, and C3 reproduced the measured ones within a relative difference of 5% in the C/E (calculation/experiment) value. Subcriticality was experimentally deduced from the combination of the excess reactivity and worth of inserted control rods as shown in Table 4.2. Furthermore, the accuracy of experimental analyses was verified within 5% through the comparison between the results of MCNPX and those of the experiments. Thus, the subcriticality obtained by MCNPX will be regarded as the reference subcriticality where it was not able to be measured with the excess reactivity and the control rod worth. In such deep subcriticality, the subcriticality by the area ratio method could compare with the reference one.

Table 4.1 Comparison between calculated and measured excess reactivities and control rod worth (Ref. [2])

Reactivity	MCNPX-2.5.0 [pcm]	Experiment [pcm]	C/E*
Excess	258 ± 10	257 ± 13	1.00 ± 0.13
C1 rod	839 ± 15	812 ± 24	1.03 ± 0.02
C2 rod	508 ± 15	506 ± 15	1.00 ± 0.03
C3 rod	135 ± 15	139 ± 4	0.97 ± 0.11

C/E*: calculation/experiment ($\beta_{\text{eff}} = 807 \pm 11$ [pcm] and $\Lambda = 30.5 \pm 0.1$ [μs] by MCNP6.1 with ENDF/B-VII.0)

Table 4.2 Subcriticality deduced by excess reactivity and control rod worth in the experiments and by MCNPX-2.5.0 in Fig. A2.1 (Ref. [2])

Case	# of fuel rods	Rod insertion pattern	Subcriticality [pcm]		C/E
			MCNPX-2.5.0	Experiment	
I-1	25	C1, C2, C3	1258 ± 11	1200 ± 36	1.05 ± 0.02
I-2	25	C1, C2, C3, S4	2099 ± 11	2011 ± 60	1.04 ± 0.01
I-3	25	C1, C2, C3, S4, S5, S6	2774 ± 11	2657 ± 80	1.04 ± 0.01
I-4	21	–	2737 ± 11	–	–
I-5	21	C1, C2, S4, S6	4858 ± 11	–	–
I-6	19	–	5299 ± 11	–	–
I-7	19	C1, C2, S4, S6	7457 ± 12	–	–

4.1.2.2 Reaction Rates

The delayed neutron fraction is defined as the ratio of the delayed neutron generation to the total neutron generation as follows:

$$\beta = \frac{\langle F^d \phi \rangle}{\langle F \phi \rangle}, \quad (4.1)$$

where ϕ is forward angular flux, $\langle \rangle$ the integration over angle, space, and energy, respectively. F , F^d , and F^p are production operators of total, delayed, and prompt neutron generations by fission reactions, respectively, as follows:

$$F = \int dE' \chi(E' \rightarrow E) \nu(E') \Sigma_f(E'), \quad (4.2)$$

$$F^d = \int dE' \chi^d(E' \rightarrow E) \nu^d(E') \Sigma_f(E'), \quad (4.3)$$

$$F^p = F - F^d, \quad (4.4)$$

where χ and χ^d are the energy spectra of total and delayed neutrons, respectively; ν and ν^d the neutron yields for total and delayed neutrons, respectively; E the energy of scattered neutrons; E' induced neutron energy; and Σ_f the macroscopic fission cross sections. And β_{eff} is defined as the ratio of the contribution of delayed neutrons to total neutrons for reactivity with the use of adjoint angular flux ϕ^+ for weighting function as follows:

$$\beta_{\text{eff}} = \frac{\langle \phi^+, F^d \phi \rangle}{\langle \phi^+, F \phi \rangle}. \quad (4.5)$$

The effective multiplication factor k_{eff} and prompt multiplication factor k_p are then expressed by the neutron fluxes as follows:

$$k_{\text{eff}} = \frac{\langle \phi^+, F \phi \rangle}{\langle \phi^+, L \phi \rangle}, \quad (4.6)$$

$$k_p = \frac{\langle \phi_p^+, F^p \phi_p \rangle}{\langle \phi_p^+, L \phi_p \rangle}, \quad (4.7)$$

where L is loss operator account for leakage, absorption, and scattering, and ϕ_p and ϕ_p^+ are the forward and adjoint fluxes taking into account prompt neutrons, respectively. In the k-ratio method [6], the multiplication factors k_{eff} and k_p are used to obtain an approximate value of β_{eff} as follows:

$$\beta_{\text{eff}, \text{ eigen}} = \frac{\langle \phi^+, (F - F^p) \phi \rangle}{\langle \phi^+, F \phi \rangle} = 1 - \frac{\langle \phi^+, F^p \phi \rangle}{\langle \phi^+, F \phi \rangle} \approx 1 - \frac{k_p}{k_{\text{eff}}}. \quad (4.8)$$

Here, two multiplication factors k_{RR} and $k_{\text{RR},p}$ by total and prompt neutrons are newly defined with the use of reaction rates, respectively, as follows:

$$k_{\text{RR}} = \frac{\langle F \phi \rangle}{\langle L' \phi \rangle}, \quad (4.9)$$

$$k_{\text{RR},p} = \frac{\langle F^p \phi_p \rangle}{\langle L' \phi_p \rangle}. \quad (4.10)$$

Since scattered neutrons are eventually absorbed in the core and the reflector or leaked out from the core, denominator in Eqs. (4.9) and (4.10) can be expressed in terms of leak out and the absorption reactions only. When numerator and denominator are interpreted as integrated reaction rates of the destruction and fission operators, respectively, the loss operator of L' is defined, taking into account leakage and absorption, as follows:

$$L' = \vec{\Omega} \cdot \nabla + \Sigma_a(E), \quad (4.11)$$

where $\vec{\Omega}$ is the direction of the neutron flight, and Σ_a the macroscopic absorption cross sections. Then, $\beta_{\text{eigen}}^{\text{RR}}$ deduced by the reaction rates was expressed approximately by substitution of the multiplication factors by Eqs. (4.9) and (4.10) in Eq. (4.8) as follows:

$$\beta_{\text{eigen}}^{\text{RR}} \approx 1 - \frac{k_{\text{RR},p}}{k_{\text{RR}}}. \quad (4.12)$$

Table 4.3 Comparison between effective multiplication factors k_{eff} by MCNPX-2.5.0 and k_{RR} in Eq. (4.9) with reaction rate calculations (Ref. [2])

Core	Case	k_{eff} (MCNPX-2.5.0)	k_{RR} (MCNPX-2.5.0)
EE1 core* (Spallation neutrons)	I-1	0.99093 ± 0.00009	0.99096 ± 0.00013
	I-2	0.98274 ± 0.00009	0.98269 ± 0.00013
	I-3	0.97627 ± 0.00009	0.97619 ± 0.00013
	I-4	0.97662 ± 0.00009	0.97655 ± 0.00012
	I-5	0.95680 ± 0.00009	0.95667 ± 0.00012
	I-6	0.95278 ± 0.00009	0.95281 ± 0.00012
	I-7	0.93358 ± 0.00009	0.93353 ± 0.00012
Th-HEU-5PE (Spallation neutrons)	II-1	0.86397 ± 0.00008	0.86387 ± 0.00012
Th-HEU-5PE (14 MeV neutrons)	II-2	0.84924 ± 0.00008	0.84923 ± 0.00012

* $k_{\text{eff}} = 1.00344 \pm 0.00009$ (at critical state by MCNP2.5.0 with ENDF/V-VII.0)

The multiplication factor k_{RR} thus deduced should be compared with the results in k_{eff} shown in Eq. (4.6) by the Monte Carlo calculations to examine the validity of k_{RR} obtained by Eq. (4.9). Reaction rate calculations were performed by MCNPX-2.5.0. Comparing the results of k_{eff} and k_{RR} in Eqs. (4.6) and (4.9), respectively, the results by the reaction rates in Eq. (4.9) revealed fairly good agreement with a difference of 10 pcm with those in Eq. (4.6) by MCNPX, as shown in Table 4.3.

The results revealed that the proposed methodology of k_{RR} with the use of reaction rates in Eq. (4.9) is appropriate through the comparison with the effective multiplication factor by the eigenvalue calculations. Note that $(n, 2n)$ and $(n, 3n)$ reactions were not considered for the estimation of the neutron productions, because there was no relevant difference between considering them or not in the uranium- and the thorium-loaded cores, respectively, in the eigenvalue calculations shown in Eq. (4.12).

4.1.3 *k*-Ratio Method

4.1.3.1 Reaction Rates by Eigenvalue Calculations

On the basis of the theoretical preparation discussed in Sect. 4.1.2, $\beta_{\text{eff, eigen}}$ and $\beta_{\text{eigen}}^{\text{RR}}$, as defined in Eqs. (4.8) and (4.12), can be obtained from the eigenvalue calculations, compared with the reference one obtained by MCNP6.1 [7]. Through the comparison with β_{eff} by MCNP6.1 shown in Table 4.4, the estimation of $\beta_{\text{eff, eigen}}$ in Eq. (4.8) demonstrated that the methodology is valid in the subcriticality level of interest. Also, the value of $\beta_{\text{eigen}}^{\text{RR}}$ in Eq. (4.12) with the use of reaction rates showed good agreement with those by MCNP6.1, as shown in Table 4.3, indicating a high precision of the reaction rates and an applicability of the *k*-ratio method with reaction rates. From the

Table 4.4 Comparison between β_{eff} by MCNP6.1, $\beta_{\text{eff,eigen}}$ by the k-ratio method in Eq. (4.8), $\beta_{\text{eigen}}^{\text{RR}}$ by the k-ratio method with reaction rate calculations (eigenvalue calculation) in Eq. (4.12), $\beta_{\text{source}}^{\text{RR}}$ by the k-ratio method with reaction rate calculations (fixed-source calculation) in Eq. (4.20), and β (eigenvalue and fixed-source calculations) in Eq. (4.1) (Ref. [2])

Case	Effective delayed neutron fraction (and delayed neutron fraction) [pcm]			
	β_{eff} (MCNP6.1)	$\beta_{\text{eff,eigen}}$ (MCNPX-2.5.0)	$\beta_{\text{eigen}}^{\text{RR}}$ (MCNPX-2.5.0)	$\beta_{\text{source}}^{\text{RR}}$ (MCNPX-2.5.0)
I-1	817 ± 11	797 ± 9	791 ± 13 (649 $\pm$ 22)	809 ± 43 (637 $\pm$ 32)
I-2	794 ± 11	801 ± 9	795 ± 13 (649 $\pm$ 22)	873 ± 45 (636 $\pm$ 33)
I-3	814 ± 11	811 ± 9	807 ± 13 (649 $\pm$ 22)	880 ± 46 (636 $\pm$ 33)
I-4	806 ± 11	810 ± 9	809 ± 13 (649 $\pm$ 22)	826 ± 55 (637 $\pm$ 36)
I-5	822 ± 11	825 ± 9	838 ± 13 (649 $\pm$ 22)	819 ± 57 (637 $\pm$ 37)
I-6	808 ± 11	824 ± 9	806 ± 13 (649 $\pm$ 22)	865 ± 81 (636 $\pm$ 44)
I-7	829 ± 11	801 ± 9	783 ± 13 (649 $\pm$ 22)	792 ± 86 (635 $\pm$ 45)
II-1	808 ± 11	795 ± 9	803 ± 9 (652 $\pm$ 24)	910 ± 11 (645 $\pm$ 14)
II-2	798 ± 11	802 ± 9	802 ± 9 (653 $\pm$ 24)	881 ± 7 (635 $\pm$ 9)

results in Table 4.4, the proposed methodology by the k-ratio method was confirmed valid on the basis of the reaction rates by the eigenvalue calculations.

