Chapter 3

Reactor Kinetics

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Abstract In static and kinetic experimental analyses, the reactivity effect of introducing a neutron guide has been examined with various materials and adjustments of the beam window. With the objective of improving the KUCA core characteristics, the implementation of the neutron guide is predicted to increase the fast neutrons in directing the fuel region. With regard to the kinetic characteristics, the subcriticality and the prompt neutron decay constant are monitored for several core configurations and detector positions. The KUCA core is equipped to make locally a hard spectrum core region with the combined use of ^{235}U fuel, a polyethylene moderator, and a Pb–Bi reflector for criticality. In this study, the first attempt is made to examine experimentally the characteristics of kinetics parameters in ADS comprised of ^{235}U -fueled and Pb–Bi-zoned core, and spallation neutrons generated by an injection of 100 MeV protons onto the solid Pb–Bi target. Online monitoring of reactivity has been deduced in real time by the inverse kinetic method on the basis of the one-point kinetic equation with measured neutron signals in the core. Here, measurements by the one-point kinetic equation are validated through the subcriticality evaluation with the PNS histogram and the methodology by the inhour equation.

Keywords α -fitting method · Pulsed-neutron source method · Inverse kinetic method · Kalman filter method

3.1 α -Fitting Method

3.1.1 *Experimental Settings*

3.1.1.1 Core Configurations

For safety reasons particular to KUCA, the tritium target is located not at the center of the core but at a peripheral position in the critical assembly. This location is very

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different from the current design of ADS where a target is located at the center of the core for effective utilization of the generated neutrons. Consequently, the introduction of a neutron guide is requisite for effectively directing the high-energy neutrons generated from the tritium target to the center of the core for experiments on ADS with 14 MeV neutrons. The neutron guide, which is very similar to the neutron shield and the beam duct [1, 2], is composed of several shielding materials, including iron, boron, polyethylene, the beam duct, and a special fuel assembly with a void.

Five major core settings are presented here: a core without the neutron guide (Fig. 3.1a; Reference core: Case 1); a core including only streaming void (SV) in the fuel region (Fig. 3.1b; SV core: Case 2); and three cores with the neutron guide including SV (Fig. 3.1c–f; neutron-guided core with beam window: Cases 3, 4, 5, and 6, respectively). Numerals 12, 20, 22, 26, and 24 correspond to fuel plates in the partial assembly/ies used to reach criticality (Fig. 3.1a–f, respectively).

Each fuel assembly was set in a $2.1'' \times 2.1''$ and 1.5 mm thick aluminum (Al) sheath; the cross section of the elements within the assembly was $2'' \times 2''$. The fuel assemblies constituting the core are reproduced in Fig. 3.2. The standard fuel assembly shown (F; Fig. 3.2a) was composed of 36 unit cells of $1/16''$ thick and a 93 wt% enriched uranium plate with Al clad and two ($1/8''$ and $1/4''$) thick polyethylene plates. The active height of the core was about 16'', with additional about 23'' and 21'' upper and lower polyethylene reflectors, respectively. Case 1 (Fig. 3.1a) was composed of 20 regular fuel assemblies and one partial fuel assembly of 12 fuel unit cells. In Cases 2–6 (Fig. 3.1b–f, respectively), the fuel region consisted of 18 regular fuel assemblies, SV assembly composed of one $5.08 \times 5.08 \times 5.08$ cm center void (Fig. 3.2b), 32 fuel unit cells, and two partial fuel assemblies. Details of the partial fuel assembly of (14, M and 16, M) used for Case 5 are presented in Fig. 3.2e. The active part was centered with the rest of the core using an Al cell identical to the fuel cell but with Al replacing the fuel plates.