4.1.3.2 Reaction Rates by Fixed-Source Calculations

From the definition of $\beta_{\text{eigen}}^{\text{RR}}$ shown in Eq. (4.12), the methodology using eigenvalue calculations is extended into a methodology using the fixed-source calculations. In the fixed-source problem, the transport equation is expressed by introducing source term s to acquire neutron flux formed in the subcritical core ϕ_{sub} as follows:

$$L \phi_{\text{sub}} = F \phi_{\text{sub}} + s, \quad (4.13)$$

where L and F indicate the loss and fission operators, respectively. To distinguish source and fission neutrons, ϕ_{sub} was assumed to be expressed in terms of ϕ_{source} and ϕ_{core} as follows:

$$\phi_{\text{sub}} = \phi_{\text{source}} + \phi_{\text{core}}, \quad (4.14)$$

where ϕ_{source} means the neutron flux corresponding to the source neutrons, and ϕ_{core} the neutron flux corresponding to the fission chain reactions. When the source problem is solved by applying $\nu = 0$ to Eq. (4.13), the multiplied angular flux is not formed, and the source term is expressed as follows:

$$s = L \phi_{\text{source}}. \quad (4.15)$$

Substituting Eq. (4.15) for Eq. (4.13), the source term is replaced by the product of L and ϕ_{source} , and the neutron balance equation is expressed as follows:

$$L \phi_{\text{sub}} = F \phi_{\text{sub}} + L \phi_{\text{source}}. \quad (4.16)$$

Here, on the basis of the manner in which effective eigenvalues are calculated with the neutron flux corresponding to fission neutrons, a pseudo multiplication factor k^{pseudo} is defined in the pseudo eigenvalue calculations with the use of neutron flux under the existence of an external neutron source as follows:

$$L \phi_{\text{core}} = \frac{1}{k^{\text{pseudo}}} F \phi_{\text{core}}. \quad (4.17)$$

Then, pseudo multiplication factors $k_{\text{RR}}^{\text{pseudo}}$ and $k_{\text{RR}}^{\text{pseudo}}$ are obtained by substituting Eqs. (4.14) and (4.16) in Eq. (4.17), with the use of reaction rates by the fixed-source calculations, in the same manner as in Eqs. (4.9) and (4.10), respectively, as follows:

$$k_{\text{RR}}^{\text{pseudo}} = \frac{\langle F \phi_{\text{core}} \rangle}{\langle L \phi_{\text{core}} \rangle} = \frac{\langle F \phi_{\text{sub}} \rangle - \langle F \phi_{\text{source}} \rangle}{\langle L' \phi_{\text{sub}} \rangle - \langle L' \phi_{\text{source}} \rangle}, \quad (4.18)$$

$$k_{\text{RR},p}^{\text{pseudo}} = \frac{\langle F^p \phi_{\text{core},p} \rangle}{\langle L \phi_{\text{core},p} \rangle} = \frac{\langle F^p \phi_{\text{sub},p} \rangle - \langle F^p \phi_{\text{source},p} \rangle}{\langle L' \phi_{\text{sub},p} \rangle - \langle L' \phi_{\text{source},p} \rangle}. \quad (4.19)$$

The $k_{\text{RR}}^{\text{pseudo}}$ ($k_{\text{RR},p}^{\text{pseudo}}$) can be calculated with two runs considering total (prompt) neutrons: for the calculation of ϕ_{sub} ($\phi_{\text{sub},p}$) with standard fixed-source calculation and for that of ϕ_{source} ($\phi_{\text{source},p}$) with the fixed-source calculation under the option of $\nu = 0$. Finally, $\beta_{\text{source}}^{\text{RR}}$ can be obtained with the reaction rates by the fixed-source calculations by substituting Eqs. (4.18) and (4.19) for Eq. (4.8) as follows:

$$\beta_{\text{source}}^{\text{RR}} \approx 1 - \frac{k_{\text{RR},p}^{\text{pseudo}}}{k_{\text{RR}}^{\text{pseudo}}}. \quad (4.20)$$

4.1.3.3 Results and Discussion

Fixed-source calculations were performed by MCNPX-2.5.0 with $1\text{E} + 09$ total histories with nuclear data libraries of ENDF/B-VII.0 and JENDL/HE-2007 [8] as shown in Table 4.5. For the case of the spallation neutrons, since neutrons over 20 MeV neutrons could be produced by the 100 MeV proton injections into the W target, JENDL/HE-2007 has a full advantage in the accuracy of the particle transport for neutrons over the energy of 20 MeV. However, because the yields for total and prompt neutrons are not provided in JENDL/HE-2007, ENDF/B-VII.0 was used for fissile and fissionable nuclei. β values calculated with the fluxes by eigenvalue and fixed-source calculations showed independency on the subcriticality, as shown in Table 4.4. However, the β values were decreased by the flux in fixed-source calculation. Then, $\beta_{\text{source}}^{\text{RR}}$ in Eq. (4.20) deduced by the fixed-source calculations indicated different values from β (fixed-source calculation), and these values were varied larger than those of β_{eff} by the eigenvalue calculations (MCNP6.1). In Cases I-1 to I-7, the increase of buckling in the core can be the reason to increase β_{eff} and $\beta_{\text{source}}^{\text{RR}}$ by the fuel rod replacement (Cases I-1, I-4, and I-6) and by the control rod insertion (Cases I-3, I-5, and I-7) because of especially increasing the leakage of prompt neutrons having higher energy compared to delayed neutrons. The neutron flux distribution was distorted by the injection of spallation neutrons having dominantly lower energy with the comparison of 14 MeV neutrons in Case II-1. This distorted flux distribution was considered to be induced by the leakage of prompt neutrons, resulting in the increase of $\beta_{\text{source}}^{\text{RR}}$.

The target results of the measured subcriticality (pcm units) in the uranium-fueled core were obtained from measured subcriticality in dollar unit multiplied by β_{eff} (MCNP6.1), $\beta_{\text{eigen}}^{\text{RR}}$, and $\beta_{\text{source}}^{\text{RR}}$ in Eqs. (4.11) and (4.19) (Table 4.4), respectively, as shown in Table 4.6. The measured subcriticality with the use of $\beta_{\text{source}}^{\text{RR}}$ in Eq. (4.20) for conversion from dollar units into pcm units by the fixed-source calculations showed good agreement with the reference subcriticality within a relative difference of 10% in the variation of the subcriticality level. And, $\beta_{\text{source}}^{\text{RR}}$ worked well for the results of measured subcriticality, comparing those of reference subcriticality. In Cases I-4 to I-7 (Table 4.6), there was a slight difference between β_{eff} and $\beta_{\text{source}}^{\text{RR}}$.

In deep subcritical cores, the measured subcriticality in the area ratio method is generally considered inaccurately obtained in pcm units to compare with reference one because an assumption is imposed on the measurements: all source neutrons induce the fission reactions and neutron signals originate from correlated neutrons to

Table 4.5 List of nuclear data libraries for calculation of $\beta_{\text{source}}^{\text{RR}}$ in particle transport simulations of the ADS experiments (Ref. [2])

	Neutrons	Protons
Spallation neutrons	JENDL/HE-2007 and ENDF/B-VII.0 (for U and Th only)	JENDL/HE-2007
14 MeV neutrons	ENDF/B-VII.0	—

Table 4.6 Results of subcriticality on the basis of effective delayed neutron fractions estimated by MCNP6.1, $\beta_{\text{eigen}}^{\text{RR}}$, and $\beta_{\text{source}}^{\text{RR}}$ (Ref. [2])

Case	Reference Subcriticality [pcm]	Measured subcriticality [pcm]		
		β_{eff} (MCNP6.1)	$\beta_{\text{eigen}}^{\text{RR}}$ (MCNPX-2.5.0)	$\beta_{\text{source}}^{\text{RR}}$ (MCNPX-2.5.0)
I-1	1200 ± 36	1147 ± 17 (1.05 ± 0.02)	1111 ± 20 (1.08 ± 0.02)	1136 ± 61 (1.06 ± 0.06)
I-2	2012 ± 60	1847 ± 27 (1.09 ± 0.02)	1850 ± 30 (1.09 ± 0.02)	2032 ± 105 (0.99 ± 0.05)
I-3	2657 ± 80	2419 ± 35 (1.09 ± 0.02)	2400 ± 40 (1.11 ± 0.02)	2615 ± 136 (1.02 ± 0.05)
I-4	2722 ± 32	2679 ± 39 (1.02 ± 0.02)	2688 ± 44 (1.02 ± 0.02)	2745 ± 183 (1.00 ± 0.07)
I-5	4891 ± 50	5036 ± 84 (0.96 ± 0.02)	5134 ± 88 (0.95 ± 0.02)	5015 ± 354 (0.97 ± 0.07)
I-6	5291 ± 54	5489 ± 85 (0.97 ± 0.02)	5475 ± 95 (0.97 ± 0.02)	5877 ± 554 (0.90 ± 0.09)
I-7	7474 ± 75	8421 ± 144 (0.89 ± 0.02)	7952 ± 149 (0.94 ± 0.02)	8050 ± 874 (0.93 ± 0.10)

(): Calculation/experiment

the fission multiplication. And, in the case of different external neutron sources for the deep subcritical core, as shown in Table 4.7, subcriticalities were compared in terms of the k_{eff} . The difference of the reference k_{eff} between spallation and 14 MeV neutrons is considered to be caused by the slight difference in the core configuration of the air gap shown in Figs A5.3 and A5.4. Here, the subcriticality with $\beta_{\text{source}}^{\text{RR}}$ was observed to reveal a comparative tendency through the comparison of the subcriticality with β_{eff} within the C/E value of 2%. The applicability of the proposed methodology was also confirmed with the variation of the external neutron sources.

Table 4.7 Comparison of k_{eff} and their C/E values estimated by MCNP6.1, $\beta_{\text{eigen}}^{\text{RR}}$, and $\beta_{\text{source}}^{\text{RR}}$ in different external neutron sources (Ref. [2])

Case	Reference k_{eff}	Measured k_{eff}		
		β_{eff} (MCNP6.1)	$\beta_{\text{eigen}}^{\text{RR}}$ (MCNPX-2.5.0)	$\beta_{\text{source}}^{\text{RR}}$ (MCNPX-2.5.0)
II-1	0.86397 ± 0.00008	0.89707 ± 0.00387 (0.96 ± 0.01)	0.89945 ± 0.00392 (0.96 ± 0.01)	0.88559 ± 0.00418 (0.98 ± 0.01)
II-2	0.84924 ± 0.00008	0.84775 ± 0.02543 (1.00 ± 0.03)	0.84516 ± 0.02581 (1.00 ± 0.03)	0.83243 ± 0.02743 (1.02 ± 0.03)

(): Calculation/experiment

Finally, these results demonstrated the fact that proper values for subcriticality determination with the area ratio technique were successfully obtained for the subcriticality estimation with the use of the proposed methodology by the fixed-source calculations.

4.2 Measurement

4.2.1 Nelson Number Method

4.2.1.1 Formulation of Rossi- α Method

In the measurement methodology for β_{eff} , the Nelson number method based on the Rossi- α method [9] provides the advantage of reducing the parameters that are considered difficult to obtain experimentally, including detection efficiency, fission rate at the core center, and the number of neutrons. The methodology assumes, however, that β_{eff} is measured near the critical state and by locating the external neutron source at the core center. It is also applicable to the PNS experiments by modifying the formulation of the Rossi- α method. In the PNS experiments, the formulation by the Rossi- α method is already available for processing neutron signals by the methodology used in the analysis with the pulsed-neutron source [10, 11].

In the Rossi- α method, the joint probability $P(t_1, t_2)$ between two neutron signals detected at times t_1 and t_2 is evaluated by categorizing the same fission chain reactions into correlated probability P_C and the different fission chain reactions and the neutron sources into uncorrelated probability P_U , as follows:

$$P(t_1, t_2) dt_1 dt_2 = P_C(t_1, t_2) dt_1 dt_2 + P_U(t_1, t_2) dt_1 dt_2. \quad (4.21)$$

P_C in the existence of the pulsed-neutron source can then be expressed as follows:

$$P_C(t_1, t_2) = g \left(\frac{\lambda_d \lambda_f \langle v_p (v_p - 1) \rangle}{2\alpha} \right) e^{-\alpha(t_2 - t_1)} dt_1 dt_2, \quad (4.22)$$

where α is the prompt neutron decay constant, $\langle v_p (v_p - 1) \rangle$ the second moment of the prompt neutron multiplicity distribution for induced prompt fission neutrons, λ_d the detection efficiency for neutron, λ_f the detection efficiency of a fission reaction, and g the correction factor taking into account the variation in the probability of detecting correlated counts originating from neutrons of different worth [9]. For uncorrelated probability P_U , the formulation of its signal is sensitive to the pulsed shape of the external neutron source [10]. In the present study, the shape was regarded as the Gaussian function, and the uncorrelated probability is represented by constant term $P_{U, \text{const}}$ and trigonometric term $P_{U, \text{trig}}$ as follows:

$$\begin{aligned}
P_U(t_1, t_2) dt_1 dt_2 &= P_{U, \text{const}}(t_1, t_2) dt_1 dt_2 + P_{U, \text{trig}}(t_1, t_2) dt_1 dt_2 \\
&= \frac{\lambda_d g * S \Lambda \sqrt{2\pi} \sigma}{(-\rho) T_0} dt_1 dt_2 \\
&\quad + \frac{\lambda_d g * S (-\rho) T_0}{2\Lambda \sqrt{2\pi} \sigma} \sum_{n=1}^{\infty} \frac{1}{\alpha^2 + \left(\frac{2n\pi}{T_0}\right)^2} e^{-\left(\frac{2n\pi}{T_0}\right)^2 \sigma^2} \cos\left\{\left(\frac{2n\pi}{T_0}\right)(t_2 - t_1)\right\} dt_1 dt_2,
\end{aligned} \tag{4.23}$$

where S is the intensity of source neutrons, T_0 the pulsed period, ρ the reactivity, Λ the generation time, σ the pulsed width, and g^* the correction factor for spatial and energy distribution of the source neutrons [9]. With the result of the Rossi- α method in the PNS experiments, the intensity of P_C and P_U is obtained from fitting by Eq. (4.21). Since P_C is predicted to decay rapidly; however, the fitting is considered difficult for obtaining both intensities together. Accordingly, the uncorrelated terms were deduced by first fitting in the region, where P_C is sufficiently decayed in Eq. (4.23), with fitting parameters B , D , and σ as follows:

$$\begin{aligned}
P_U(t_1, t_2) dt_1 dt_2 &= B dt_1 dt_2 \\
&\quad + D \sum_{n=1}^{1000} \frac{1}{\alpha^2 + \left(\frac{2n\pi}{T_0}\right)^2} e^{-\left(\frac{2n\pi}{T_0}\right)^2 \sigma^2} \cos\left\{\left(\frac{2n\pi}{T_0}\right)(t_2 - t_1)\right\} dt_1 dt_2,
\end{aligned} \tag{4.24}$$

where B is the fitting parameter for the constant value shown in Eq. (4.23) as follows:

$$B = \frac{\lambda_d g * S \Lambda \sqrt{2\pi} \sigma}{(-\rho) T_0}, \tag{4.25}$$

and the upper value of summation was set as 1000 leaving a margin from the saturation of the fitting results by setting about 300 in the upper value. With the fitting results of B , D , and σ , P_C is deduced by subtracting uncorrelated terms from the result of the Rossi- α method in the PNS experiments shown in Eq. (4.21) as follows:

$$\begin{aligned}
P_C(t_1, t_2) dt_1 dt_2 &= P(t_1, t_2) dt_1 dt_2 \\
&\quad - \left[B dt_1 dt_2 + D \sum_{n=1}^{1000} \frac{1}{\alpha^2 + \left(\frac{2n\pi}{T_0}\right)^2} e^{-\left(\frac{2n\pi}{T_0}\right)^2 \sigma^2} \cos\left\{\left(\frac{2n\pi}{T_0}\right)(t_2 - t_1)\right\} dt_1 dt_2 \right],
\end{aligned} \tag{4.26}$$

Here, the intensity of correlated probability C is obtained by fitting Eq. (4.26) as follows:

$$P_C(t_1, t_2) dt_1 dt_2 = C e^{-\alpha(t_2 - t_1)} dt_1 dt_2, \tag{4.27}$$

where C is expressed as follows:

$$C = g \left(\frac{\lambda_d \lambda_f \langle v_p (v_p - 1) \rangle}{2\alpha} \right). \quad (4.28)$$

4.2.1.2 Estimation of β_{eff}

For the estimation of β_{eff} , the parameters of B and C obtained by fitting with Eqs. (4.24) and (4.28), respectively, are used with the value of α , which is defined with the use of prompt multiplication factor k_p and neutron lifetime l , as follows:

$$\alpha = \frac{\beta_{\text{eff}} - \rho}{\Lambda} = \frac{1 - k_p}{l} = \frac{1 - (1 - \beta_{\text{eff}})k_{\text{eff}}}{l} = \frac{\beta_{\text{eff}}}{l} \frac{1 - \rho_{\$}}{1 - \rho_{\$}\beta_{\text{eff}}}, \quad (4.29)$$

where k_{eff} is the effective multiplication factor and $\rho_{\$}$ the reactivity in dollar units. Further, λ_f and Λ can be rewritten with the use of β_{eff} and k_{eff} as follows:

$$\lambda_f = \frac{1}{l_f} = \frac{k_p}{l \langle v_p \rangle} = \frac{(1 - \beta_{\text{eff}})k_{\text{eff}}}{l \langle v_p \rangle}, \quad (4.30)$$

$$\Lambda = \frac{l}{k_{\text{eff}}}, \quad (4.31)$$

where l_f is the mean time between fission iterations and $\langle v_p \rangle$ is the average number of prompt neutrons released per fission. With the use of Eqs. (4.29), (4.30), and (4.31), the intensity of correlated probability C in Eq. (4.27) can be expressed as follows:

$$\begin{aligned} C &= \frac{g \lambda_d \langle v_p (v_p - 1) \rangle k_p}{2\alpha l \langle v_p \rangle} = \frac{g \lambda_d \langle v_p (v_p - 1) \rangle k_p}{2\alpha \Lambda \langle v_p \rangle k_{\text{eff}}} \\ &= \frac{g \lambda_d \langle v_p (v_p - 1) \rangle (1 - \beta_{\text{eff}})k_{\text{eff}}}{2(1 - \rho_{\$})\beta_{\text{eff}} \langle v_p \rangle k_{\text{eff}}} \\ &= \frac{g \lambda_d \langle v_p (v_p - 1) \rangle (1 - \beta_{\text{eff}})}{2(1 - \rho_{\$})\beta_{\text{eff}} \langle v_p \rangle} \end{aligned} \quad (4.32)$$

Also, B shown in Eq. (4.25) can be rewritten with the use of $\rho_{\$}$ and α , as follows:

$$B = \frac{\lambda_d g * S \Lambda \sqrt{2\pi} \sigma}{(-\rho_{\$}) T_0} = \frac{\lambda_d g * S (1 - \rho_{\$}) \sqrt{2\pi} \sigma}{(-\rho_{\$}) T_0 \alpha}. \quad (4.33)$$

Here, the Nelson number is defined with the combination of α , B in Eq. (4.33) and C in Eq. (4.32) as follows:

$$N = \left(\frac{2\sqrt{2\pi} \sigma g^* S \langle v_p \rangle}{g \langle v_p (v_p - 1) \rangle T_0} \right) \left(\frac{C}{\alpha B} \right) = \frac{(1 - \beta_{\text{eff}})(-\rho_{\text{S}})}{\beta_{\text{eff}}(1 - \rho_{\text{S}})^2}, \quad (4.34)$$

and correction factors g and g^* were obtained with the following calculations [6]:

$$g = \frac{\int \mathbf{P}(\mathbf{r}) d\mathbf{r} \int \mathbf{P}(\mathbf{r}) \mathbf{I}^2(\mathbf{r}) d\mathbf{r}}{(\int \mathbf{P}(\mathbf{r}) \mathbf{I}(\mathbf{r}) d\mathbf{r})^2}, \quad (4.35)$$

$$g^* = \frac{\int \mathbf{S}(\mathbf{r}) \mathbf{I}_q(\mathbf{r}) d\mathbf{r} \int \mathbf{P}(\mathbf{r}) d\mathbf{r}}{\int \mathbf{P}(\mathbf{r}) \mathbf{I}(\mathbf{r}) d\mathbf{r} \int \mathbf{S}(\mathbf{r}) d\mathbf{r}}, \quad (4.36)$$

$$\mathbf{P}(\mathbf{r}) = \int \Sigma_f(\mathbf{r}, E) \phi(\mathbf{r}, E) dE, \quad (4.37)$$

$$\mathbf{I}(\mathbf{r}) = \int \chi(\mathbf{r}, E) \phi^+(\mathbf{r}, E) dE, \quad (4.38)$$

$$\mathbf{I}_q(\mathbf{r}) = \int \chi_q(\mathbf{r}, E) \phi^+(\mathbf{r}, E) dE, \quad (4.39)$$

where ϕ and ϕ^+ are the forward and adjoint fluxes, respectively, Σ_f the macroscopic fission cross section, and χ and χ_q the fission spectrum and the spectrum of the external neutron source, respectively. From the relation between N and β_{eff} shown in Eq. (4.24), β_{eff} is obtained experimentally as follows:

$$\beta_{\text{eff}} = \frac{-\rho_{\text{S}}}{N(1 - \rho_{\text{S}})^2 - \rho_{\text{S}}}. \quad (4.40)$$

The procedure for the deduction of β_{eff} in the PNS experiments is shown in Fig. 4.1.

Fig. 4.1 Procedure for deduction of β_{eff} in PNS experiments (Ref. [12])

4.2.2 Experimental Settings

4.2.2.1 ADS Experiments

The A-core was used for the measurement of β_{eff} in the ADS experiments with 100 MeV protons [12]. The core comprised 25 fuel rods surrounded by polyethylene reflectors as shown in Fig. A2.1. Each fuel rod (1/8" p60EUEU) was made up of an HEU (2" $\times$ 2" and 1/16" thick) and a polyethylene moderator (PE; 2" $\times$ 2" and 1/8" thick) in an Al sheath 54 $\times$ 54 $\times$ 1524 mm as shown in Fig. A2.2. The core spectrum was a relatively hard one with an H/U (hydrogen/uranium) ratio of approximately 50 in a thermal reactor.

A proton accelerator (FFAG accelerator) was operated to inject 100 MeV protons (beam spot 50 mm, intensity 50 pA (Cases II-1 to II-3 in Fig. A2.3a–c) and 75 pA (Cases II-4 to II-7 in Figs. A2.3d, g) and pulsed frequency 20 Hz) onto a tungsten target (W) (50 mm diam. and 12 mm thick) located at position (15, H; Fig. A2.1) to generate spallation neutrons.

Time evolution according to the injection of spallation neutrons was obtained from the signals of a BF₃ detector installed diagonally to the target at position (10, U; Fig. A2.1a) and an optical fiber detector [3] (coated with a powdered mixture of ⁶LiF (95% enrichment) for detection based on ⁶Li(*n*, *t*)⁴He reactions and ZnS(Ag) for scintillation in the core) installed at position (15–16, M-N; Fig. A2.1). The PNS experiments were conducted for 10 min in each case to acquire neutron signals for data analyses by the area ratio method and the Rossi- α method. The pulsed width of the neutron source was deduced by fitting for each detector with Eq. (4.23), as shown in Table 4.8. Here, the reason being the pulsed width of the BF₃ detector larger than that of the optical fiber is considered as follows: the pulsed width of the source neutrons is large during the transport into detectors; the transported source neutrons are detected, having the width depending on the distance from the neutron source.