The purpose of the neutron guide was to reproduce the conditions of a high-energy neutron beam entering the fuel region from an isotropic source. Thus, the role of the SV assemblies was to direct the highest possible number of the high-energy neutrons generated in the target to the center of the fuel region, in order to improve neutron multiplication. Moreover, to reproduce a high-energy source, it was necessary to reduce the thermal component of the external neutron source, i.e., the neutrons were moderated before they reached the fuel region. This was achieved by shielding unnecessary fast neutrons and by capturing parasite thermal neutrons. For deflecting unnecessary fast neutrons, the close vicinity in front of the target included an ion block around the guide void to shield the fast neutrons by inelastic scattering. For capturing parasite thermal neutrons, polyethylene blocks containing 10 wt% boron around the guide void were included around the Fe shielding near the target and in the two rows next to the assemblies. The rest of the neutron guide consisted of polyethylene assemblies and one void space. The detailed composition of the neutron guide was presented in Ref. [1].

14 MeV neutrons were produced with a yield of about $8 \times 10^8 \text{ s}^{-1}$ from the tritium target in pulsed mode. The duty ratio and the duration of irradiation were adjusted

(a) Core configuration (Reference core: Case 1)

(b) Case 2

(c) Case 3

Fig. 3.1 ADS core configurations with 14 MeV neutrons shown in Table 3.1(Ref. [2])

22 Partial fuel (3/8"P22EU) s' Void × 1 + polyethylene

(d) Case 4

26 Partial fuel (3/8"P26EU)

(e) Case 5

24 Partial fuel (3/8"P24EU) ○ 1/2% BF₃ detector

(f) Case 6

Fig. 3.1 (continued)

(a) Standard fuel assembly (3/8''P36EU)

(b) SV fuel assembly

(c) Partial fuel assembly (26)

Fig. 3.2 Fuel assemblies (Ref. [2])

depending on the subcriticality and the requisite measurements. The duty ratio was limited to 1% as a design limit.

3.1.1.2 Kinetics Parameters

The principle of the optical fiber neutron detector [3, 4] is to have neutrons interact with a neutron converter material. The reaction product can then produce photons in a scintillating material which is extracted through a plastic optical fiber, multiplied into a photo-multiplier, and converted to electrical signals. In the present experiments, detectors were formed by a mixture of lithium-6 (^6Li) enriched LiF and ZnS(Ag) scintillator pasted at the tip of a 1 mm diameter plastic optical fiber. ^6Li was selected for its large $^6\text{Li}(n, t)^4\text{He}$ cross sections for thermal neutrons.

The main advantages of the optical fiber neutron detector are not only its relative simplicity and low building cost, but also its very small size, which allows it to be used in small cores such as those in KUCA with negligible perturbation. The drawbacks include low sensibility because of a very small quantity of reacting material and the treatment of the signal required to remove as much as possible of the noise from γ -ray interferences without losing too much of the valuable signal. A schema of the detection settings and size references is shown in Fig. 3.3. Levels of the thermal neutron flux in the measurements were between 10^6 and $10^4 \text{ s}^{-1} \text{ cm}^{-2}$, for the duration of irradiation in hours.

To provide information on detector position dependency of the measurements, each core was set with three detectors: Fiber #1 in the fuel region, Fiber #2 in the boundary region between the standard and partial fuel assemblies, and Fiber #3 in the neutron guide region (Fig. 3.4). Moreover, another fiber with a ThO_2 scintillator being fission reactive to the high-energy neutrons was used as a monitor of source intensity of 14 MeV neutrons. The quantity of neutron converter and scintillating material, in the order of milligram, cannot be made identical for all detectors and, thus, precludes making an absolute comparison between count rates.

Fig. 3.3 Schema of an optical fiber detection system (Ref. [2])

Fig. 3.4 Top view of the configuration of A-core kinetic experiments shown in Table 3.1 (Ref. [2])

Fig. 3.5 Comparison of measured and calculated prompt neutron decay constants (Ref. [2])

Using prompt neutron performance (Fig. 3.5), the prompt neutron decay constant was deduced by the least-square fitting of the time distribution of the reaction rates to an exponential function over the time optimal duration. Subcriticality was deduced from the prompt neutron decay constant by the extrapolated area ratio method [5]. For normalization and comparison between experimental integral results, the duration of irradiation and the duty ratio of the beam were used.