4.2.2.2 Subcriticality

Subcriticality was obtained by full insertion of control and safety rods, and by the substitution of the fuel assembly for polyethylene rods, as shown in Table 4.8. The excess reactivity and control rod worth (C1, C2, and C3) were measured by the positive period method and the rod drop method, respectively. In Cases II-1 to II-3, the subcriticality was experimentally deduced with the combined use of control rod worth and its calibration curve obtained by the positive period method. Moreover, in Cases II-4 to II-7, some of the fuel rods "F" (Fig. A2.1) were replaced by polyethylene reflectors and configured as shown in Figs. A2.3d–f. The subcriticality in dollar units was acquired experimentally by the extrapolate area ratio method [1]. The subcriticality level then ranged between 1300 and 7500 pcm.

Table 4.8 Deduced pulsed width and measured subcriticality in dollar units by fuel rod substitution and control rod insertion (Ref. [12])

Case	# of fuel rods	Rod Insertion pattern	Pulsed width in optical fiber and (BF ₃) detectors [s]	Subcriticality [\$] (methodology)
II-1	25	C1, C2, C3	(2.10 ± 0.05)E-04 ((4.30 ± 0.20)E-04)	1.20 ± 0.03 (control rod worth)
II-2	25	C1, C2, C3, S4	(1.25 ± 0.05)E-04 ((3.10 ± 0.20)E-04)	2.01 ± 0.06 (control rod worth)
II-3	25	C1, C2, C3, S4, S5, S6	(1.00 ± 0.05)E-04 ((2.90 ± 0.20)E-04)	2.66 ± 0.07 (control rod worth)
II-4	21	–	(1.00 ± 0.05)E-04 ((2.70 ± 0.10)E-04)	3.32 ± 0.02 (area ratio method)
II-5	21	C1, C2, S4, S6	(5.12 ± 0.25)E-05 ((2.20 ± 0.05)E-04)	6.13 ± 0.05 (area ratio method)
II-6	19	–	(5.00 ± 0.42)E-05 ((2.75 ± 0.02)E-04)	6.80 ± 0.05 (area ratio method)
II-7	19	C1, C2, S4, S6	(5.00 ± 0.42)E-05 ((2.60 ± 0.02)E-04)	10.16 ± 0.09 (area ratio method)

4.2.3 Results and Discussion

4.2.3.1 Parameters in Rossi-α Method

The PNS experiments were carried out for the neutron noise analyses by the Rossi-α method. Moreover, the PNS histogram was obtained to acquire α values to supplement neutron noise analyses, as shown in Fig. 4.2. To obtain the intensity of the second term in Eq. (4.23), fitting based on Eq. (4.24) is required in the region where

Fig. 4.2 PNS histogram of optical fiber detector in Case II-1 (Ref. [12])

Fig. 4.3 Correlated and uncorrelated probabilities of fiber detector in Case II-1 by Rossi- α method in PNS experiment (Ref. [12])

the correlation probability is negligible. From the PNS histogram shown in Fig. 4.2, the decay of the correlation probability was confirmed at 0.025 s, and intensity B was obtained by fitting for the joint probability by the Rossi- α method in the PNS experiments, on the basis of Eq. (4.25) in the region between 0.025 and 0.125 s. The intensity of the correlation term in Eq. (4.22) was then obtained by subtracting the uncorrelated term from the joint probability in Eq. (4.26) and fitting by Eq. (4.27).

The uncorrelated probability decreased rapidly, compared with the correlated probability as shown in Fig. 4.3. Instead of decreasing exponentially, the correlated probability increased in the vicinity of zero, the reason being the overestimation of the uncorrelated probability in the region arising by varying the shape of the external neutron source from the Gaussian to the pulsed, because the formulation of uncorrelated probability has the sensitivity to the shape of the external neutron source.

In the fitted parameters, α and B values differed from each other when the subcriticality was varied, as shown in Figs. 4.4a, b, respectively, because of the increase of the subcriticality in the α value and the decrease of neutrons in B . For the intensity of correlated probability C , the optical fiber indicated an asymptotic tendency of monotonic decrease, by the change in the subcriticality attributed to control rod insertion and fuel rod replacement in Cases II-1 to II-7, as shown in Fig. 4.4c; the intensity of fission reactions decreased by the subcriticality. The BF_3 detector, however, showed the increasing tendency in Cases II-4, II-5, and II-6. Since the low correlated probability of the fission neutrons could be predicted in BF_3 installed outside the core, the accuracy of the reconstruction of correlated probability is considered low.

4.2.3.2 Estimation of β_{eff}

To obtain correction factors g and g^* by Eqs. (4.30) and (4.31), respectively, the calculations of reaction rates and adjoint flux were performed by MCNP6.1 together

Fig. 4.4 Parameters obtained by the Rossi- α method in PNS experiments by varying subcriticality (Ref. [12])

(a) Prompt neutron decay constant α

(b) Intensity of uncorrelated probability B

(c) Intensity of correlated probability C

Table 4.9 Calculated correction factors g and g^* by Eqs. (4.30) and (4.31) with JENDL-4.0 and JENDL/HE-2007 (or ENDF/B-VII.0 and JENDL/HE-2007), respectively (Ref. [12])

Case	g	g^*
1	1.03 ± 0.01 (1.03 ± 0.01)	$(4.29 \pm 0.01)\text{E-03}$ ($(4.32 \pm 0.01)\text{E-03}$)
2	1.03 ± 0.01 (1.03 ± 0.01)	$(4.28 \pm 0.01)\text{E-03}$ ($(4.31 \pm 0.01)\text{E-03}$)
3	1.04 ± 0.01 (1.04 ± 0.01)	$(4.18 \pm 0.01)\text{E-03}$ ($(4.19 \pm 0.01)\text{E-03}$)
4	1.03 ± 0.01 (1.03 ± 0.01)	$(3.84 \pm 0.01)\text{E-03}$ ($(3.83 \pm 0.01)\text{E-03}$)
5	1.03 ± 0.01 (1.03 ± 0.01)	$(3.74 \pm 0.01)\text{E-03}$ ($(3.78 \pm 0.01)\text{E-03}$)
6	1.03 ± 0.01 (1.03 ± 0.01)	$(2.84 \pm 0.02)\text{E-03}$ ($(2.84 \pm 0.01)\text{E-03}$)
7	1.03 ± 0.01 (1.03 ± 0.01)	$(2.75 \pm 0.01)\text{E-03}$ ($(2.76 \pm 0.01)\text{E-03}$)

with JENDL-4.0 [13] or ENDF/B-VII.0 (uranium and boron) and JENDL/HE-2007 (all nuclides except uranium and boron); total histories for adjoint flux and reaction rates were $1\text{E} + 07$ and $1\text{E} + 06$, respectively; the statistical error of the calculations was less than 1%. The adjoint flux was manually obtained for three-dimensional calculations as follows: an external neutron source in the Watt spectrum of ^{235}U was set inside an HEU plate; the reaction rates for the response of $\nu\Sigma_f$, which is discussed as more appropriate than Σ_f [14] to estimate the adjoint flux, were tallied over the core with NONU option to avoid the neutron multiplication; these reaction rates approximately corresponded to the adjoint flux at the position of the HEU plate; this fixed-source problem was repeated by changing the position of the external neutron source in HEU plate. The correction factor g remained constant on subcriticality, and conversely, the decreasing tendency was indicated on g^* values by varying the subcriticality, as shown in Table 4.9. Here, a slight difference was observed in the correction factors g and g^* between the selection of cross sections, and g and g^* estimated with JENDL-4.0 and JENDL/HE-2007 were used for the β_{eff} measurements.

With the results shown in Table 4.8 and fitted parameters α , B and C , β_{eff} values were deduced by Eq. (4.40) in the ADS experiments. The result of measured β_{eff} is compared with that of calculated β_{eff} by MCNP6.1 as shown in Table 4.10, revealing that the result of the optical fiber at the core center indicates acceptable accuracy of the results by MCNP6.1 within a relative difference of about 13% in the subcritical range of the ADS operation (Cases II-1 to II-7) around $k_{\text{eff}} = 0.93$. With the BF_3 detector, the difference between the measured and calculated β_{eff} was very large. The resulting low accuracy is attributable to the variation in the shape of the pulsed-neutron source and the extraction of correlated probability with pulsed width of neutron source based on Eq. (4.24). Thus, the emphasis was placed where the optical fiber detector located at the core center showed good accuracy because the effect of the higher mode in flux distribution is not effective and the correlated neutrons

Table 4.10 Comparison between measured and calculated β_{eff} values (Ref. [12])

Case	k_{eff}^*	Effective delayed neutron fraction β_{eff} [pcm]			
	MCNP6.1	MCNP6.1	Area ratio method	Optical fiber	BF ₃
II-1	0.99093	817 ± 11	763 ± 21 (6.6 ± 0.2)	870 ± 81 (−6.5 ± 0.6)	911 ± 29 (−11.5 ± 0.4)
II-2	0.98274	794 ± 11	874 ± 26 (−10.0 ± 0.3)	876 ± 58 (−10.3 ± 0.7)	845 ± 33 (−6.5 ± 0.3)
II-3	0.97627	814 ± 11	919 ± 24 (−12.3 ± 0.3)	855 ± 45 (−5.0 ± 0.3)	725 ± 36 (11.0 ± 0.6)
II-4	0.97662	801 ± 11	721 ± 5 (10.0 ± 0.2)	903 ± 109 (−12.7 ± 1.5)	704 ± 28 (12.1 ± 0.5)
II-5	0.95680	826 ± 12	737 ± 6 (10.8 ± 0.2)	915 ± 75 (−10.8 ± 0.9)	303 ± 7 (63.3 ± 1.7)
II-6	0.95278	817 ± 11	729 ± 6 (10.8 ± 0.2)	921 ± 84 (−12.7 ± 1.2)	189 ± 19 (76.9 ± 7.8)
II-7	0.93358	815 ± 11	729 ± 6 (14.3 ± 0.3)	811 ± 72 (−0.4 ± 0.1)	162 ± 14 (90.1 ± 7.1)

(): Relative difference (%); *Standard deviation of 9 pcm

to the same family in fission chain is largely detected. Inversely, when the detector is located outside of the core, the measurement accuracy is considered deteriorated by the difficulty in the extraction of the correlated term related to correlated neutron detections with Eq. (4.26). To estimate the measurement accuracy of β_{eff} by proposed methodology, β_{eff} was compared with the simple estimation by the area ratio method and the relation between reactivity in dollar and pcm units. With the use of the subcriticality in Table 4.8 and k_{eff} in Table 4.10, β_{eff} can be estimated as follows:

$$\beta_{\text{eff}}^{\text{Area ratio method}} = \frac{\rho}{\rho_{\$}} = \frac{1}{\rho_{\$}} \left(1 - \frac{1}{k_{\text{eff}}} \right). \quad (4.41)$$

Here, measured β_{eff} by proposed methodology indicated comparable or more accurate results in the comparison with β_{eff} by Eq. (4.41) (named Area ratio method) in Cases II-1 to II-7 for the optical fiber detector, as shown in Table 4.10. Thus, the applicability of the measurement methodology for β_{eff} was demonstrated in the subcritical range of the ADS operation around $k_{\text{eff}} = 0.93$. Furthermore, to obtain good accuracy of β_{eff} values with the proposed methodology, improvement of correlated probability is attainable with the use of a specific shape (Gaussian distribution) of the external neutron source.

4.3 Evaluation of $\beta_{\text{eff}}/\Lambda$

4.3.1 Experimental Settings

4.3.1.1 Core Configurations

ADS experiments were carried out in a uranium-lead (U-Pb) core with 14 MeV neutrons (Fig. A3.1) and spallation neutrons (Fig. A3.2) [15]. The core comprised normal fuel rods ($1/8''\text{p60EUEU}$) of highly enriched uranium (HEU: $2'' \times 2'' \times 1/16''$) and a polyethylene moderator (p: $2'' \times 2'' \times 1/8''$) in an aluminum sheath $2.1'' \times 2.1'' \times 60''$, as shown in Fig. A3.3a, and U-Pb fuel rods composed of an HEU plate and a Pb plate (Pb: $2'' \times 2'' \times 1/8''$) in the center, as shown in Fig. A3.3b. The core spectrum was approximately hard in the central region for the spectrum of the actual ADS, and the driver (normal fuel) region has an H/U (hydrogen/uranium) ratio of approximately 50 in the thermal reactor.

14 MeV neutrons were generated by the injection of a deuteron beam (intensity 0.3 mA, pulsed width 10 μs , and pulsed frequency 20 or 50 Hz) onto a tritium target located at (14–15, Y; Fig. A3.1). A proton accelerator (FFAG accelerator) was operated to inject 100 MeV protons (beam spot 50 mm, intensity 10 pA, and pulsed frequency 20 Hz) onto a lead-bismuth (Pb–Bi) target (50 mm diam. and 18 mm thick) located at (15, D; Fig. A3.2) to generate spallation neutrons.

Time evolution according to the injection of external neutrons was obtained from the signals of four BF_3 detectors set around the core. Furthermore, in the case of spallation neutrons, an optical fiber type detector containing a Eu:LiCaAlF_6 scintillator [16] was additionally installed at location (10, U; Fig. A3.2) symmetrical to $\text{BF}_3\#2$ so as to examine the influence of the detector on measured subcriticality.

Subcriticality was obtained by full insertion of control and safety rods, and by the substitution of the fuel assembly for polyethylene rods, as shown in Table 4.11a. In Cases I-1 to I-5, the subcriticality was experimentally deduced with the combined use of control rod worth and its calibration curve obtained by the positive period method. Moreover, in Cases II to VII (Table 4.11b), some of the fuel rods “F” (Figs. A3.1 and A3.2) were substituted for polyethylene reflectors and configured as shown in Figs. A3.6 and A3.7. The subcriticality in dollar units was acquired experimentally by the extrapolated area ratio method [1]. Also, α was obtained by the α -fitting method in PNS experiments. The subcriticality level ranged between 500 and 7500 pcm.

4.3.1.2 Numerical Analyses

Numerical analyses were conducted by using MCNP6.1 together with ENDF/B-VII.1 (total histories $5\text{E} + 08$; $5\text{E} + 05$ histories per cycle and $1\text{E} + 03$ active cycles) and by PARTISN [17] (with mesh size less than $10 \times 10 \times 10$ mm; transport cross section instead of P_L scattering treatment; EO_{16} quadrature for S_N [18]; Fig. 4.5). In PARTISN analyses, seven effective cross Sects. (7-energy group) were generated as

Table 4.11 Subcriticality variation and pulsed-neutron frequency of 14 MeV neutrons and spallation neutrons (Ref. [15])**(a) Cases I-1 to I-5**

Case	# of fuel plates	Rod insertion pattern	Pulsed frequency (Hz)	
			14 MeV neutrons	Spallation neutrons
I-1	4560	C1	20	20
I-2	4560	C1, C3	20	20
I-3	4560	C1, C2, C3	20	20
I-4	4560	C1, C3, S5	–	20
I-5	4560	C1, C3, S4, S5	–	20

(b) Cases II to VII

Case	# of fuel plates	Rod insertion*	Pulsed frequency [Hz]	
			14 MeV neutrons	Spallation neutrons
II	4440	–	20	20
III	4200	–	20	20
IV	4080	–	50	20
V	3960	–	20	20
VI	3840	–	50	20
VII	3600	–	20	20

*Full withdrawal of all control (C1, C2, and C3) and safety (S4, S5, and S6) rods

described in Ref. [19] with the SCALE6.2 code system [20], as shown in Fig. 4.6. The validation of the numerical analyses was confirmed through comparison between measured reactivity and calculated reactivity. Here, excess reactivity and control rod worth (C1, C2, and C3) were measured by the positive period method and the rod drop method, respectively, and calculated as follows:

$$\rho_{\text{excess}}^{\text{cal}} = \left(1 - \frac{1}{k_{\text{eff}}^{\text{clean}}}\right) - \left(1 - \frac{1}{k_{\text{eff}}^{\text{critical}}}\right) = \frac{1}{k_{\text{eff}}^{\text{critical}}} - \frac{1}{k_{\text{eff}}^{\text{clean}}}, \quad (4.42)$$

$$\rho_{\text{rod}}^{\text{cal}} = \left(1 - \frac{1}{k_{\text{eff}}^{\text{critical}}}\right) - \left(1 - \frac{1}{k_{\text{eff}}^{\text{rod}}}\right) = \frac{1}{k_{\text{eff}}^{\text{rod}}} - \frac{1}{k_{\text{eff}}^{\text{critical}}}, \quad (4.43)$$

where $\rho_{\text{excess}}^{\text{cal}}$ and $\rho_{\text{rod}}^{\text{cal}}$ are the calculated excess reactivity and control rod worth, respectively, $k_{\text{eff}}^{\text{critical}}$ the value of the effective multiplication factor at the critical state so as to reduce the calculation bias induced by the nuclear data, $k_{\text{eff}}^{\text{clean}}$ the value of effective multiplication factor at the withdrawal of all control rods, and $k_{\text{eff}}^{\text{rod}}$ the value of effective multiplication factor at the insertion of control rod C1, C2, or C3 at the critical state. The difference was confirmed at less than 5% in control rod worth by MCNP6.1, as shown in Table 4.12, although large uncertainty was observed in excess reactivity attributed to estimating small reactivity, considering valid for subsequent analyses. For PARTISN calculations, while the C/E (calculation/experiment) ratio

Fig. 4.5 Calculation geometry with grid mesh in PARTISN (Ref. [15])

Fig. 4.6 Calculation geometry (2-D) for homogenization with SCALE/NEWT (Ref. [15])

Table 4.12 Comparison between measured and calculated excess reactivity and control rod worth (Ref. [15])

Reactivity	Experiment [pcm]	MCNP6.1 [pcm]	PARTISN [pcm]
Excess	47 ± 5	37 ± 5 (0.77 ± 0.12)	—
C1 rod	840 ± 2	872 ± 6 (1.04 ± 0.01)	983 (1.17 ± 0.01)
C2 rod	646 ± 1	665 ± 6 (1.03 ± 0.01)	806 (1.25 ± 0.01)
C3 rod	226 ± 13	228 ± 6 (1.01 ± 0.01)	229 (1.01 ± 0.06)

(): C/E (calculation/experiment) value

was 1.25 at most, the reproducibility was considered acceptable by comparison with experiments involving variations of subcriticality shown in Table 4.13.

4.3.2 Kinetics Parameters

4.3.2.1 Subcriticality in Dollar Units

The measured $\rho_{\$}$ was compared with the calculated $\rho_{\$}^{\text{MCNP}}$ by using MCNP6.1 and deduced as follows:

$$\begin{aligned}\rho_{\$}^{\text{MCNP}} &= \frac{1}{\beta_{\text{eff}}^{\text{MCNP}}} \left\{ \left(1 - \frac{1}{k_{\text{eff}}^{\text{critical}}} \right) - \left(1 - \frac{1}{k_{\text{eff}}^{\text{case}}} \right) \right\} \\ &= \frac{1}{\beta_{\text{eff}}^{\text{MCNP}}} \left(\frac{1}{k_{\text{eff}}^{\text{case}}} - \frac{1}{k_{\text{eff}}^{\text{critical}}} \right),\end{aligned}\quad (4.44)$$

where $k_{\text{eff}}^{\text{case}}$ is the effective neutron multiplication factor in each of the cases shown in Table 4.13, and $\beta_{\text{eff}}^{\text{MCNP}}$ the effective delayed neutron fraction obtained by MCNP6.1 corresponding to each case. In the comparison between $\rho_{\$}$ and $\rho_{\$}^{\text{MCNP}}$ by varying the neutron source shown in Figs. 4.7 and 4.8, $\rho_{\$}^{\text{MCNP}}$ values were comparable until deep subcriticality.

Also, notable is that no spatial effect of detector location was observed except for that of BF3#4 detector. The results by BF3#4 detector placed near the neutron source were not reliable since $\rho_{\$}$ values were significantly overestimated in deep subcriticality (over 6\$) in both 14 MeV and spallation external neutron sources. The large error in deep subcriticality of 9\$ in BF3#1 and BF3#2 was caused by the low count rate of delayed neutrons. As an examination of detector type dependency on $\rho_{\$}$, LiFCAF fiber detector indicated almost the same $\rho_{\$}$ value compared with that by BF3#2, validating the measurement results and capability of the λ -mode calculation for $\rho_{\$}$.

4.3.2.2 Prompt Neutron Decay Constant

Measured α was compared with calculated one (α^{MCNP}) by using MCNP6.1 and deduced as follows:

$$\alpha^{\text{MCNP}} = \frac{1}{\Lambda^{\text{MCNP}}} \left\{ \left(\frac{1}{k_{\text{eff}}^{\text{case}}} - \frac{1}{k_{\text{eff}}^{\text{critical}}} \right) - \beta_{\text{eff}}^{\text{MCNP}} \right\}, \quad (4.45)$$

where Λ^{MCNP} is generation time obtained by MCNP6.1. In addition to α^{MCNP} , the prompt neutron decay constant by the ω -mode calculation with PARTISN (α^{PARTISN}) was added to compare the difference between λ -mode and ω -mode calculations, as

Table 4.13 Comparison between subcriticality of experiments and calculations (MCNP6.1 and PARTISN) (Ref. [15])

(a) Cases I-1 to I-5					
Case	Experiment	MCNP6.1	Difference* [pcm]	PARTISN	Difference* [pcm]
I-1	0.99292 ± 0.00004	0.99172 ± 0.00008	-43	0.98957	-259
I-2	0.99070 ± 0.00013	0.98948 ± 0.00010	-45	0.98723	-272
I-3	0.98440 ± 0.00013	0.98301 ± 0.00011	-63	0.97858	-514
I-4	0.98848 ± 0.00018	0.98725 ± 0.00011	-46	0.98532	-242
I-5	0.98034 ± 0.00018	0.97882 ± 0.00012	-78	0.97545	-422
(b) Cases II to VII					
Case	MCNP6.1		PARTISN	Difference** [pcm]	
II	0.99553 ± 0.00004		0.99343	211	
III	0.98344 ± 0.00004		0.98169	178	
IV	0.96918 ± 0.00004		0.96784	138	
V	0.96096 ± 0.00004		0.96048	49	
VI	0.95661 ± 0.00004		0.95585	79	
VII	0.93272 ± 0.00004		0.93334	-66	

*Difference between experiment and calculation ($(1 - \frac{k_{\text{calc}}}{k_{\text{eff}}}) \times 10^5$ [pcm])

**Difference of k_{eff} between MCNP6.1 and PARTISN ($(1 - \frac{k_{\text{eff}}^{\text{PARTISN}}}{k_{\text{eff}}^{\text{MCNP6.1}}}) \times 10^5$ [pcm])

Fig. 4.7 Comparison between measured and calculated (MCNP6.1 with ENDF/B-VII.1) subcriticalities in dollar units with 14 MeV neutrons (Ref. [15])

Fig. 4.8 Comparison between measured and calculated (MCNP6.1 with ENDF/B-VII.1) subcriticalities in dollar units with spallation neutrons (Ref. [15])

shown in Figs. 4.9 and 4.10. Here, the error of the measurement was evaluated by the fitting error. The large error was obtained in the result by BF₃#4 with spallation neutrons in Fig. 4.10 since the PNS histogram was largely influenced by the decay of the neutron source. As in ρ_s , no dependence on any external neutron source was observed in the measurements, indicating that the decay of neutron flux in the fundamental mode was measured correctly. Also, the results from all detectors were equivalent, except from BF₃#4. Furthermore, the difference in measurement methodology was around 5% between the α -fitting method and the Feynman- α method as described in Ref. [20], demonstrating that the measurement was valid. In comparing α^{MCNP} with measured α , α^{MCNP} was overestimated by 1100 1/s ($k_{\text{eff}} = 0.97$). Conversely, α^{PARTISN} agreed with the measured ones, demonstrating that the λ -mode calculation [21] has the possibility to be incapable of evaluating the α value even for target subcriticality in ADS operations through the comprehensive comparisons.

Fig. 4.9 Comparison between measured and calculated (MCNP6.1 with ENDF/B-VII.1 and PARTISN) prompt neutron decay constants in 14 MeV neutrons (Ref. [15])

Fig. 4.10 Comparison between measured and calculated (MCNP6.1 with ENDF/B-VII.1 and PARTISN) prompt neutron decay constants in spallation neutrons (Ref. [15])

4.3.3 Results and Discussion

4.3.3.1 Evaluation of $\beta_{\text{eff}}/\Lambda$

$\beta_{\text{eff}}/\Lambda$ as representative of the cores were experimentally deduced by combining α and ρ_s at BF₃#2 detector as follows:

$$\left(\frac{\beta_{\text{eff}}}{\Lambda}\right)_{\text{EXP}} = \left(\frac{\alpha^{\text{EXP}}}{1 + \rho_s^{\text{EXP}}}\right). \quad (4.46)$$

Also, calculated $\beta_{\text{eff}}/\Lambda$ values were obtained by combining adjoint-weighted effective delayed neutron fraction $\beta_{\text{eff}}^{\text{PARTISN}}$ by PARTISN, and the adjoint-weighted generation time Λ^{PARTISN} was defined as follows:

$$\beta_{\text{eff}}^{\text{PARTISN}} = \frac{\langle \phi^+(g') \chi_d(g') \nu_d(g) \Sigma_f(g) \phi(g) \rangle}{\langle \phi^+(g') \chi(g') \nu(g) \Sigma_f(g) \phi(g) \rangle}, \quad (4.47)$$

$$\Lambda^{\text{PARTISN}} = \frac{\left\langle \frac{1}{v(g)} \phi^+(g) \phi(g) \right\rangle}{\langle \phi^+(g') \chi(g') \nu(g) \Sigma_f(g) \phi(g) \rangle}, \quad (4.48)$$

where ϕ and ϕ^+ are the forward and adjoint fluxes in λ -mode or ω -mode calculations, respectively, χ and χ_d the total and delayed neutron fission spectra, respectively, ν and ν_d the number of total and delayed neutrons by fission reaction, respectively, Σ_f the effective fission cross section, v the neutron velocity, $\langle \rangle$ the integration over energy group and phase space, and g and g' the energy groups. Here, $\chi_d(g')$ was obtained by the extraction of nuclear data of ^{235}U from ENDF/B-VII.1 by NJOY99 [22]. Also, $\nu_d(g)$ was defined as follows:

$$\nu_d(g) = \nu(g) - \nu_p(g), \quad (4.49)$$

where $\nu_p(g)$, which was obtained as in the procedure for $\chi_d(g')$, is the number of prompt neutrons by the fission reaction. Furthermore, $\beta_{\text{eff}}/\Lambda$ values by using MCNP6.1 were obtained by combining kinetics parameters (evaluated with KOPT option in MCNP6.1).

By varying subcriticality, the value of $\beta_{\text{eff}}/\Lambda$ tended to decrease with deep subcriticality as shown in Fig. 4.11. Interestingly, $\beta_{\text{eff}}/\Lambda$ differed even when subcriticality was the same, although the core size was different in Cases I-2 and III. Also, the nonsignificant difference of $\beta_{\text{eff}}/\Lambda$ values was observed indicated between 14 MeV and spallation neutron sources in measurements. Thus, $\beta_{\text{eff}}/\Lambda$ was considered independent of any external neutron source, indicating that the measurement was conducted correctly to extract fundamental-mode components in the time evolution of neutron flux. In the comparison between calculations, the λ -mode calculation by PARTISN showed a bias compared with that by MCNP6.1, and the results were

Fig. 4.11 $\beta_{\text{eff}}/\Lambda$ value for α value in ω -mode for subcriticality variation (Ref. [15])

almost the same as the measured ones at shallow subcritical states (Cases I-1 to I-5). At deep subcriticality, however, the λ -mode calculations showed a large difference as compared with the experiments. Conversely, ω -mode calculations correctly evaluated the experiments in the whole range except for Case III, indicating that the kinetics parameters were not correctly evaluated by the λ -mode calculation even for target subcriticality in ADS for Case IV ($k_{\text{eff}} = 0.97$).

Here, ρ_s^{MCNP} by λ -mode calculation showed agreement with measured ρ_s even in deep subcritical states, as shown in Figs. 4.7 and 4.8. Conversely, as shown in Figs. 4.9 and 4.10, from the comparison of α values, it was observed that the large discrepancy between experiments and calculations as subcriticality becomes deeper. The discrepancy in the comparison between the experiment and the calculation was considered attributable to Λ value for estimating calculated α value by Eq. (4.45). The difference of Λ value between λ -mode and ω -mode calculations could be also explained by comparing between ρ_s and $\beta_{\text{eff}}/\Lambda$ since ρ_s values agreed between the experiment and the λ -mode calculation more accurately than the results of $\beta_{\text{eff}}/\Lambda$ (Fig. 4.11).

4.3.3.2 Discussion

The discussion is focused on the variation of each kinetics parameter by the calculations toward the subcriticality. β_{eff} and Λ were compared in terms of the λ -mode or the ω -mode calculation as shown in Fig. 4.12. The values of β_{eff} were independent of subcriticality in ω -mode calculations; however, λ -mode calculations showed a slightly increased tendency toward subcriticality. Notably, Λ values revealed a different tendency compared with β_{eff} values under λ -mode and ω -mode calculations. Furthermore, β_{eff} values were equivalent in Cases I-5 and IV; by contrast, however, Λ values were different even at almost the same subcritical states, indicating that Λ is sensitive to the size of the core and to the insertion of control rods.

Fig. 4.12 Relationship between calculated β_{eff} and Λ for subcriticality variation (Ref. [15])

In qualitative calculation property, the ω -mode calculation involves $-\alpha/v$ value in absorption term compared to the λ -mode calculation, indicating softer neutron spectrum by decreasing the total absorption cross section of $-\alpha/v$ (since lower the energy, the more increase $-\alpha/v$ value). Furthermore, the adjoint neutron spectrum is estimated softer by the λ -mode calculation to induce fission reactions, resulting in the overestimation of β_{eff} value in the λ -mode calculation since the importance of delayed neutrons is emphasized by the adjoint neutron flux. Furthermore, the different estimation of adjoint neutron spectra is also considered to influence the value of Λ since observation showed an overestimation of β_{eff} and underestimation of Λ in the λ -mode calculation. Accordingly, calculated k_{eff} by the λ -mode calculation is implied to be different from actual neutron multiplication factor (in the subcritical system) by the combined use of α , β_{eff} , and Λ since the spectrum calculation was inadequate in the λ -mode calculations for super-critical and subcritical cores.

Investigation of the cause in the variation of Λ is considered limited since the number of energy groups is insufficient for characterizing the neutron spectra of ϕ and ϕ^+ . Thus, further investigation with a higher number of energy groups is needed.

4.4 Neutron Generation Time

4.4.1 Experimental Settings

4.4.1.1 Core Configuration

Critical cores set in the A-core (Figs. A4.1 and A4.4) have polyethylene moderator and reflector rods, and three different fuel assemblies: normal “F” (Fig. A4.2a), partial “8”, and “4” (Figs. A4.2b and A4.5) corresponding to Figs. A4.1 and A4.4, respectively. The normal fuel assembly “F” is composed of 60 unit cells, and upper and lower polyethylene blocks about 23" and 21" long, respectively, in an Al sheath ($2.1 \times 2.1 \times 60''$). A unit cell is composed of two HEU fuel plates $2 \times 2''$ square and $1/8''$ thick ($1/16'' \times 2$), polyethylene (p) plate $2 \times 2''$ square and $1/8''$ thick, for normal fuel plate “F.” Numeral “8” represents a partial fuel assembly composed of eight unit cells, with two HEU fuel plates and a polyethylene plate as in the normal fuel assembly, providing 52 unit cells of two Al plates $2 \times 2''$ square and $1/8''$ thick ($1/16'' \times 2$), and $1/8''$ polyethylene plates. Also, the numeral “4” corresponds to four unit cells of fuel assembly with 56 unit cells composed of Al and polyethylene plates.

4.4.1.2 Experiments

In the two critical cores [23] shown in Figs. A4.1 and A4.4, criticality was reached at positions of control (C1, C2, and C3) and safety (S4, S5, and S6) rods for the total number of HEU fuel plates: 3016 and 3008, as shown in Tables 4.14 and 4.15, respec-

Table 4.14 Positions of control and safety rods at critical state [mm] (# of HEU plates: 3016; critical core in Fig. A4.1) (Ref. [23])

Control rod			Safety rod		
C1	C2	C3	S4	S5	S6
1200.00	1200.00	630.01	1200.00	1200.00	1200.00

(1200 [mm]: Position of upper limit)

Table 4.15 Positions of control and safety rods at critical state [mm] (# of HEU plates: 3008; critical core in Fig. A4.4) (Ref. [23])

Control rod			Safety rod		
C1	C2	C3	S4	S5	S6
1200.00	723.31	1200.00	1200.00	1200.00	1200.00

(1200 [mm]: Position of upper limit)

tively. Excess reactivity and control rod worth in pcm units were then experimentally obtained by the positive period method and the rod drop method, respectively, with the use of two kinetics parameters β_{eff} and ℓ estimated by numerical calculations. Here, excess reactivity obtained experimentally was about 250 and 100 pcm in Figs. A4.1 and A4.2, respectively, and control rod worth of C1, C2 (Fig. A4.1 or C3 in Fig. A4.2), and C3 (Fig. A4.2 or C2 in Fig. A4.1) was about 880, 150, and 520 pcm, respectively.

In the ADS core (3000 HEU fuel plates) at the subcritical state [23], deuteron beams were injected onto a tritium target set at (14–15, X) in Fig. A4.3a. In the same subcritical core, another external neutron source of 100 MeV proton beams was injected onto a lead-bismuth (Pb–Bi) target set at (15, H) in Fig. A4.6a. Two different external neutron sources (14 MeV neutrons; 100 MeV protons: spallation neutrons) were separately injected into the subcritical core: deuteron beams, 0.1 mA current, 20 Hz repetition rate, 97 μs width, and $1 \times 10^5 \text{ s}^{-1}$ neutron yield; 100 MeV protons, 0.1 nA current, 20 Hz repetition rate, 100 ns width, and $1 \times 10^7 \text{ s}^{-1}$ neutron yield. The Pb–Bi target was 50 mm in diameter and 18 mm thick. In a series of ADS experiments and during the injection of an external neutron source, α and ρ_s were measured by the least-square fitting method and the extended area ratio method [1], respectively.

Moreover, to examine the effects of detector position dependence [24] and external neutron source spectrum on measurement results, three optical fiber detectors [16] were set at (14–15, L–M); (13–14, K–L); (12–13, J–K) in Fig. A4.3a and (15–16, O–Q); (16–17, Q–R); (18–19, S–T) in Fig. A4.6a; also, three BF-3 detectors were at (15, H); (11, I); (11, M) in Fig. A4.3a and (15, U); (11, T); (10, O) in Fig. A4.6a.

4.4.1.3 Kinetics Parameters

For the α -fitting method [25], α is easily deduced by combining subcriticality ($-\rho_{\text{pcm}}$) in pcm units, β_{eff} and Λ as follows:

$$\alpha = \frac{\beta_{\text{eff}} - \rho_{\text{pcm}}}{\Lambda}. \quad (4.50)$$

The relationship between ρ_{pcm} and $\rho_{\text{\$}}$ is expressed with the use of β_{eff} as follows:

$$\rho_{\text{pcm}} = \rho_{\text{\$}} \beta_{\text{eff}}. \quad (4.51)$$

Substituting Eq. (4.51) for Eq. (4.50), $\beta_{\text{eff}}/\Lambda$ can be expressed as follows:

$$\frac{\beta_{\text{eff}}}{\Lambda} = \frac{\alpha}{1 - \rho_{\text{\$}}}. \quad (4.52)$$

The value of $\beta_{\text{eff}}/\Lambda$ is easily deduced from the experimental results of α and $\rho_{\text{\$}}$.

4.4.2 Results and Discussion

4.4.2.1 Eigenvalue Calculations

For transport, numerical calculations were performed by the Monte Carlo transport code, MCNP6.1 together with nuclear data libraries, JENDL-4.0 and ENDF/B-VII.1. Here, in MCNP6.1, since the effects of reactivity by neutron detectors (optical fiber detectors, BF-3 detectors, fission chambers, and uncompensated ionization chambers) and control (safety) rods are not negligible, neutron detectors and control (safety) rods were modeled precisely in the simulated geometry and transport calculations. The precision of numerical criticality in pcm units was attained by the eigenvalue calculations with a total of 5×10^8 histories (1×10^5 histories per cycle, 5×10^3 active cycles, and 1×10^2 skip cycles) and a statistical error of about 4 pcm. Also, main kinetics parameters, β_{eff} , Λ and Rossi- α (termed $\beta_{\text{eff}}/\Lambda$ in MCNP6.1) values were obtained by the eigenvalue calculations, when obtaining the effective multiplication factor by the k-code option.

To confirm the numerical precision of eigenvalue calculations by MCNP6.1, excess reactivity and control rod worth (C1, C2, and C3) were compared with those obtained from the experiments by the positive period method and the rod drop method, respectively. The MCNP eigenvalue calculations in the critical state are not always unit, differing from the experimental results in critical state obtained by nuclear data accuracy and uniformed number density approximation in core materials, although a critical core configuration is more closely simulated by MCNP. Thus, excess reactivity $\rho_{\text{excess}}^{\text{cal}}$ in pcm units was numerically deduced by the difference between two

effective multiplication factors k_{critical} and $k_{\text{super-critical}}$, in critical and super-critical (clean) cores, respectively, as follows:

$$\rho_{\text{excess}}^{\text{cal}} = \frac{1}{k_{\text{critical}}} - \frac{1}{k_{\text{super-critical}}}. \quad (4.53)$$

Also, control rod worth was numerically deduced by the difference between the critical and the subcritical (rod insertion) cores, as in Eq. (4.53).

Using the kinetics parameters obtained by MCNP6.1, experimental reactivity ρ^{exp} was deduced and compared with numerical reactivity ρ^{cal} obtained by JENDL-4.0 and ENDF/B-VII.1, as shown in Tables 4.16 and 4.17, with the consideration of statistical errors of neutron counts obtained from the neutron detectors and processing of error propagation caused by experimental analyses. For the critical cores (3016 and 3008 HEU plates in Tables 4.14 and 4.15) shown in Figs. A4.1 and A4.4, respectively, numerical reactivity obtained by JENDL-4.0 revealed a fairly good agreement with the experimental reactivity within a relative difference of around 5% through the C/E (calculation/experiment) value of excess reactivity and control rod worth, as shown in Tables 4.16 and 4.18. In terms of ENDF/B-VII.1, the difference between numerical and experimental reactivity was relatively large over 10%, as shown in Tables 4.17 and 4.19. The difference between β_{eff} values by JENDL-4.0 and ENDF/B-VII.1 was attributable to numerical results by the MCNP calculations in fraction β_i ($i = 1-6$: precursor group of delayed neutrons) and decay constant λ_i of delayed neutrons, as shown in Figs. 4.13 and 4.14, respectively.

Table 4.16 Comparison between measured and calculated reactivity [pcm] with JENDL-4.0 (# of HEU plates: 3016; critical core in Fig. A4.1 and Table 4.14) (Ref. [23])

	$\rho_{\text{J40}}^{\text{cal}}$	$\rho_{\text{J40}}^{\text{exp}}$	C/E ($\rho_{\text{J40}}^{\text{cal}}/\rho_{\text{J40}}^{\text{exp}}$)
Excess reactivity	259.9 ± 5.7	271.7 ± 0.1	0.96 ± 0.02
C1 rod worth	876.1 ± 5.7	867.4 ± 3.9	1.01 ± 0.01
C2 rod worth	154.8 ± 5.7	142.8 ± 3.5	1.08 ± 0.05
C3 rod worth	529.2 ± 5.7	506.4 ± 4.1	1.05 ± 0.01

MCNP6.1 with JENDL-4.0: $\beta_{\text{eff}} = 813 \pm 10$ [pcm], $\Lambda = 31.67 \pm 0.07$ [μs]

Table 4.17 Comparison between measured and calculated reactivity [pcm] with ENDF/B-VII.1 (# of HEU plates: 3016; critical core in Fig. A4.1 and Table 4.14) (Ref. [23])

	$\rho_{\text{E71}}^{\text{cal}}$	$\rho_{\text{E71}}^{\text{exp}}$	C/E ($\rho_{\text{E71}}^{\text{cal}}/\rho_{\text{E71}}^{\text{exp}}$)
Excess reactivity	253.9 ± 5.6	229.8 ± 0.3	1.10 ± 0.02
C1 rod worth	862.5 ± 5.7	718.8 ± 3.2	1.20 ± 0.01
C2 rod worth	138.2 ± 5.6	118.3 ± 2.9	1.17 ± 0.05
C3 rod worth	513.1 ± 5.7	419.6 ± 4.5	1.22 ± 0.01

MCNP6.1 with ENDF/B-VII.1: $\beta_{\text{eff}} = 801 \pm 10$ [pcm], $\Lambda = 31.26 \pm 0.07$ [μs]

Table 4.18 Comparison between measured and calculated reactivity [pcm] with JENDL-4.0 (# of HEU plates: 3008; critical core in Fig. A4.4 and Table 4.15) (Ref. [23])

	ρ_{J40}^{cal}	ρ_{J40}^{exp}	C/E ($\rho_{J40}^{cal}/\rho_{J40}^{exp}$)
Excess reactivity	106.1 ± 5.7	100.2 ± 1.7	1.06 ± 0.06

MCNP6.1 with JENDL-4.0: $\beta_{eff} = 812 \pm 10$ [pcm], $\Lambda = 31.57 \pm 0.07$ [μ s]

Table 4.19 Comparison between measured and calculated reactivity [pcm] with ENDF/B-VII.1 (# of HEU plates: 3008; critical core in Fig. A4.4 and Table 4.15) (Ref. [23])

	ρ_{E71}^{cal}	ρ_{E71}^{exp}	C/E ($\rho_{E71}^{cal}/\rho_{E71}^{exp}$)
Excess reactivity	97.6 ± 5.6	84.7 ± 1.5	1.15 ± 0.07

MCNP6.1 with ENDF/B-VII.1: $\beta_{eff} = 802 \pm 10$ [pcm], $\Lambda = 31.30 \pm 0.07$ [μ s]

Fig. 4.13 Comparison between fraction β_i ($i = 1-6$) by JENDL-4.0 and ENDF/B-VII.1 (# of HEU plates: 3016; critical state in Fig. A4.1 and Table 4.14) (Ref. [23])

Fig. 4.14 Comparison between decay constant λ_i ($i = 1-6$) by JENDL-4.0 and ENDF/B-VII.1 (# of HEU plates: 3016; critical state in Fig. A4.1 and Table 4.14) (Ref. [23])

As shown in Tables 4.16, 4.17, 4.18 and 4.19, numerical results (eigenvalue calculations) by MCNP6.1 with JENDL-4.0 demonstrated good agreement with the experimental data of reactivity in the critical cores. JENDL-4.0 was taken as a reference library by comparing it with ENDF/B-VII.1.

4.4.2.2 Experimental Analyses of $\beta_{\text{eff}}/\Lambda$

Kinetics parameters β_{eff} , Λ and Rossi- α , and k_{eff} values were obtained by the MCNP eigenvalue calculations and compared with those of JENDL-4.0 and ENDF/B-VII.1, for critical and near-critical states, as shown in Tables 4.20, 4.21, and 4.22. For the two critical cores shown in Figs. A4.1 and A4.4, JENDL-4.0 demonstrated a small difference between the super-critical (Table 4.20) and the critical (Table 4.21) states,

Table 4.20 Comparison between kinetic parameters by MCNP6.1 with JENDL-4.0 and ENDF/B-VII.1 (# of HEU plates: 3016 in Fig. A4.1) (Ref. [23])

	Parameter	JENDL-4.0	ENDF/B-VII.1
Super-critical core (clean core)	β_{eff} [pcm]	813 ± 10	801 ± 10
	Λ [μs]	31.67 ± 0.07	31.26 ± 0.07
	Rossi- α [s^{-1}]	256.89 ± 3.11	256.18 ± 3.11
Critical core (partial insertion of C3 rod)	β_{eff} [pcm]	818 ± 10	800 ± 10
	Λ [μs]	30.91 ± 0.07	30.69 ± 0.07
	Rossi- α [s^{-1}]	264.55 ± 3.23	260.66 ± 3.17

Table 4.21 Comparison between kinetic parameters by MCNP6.1 with JENDL-4.0 and ENDF/B-VII.1 (# of HEU plates: 3008 in Fig. A4.4) (Ref. [23])

	Parameter	JENDL-4.0	ENDF/B-VII.1
Super-critical core (clean core)	β_{eff} [pcm]	812 ± 10	802 ± 10
	Λ [μs]	31.57 ± 0.07	31.30 ± 0.07
	Rossi- α [s^{-1}]	257.22 ± 3.11	256.35 ± 3.11
Critical core (partial insertion of C2 rod)	β_{eff} [pcm]	821 ± 10	801 ± 10
	Λ [μs]	31.33 ± 0.07	30.03 ± 0.07
	Rossi- α [s^{-1}]	262.22 ± 3.17	258.09 ± 3.15

Table 4.22 Comparison between kinetic parameters by MCNP6.1 with JENDL-4.0 and ENDF/B-VII.1 (# of HEU plates: 3000; subcritical core in Figs. A4.3a and A4.6a) (Ref. [23])

	Parameter	JENDL-4.0	ENDF/B-VII.1
Subcritical core (clean core)	β_{eff} [pcm]	806 ± 10	796 ± 10
	Λ [μs]	31.71 ± 0.07	31.42 ± 0.07
	Rossi- α [s^{-1}]	254.05 ± 3.07	253.35 ± 3.09

because of the slight difference in the number of fuel plates: 3016 and 3008. Also, in terms of ENDF/B-VII.1, the three kinetics parameters were almost the same in the two states.

At the subcritical state with 3000 fuel plates, three kinetics parameters showed a meaningful change from the critical state, although the subcriticality was very small and around the criticality (Table 4.22). Kinetic parameters α and $\rho_{\S}$ were then obtained by the extended area ratio method and the least-square fitting method, respectively, by varying the external neutron source: 14 MeV neutrons (Table 4.23) and spallation neutrons (Table 4.24). Also, as shown in Tables 4.23 and 4.24, detector position dependency was revealed interestingly: the results of Fiber #1 and BF-3 #3 were rather good, and on the contrary, those of Fiber #2, BF-3 #1, and BF-3 #3 looked problematic.

Table 4.23 Measured prompt neutron decay constants α [s^{-1}] deduced by least-square fitting method, subcriticality $\rho_{\S}$ [$\S$] (dollar units) by extended area ratio method, and $\beta_{\text{eff}}/\Lambda$ [s^{-1}] by α -fitting method (# of HEU plates: 3000; subcritical core with 14 MeV neutrons in Fig. A4.3a) (Ref. [23])

Detector	α [s^{-1}]	$\rho_{\S}$ [$\S$]	$(\beta_{\text{eff}}/\Lambda)^{\text{exp}}$ [s^{-1}]	C/E
BF-3 #1	294.78 ± 29.27	0.0228 ± 0.0070	288.20 ± 93.30	0.88 ± 0.29
BF-3 #2	226.96 ± 23.31	0.0158 ± 0.0054	223.44 ± 79.47	1.14 ± 0.41
BF-3 #3	258.58 ± 12.34	0.0194 ± 0.0027	253.66 ± 37.32	1.00 ± 0.15
Fiber #1	248.38 ± 39.04	0.0207 ± 0.0056	243.36 ± 72.07	1.04 ± 0.31
Fiber #2	229.95 ± 14.51	0.0200 ± 0.0032	225.45 ± 38.73	1.13 ± 0.19
Fiber #3	273.99 ± 26.24	0.0261 ± 0.0056	267.02 ± 62.56	0.95 ± 0.22

MCNP6.1 with JENDL-4.0: $\beta_{\text{eff}} = 806 \pm 10$ [pcm], $\Lambda = 31.71 \pm 0.07$ [μs], $(\beta_{\text{eff}}/\Lambda)_{\text{J40}}^{\text{cal}}$ (MCNP6.1) = 254.05 ± 3.07 [s^{-1}]; C/E = $(\beta_{\text{eff}}/\Lambda)_{\text{J40}}^{\text{cal}} / (\beta_{\text{eff}}/\Lambda)^{\text{exp}}$

Table 4.24 Measured prompt neutron decay constants α [s^{-1}] deduced by least-square fitting method, subcriticality $\rho_{\S}$ [$\S$] (dollar units) by extended area ratio method, and $\beta_{\text{eff}}/\Lambda$ [s^{-1}] by α -fitting method (# of HEU plates: 3000; subcritical core with spallation neutrons in Fig. A4.6a) (Ref. [23])

Detector	α [s^{-1}]	$\rho_{\S}$ [$\S$]	$(\beta_{\text{eff}}/\Lambda)^{\text{exp}}$ [s^{-1}]	C/E
BF-3 #1	276.25 ± 7.17	0.0900 ± 0.0011	253.43 ± 7.25	1.00 ± 0.03
BF-3 #2	300.62 ± 7.03	0.0966 ± 0.0011	274.13 ± 7.15	0.93 ± 0.03
BF-3 #3	259.96 ± 6.00	0.0899 ± 0.0011	238.53 ± 6.18	1.07 ± 0.03
Fiber #1	283.79 ± 15.38	0.0973 ± 0.0025	258.62 ± 15.49	0.98 ± 0.06
Fiber #2	360.05 ± 64.85	0.1139 ± 0.0087	323.24 ± 63.19	0.79 ± 0.15
Fiber #3	269.83 ± 26.36	0.1392 ± 0.0039	236.85 ± 24.04	1.07 ± 0.11

MCNP6.1 with JENDL-4.0: $\beta_{\text{eff}} = 806 \pm 10$ [pcm], $\Lambda = 31.71 \pm 0.07$ [μs], $(\beta_{\text{eff}}/\Lambda)_{\text{J40}}^{\text{cal}}$ (MCNP6.1) = 254.05 ± 3.07 [s^{-1}]; C/E = $(\beta_{\text{eff}}/\Lambda)_{\text{J40}}^{\text{cal}} / (\beta_{\text{eff}}/\Lambda)^{\text{exp}}$

On the basis of the experimental results of α and ρ_s , the value of $(\beta_{eff}/\Lambda)^{exp}$ was experimentally deduced by Eq. (4.52) for 14 MeV neutrons (Table 4.23) and spallation neutrons (Table 4.24), and compared with that of $(\beta_{eff}/\Lambda)_{140}^{cal}$ obtained by MCNP6.1 together with JENDL-4.0. With the 14 MeV neutrons, since subcriticality in pcm units was found to be near critical, about 16.7 ± 4.5 pcm, the value of $(\beta_{eff}/\Lambda)_{140}^{cal}$ was observed the same as that of α . The result was considered valid at a shallow subcritical state. Among optical fibers and BF-3 detectors, Fiber #1 revealed a fairly good agreement with the MCNP calculation within a relative difference of 4% in the C/E value (Table 4.23), which was attributable to the location of Fiber #1 near the center of the core (Fig. A4.1): the position dependence caused by the spatial effect was very small on the experimental results of α and ρ_s . Also, with the spallation neutrons (Table 4.24), Fiber #1 demonstrated the same accuracy about 2% in the C/E value as with 14 MeV neutrons, although the subcriticality in pcm units was about 78.4 ± 2.2 pcm. The value of $(\beta_{eff}/\Lambda)^{exp}$ was found to be experimentally valid in the deduction of kinetics parameters, through a comparison between the experiments and the MCNP6.1 calculations with JENDL-4.0 (Tables 4.23 and 4.24).

4.4.2.3 Discussion

Experimental analyses of the ADS with spallation neutrons at KUCA have clearly demonstrated that the value of β_{eff} has a little effect on the evaluation of subcriticality in pcm units converted from that in dollar units, when compared with the results of numerical subcriticality in pcm units [2]. In the present study, considering that the values of β_{eff} are almost the same in the near-critical configurations, particular attention was directed to kinetic parameter Λ obtained by combining α and ρ_s .

The values of β_{eff} by MCNP6.1 with JENDL-4.0 in the super-critical (3008 HEU fuel plates) and subcritical (3000 plates) configurations were 812 ± 10 and 806 ± 10 pcm as shown in Tables 4.21 and 4.22, respectively. Since the values of β_{eff} are almost the same and within the allowance of experimental uncertainty, the ratio of $(\beta_{eff}/\Lambda)_{super-critical}^{cal}$ and $(\beta_{eff}/\Lambda)_{subcritical}^{cal}$ in the near-critical configurations is approximated as follows:

$$(\beta_{eff}/\Lambda)_{super-critical}^{cal}/(\beta_{eff}/\Lambda)_{subcritical}^{cal} \approx (\Lambda_{subcritical}/\Lambda_{super-critical})^{cal}. \quad (4.54)$$

Here, Eq. (4.54) is defined as “Lambda ratio” that is a relative value of Λ in the near-critical configurations. From the results of Tables 4.23 and 4.24, assuming that the value of $(\beta_{eff}/\Lambda)_{subcritical}^{cal}$ in the subcritical state by MCNP6.1 is equal to that of $(\beta_{eff}/\Lambda)^{exp}$ by the experiment, Eq. (4.54) can finally be written as follows, by substituting $(\beta_{eff}/\Lambda)_{subcritical}^{cal}$ for $(\beta_{eff}/\Lambda)^{exp}$ and applying again the assumption with respect to β_{eff} in Eq. (4.54):

$$(\beta_{eff}/\Lambda)_{super-critical}^{cal}/(\beta_{eff}/\Lambda)^{exp} \approx \Lambda^{exp}/\Lambda_{super-critical}^{cal}. \quad (4.55)$$

On the basis of the validity of $(\beta_{\text{eff}}/\Lambda)^{\text{exp}}$ mentioned in Sect. 4.4.2, Eq. (4.55) can be interpreted as a kind of Lambda ratio between the experiments and the calculations, with the combined use of $(\beta_{\text{eff}}/\Lambda)^{\text{cal}}_{\text{super-critical}}$ and $(\beta_{\text{eff}}/\Lambda)^{\text{exp}}$. Actually, the value of $(\beta_{\text{eff}}/\Lambda)^{\text{exp}}$ obtained by the ADS experiments at KUCA can be used as an index of Λ in the near-critical configurations, and is, indeed, expected to be applied for the validation of Λ , by stochastic and deterministic calculations, at near-critical configurations in a thermal neutron spectrum core.

4.5 Conclusion

A calculation methodology of $\beta_{\text{source}}^{\text{RR}}$ by the k-ratio method with an external neutron source has been proposed for the subcriticality estimation. Subcriticality measurements were carried at the KUCA-A cores to examine its applicability of the proposed calculation methodology by varying the subcriticality and the external neutron source. To confirm the validity of the proposed methodology, the eigenvalue calculations were firstly performed to estimate the multiplication factor k_{RR} with reaction rates by MCNPX. Further, $\beta_{\text{eigen}}^{\text{RR}}$ by the k-ratio method with reaction rates was compared with the reference ones obtained by MCNP6.1, respectively. For the estimation of subcriticality by the fixed-source calculations, $\beta_{\text{source}}^{\text{RR}}$ was observed to be dependent on the variation of subcriticality and external neutron source. Finally, the subcriticality with $\beta_{\text{source}}^{\text{RR}}$ was acquired well ranging between about 0.99 and 0.97 in k_{eff} and revealed the comparative tendency in the deep subcriticality through the estimation with the use of $\beta_{\text{source}}^{\text{RR}}$ obtained by the proposed calculation methodology.

In the methodology for estimating the value of β_{eff} , the Rossi- α method was applied to the neutron noise analyses at the PNS experiments with pulsed spallation neutrons. With the fitting curve obtained by neutron noise analyses, the signals from two detectors installed at the center of the core (optical fiber detector) and outside the core (BF_3 detector) and the correction factors, β_{eff} was measured and compared with the value calculated by MCNP6.1. The optical fiber detector located at the core center showed that the accuracy of the measured value, compared with the calculated one, was within a relative difference of about 13% in the subcritical range of the ADS operation around $k_{\text{eff}} = 0.93$. The result with BF_3 detector installed outside the core was not compared with the calculated one because of the low accuracy attributed to the uncorrelated probability. The applicability of the measurement methodology based on the Rossi- α method was demonstrated by the comparison between calculated and measured β_{eff} values in ADS experiments with spallation neutrons.

ADS experiments were carried out to evaluate $\beta_{\text{eff}}/\Lambda$ values by varying detector type, detector position, external neutron source, and subcriticality. The capability of λ -mode and ω -mode calculations was examined by comparing directly measured ρ_s , α , and $\beta_{\text{eff}}/\Lambda$ in the PNS experiment with the target subcriticality of ADS ranging between 500 and 7500 pcm. The measurement of ρ_s and α indicated slight dependence on an external neutron source but not on any spatial effect except for the BF_3 detector located near the neutron source. In the experimental analyses, calculated ρ_s (λ -mode and ω -mode) showed good agreement with measured ρ_s within the whole range of subcriticality; however, the value of α by the λ -mode calculation showed a difference in the experiment at deep subcriticality. Conversely, α obtained by the ω -mode calculation agreed with the experiments. The calculated results of $\beta_{\text{eff}}/\Lambda$ were compared with the measured ones to examine their capability under subcriticality variation; consequently, an agreement was observed between the experiments and ω -mode calculations under a wide range of subcriticality. Notably, however, the λ -mode calculations differed from the experiments even under slight subcriticality, implying the necessity of introducing ω -mode calculations in ADS design for evaluating actual neutron multiplication factor in the subcritical system and kinetics parameters.

Main kinetics parameters, α and ρ_s , were experimentally obtained from the KUCA core, and β_{eff} and Λ , were numerically validated by MCNP6.1, at the near-critical configurations: super-critical and subcritical states. The experimental value of $(\beta_{\text{eff}}/\Lambda)^{\text{exp}}$ was then available for use as an index of Λ in the near-critical configurations, with an attempt at the validation of Λ by the numerical calculations. From the results of experimental and numerical analyses, the importance of the experimental value of $(\beta_{\text{eff}}/\Lambda)^{\text{exp}}$ was emphasized for the verification of Λ , since the kinetics parameters were successfully obtained from the clean cores of near-critical configurations (super-critical and subcritical states) in the thermal spectrum core.

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