3.1.2 Numerical Simulations

The numerical calculations were executed with the Monte Carlo multi-particle transport code, MCNP-4C3 [6]. The effect on neutronic parameters resulting from the difference between nuclear data libraries was evaluated by considering JENDL-3.3 [7] and ENDF/B-VI.2 [8] for transport. Dosimetry files JENDL-3.1 [9] and ENDF/B-V were used for reaction rate calculation, regardless of the library used for transport. The source was represented by a 14 MeV neutron punctual isotropic source. Since the effects of their reactivity are not negligible, the irradiation samples and the In wire were included in the simulated geometry and transport calculation; reaction rates were deduced from tallies taken in a similar way as in the static experiments. Although better in the core region, an overall statistical error of 5% was retained in the reaction rate in the presented results. The results of eigenvalue calculations were obtained after 2,000 active cycles of 10,000 histories each. The deduced subcriticalities have statistical errors of 0.02 % $\Delta k/k$.

Kinetics calculations were conducted using MCNP-4C3 with JENDL-3.3. Since such calculations are known to have a tendency to overestimate effective multiplication factors, reactivity adjustment was taken into consideration before conducting source calculations: the density of the fuel was artificially reduced by 5%. This adjustment was estimated in the reflected core such that a critical state calculation gives a k_{eff} equivalent to 1 (in effect 0.99985 ± 0.00025).

For the kinetics calculations, time tallies were taken at the equivalent detector position, without the actual inclusion of the detector in the calculation geometry. The following is an alternative for the preceding highlighted part: Point detectors with a sphere of exclusion 1 mm in diameter, as well as a flux tally in a 5 cm long and 1 mm in diameter cylindrical volume, were centered at the detector position for estimating the validity of the use of the point detector tally and the optimization of calculation time.

3.1.3 Results and Discussion

3.1.3.1 Subcriticality

Positions of the control and safety rods in Cores 1–9 are shown in Table 3.1. The results of measured and calculated subcriticalities are presented in Table 3.2. A comparison of the experiments and the calculations demonstrates the ability of MCNP calculations to reproduce subcriticality levels within 10% C/E. The simultaneous changes in the irradiation samples, the fuel mass, and the core geometry limit precise conclusions on possible influences on the precision of calculations. The apparent better C/E values for ENDF/B-VI.2 are not conclusive when a 5% relative experimental error is taken into account. Namely, in Cases 1 and 3, there was found to be a discrepancy of about 10% error between two libraries, due to no irradiation sample in (15, K; Fig. 3.1c) and a large size window in front of fuel

Table 3.1 Control and safety rod positions in Cores 1–9 (dimension in mm) (Ref. [2])

Core	Case	C1	C2	C3	S4	S5	S6
1	1	0.00	0.00	0.00	1200.00	1200.00	1200.00
2	2	0.00	0.00	0.00	1200.00	1200.00	1200.00
3	3	0.00	0.00	0.00	1200.00	1200.00	1200.00
4	4	0.00	0.00	0.00	1200.00	1200.00	1200.00
5	5	0.00	0.00	0.00	1200.00	1200.00	1200.00
6	10	0.00	0.00	0.00	1200.00	1200.00	1200.00
	11	0.00	0.00	0.00	0.00	0.00	0.00
7	12	1200.00	1200.00	1200.00	1200.00	1200.00	1200.00
	13	0.00	0.00	0.00	1200.00	1200.00	1200.00
	14	0.00	0.00	0.00	0.00	0.00	0.00
8	15	1200.00	1200.00	1200.00	1200.00	1200.00	1200.00
	16	0.00	0.00	0.00	1200.00	1200.00	1200.00
	17	0.00	0.00	0.00	0.00	0.00	0.00
9	18	1200.00	1200.00	1200.00	1200.00	1200.00	1200.00

0.00: Lower limit (Full insertion), 1200.00: Upper limit (Full withdrawal)

Table 3.2 Comparison of measured and calculated subcriticalities (Ref. [2])

Case	Experiment (% $\Delta k/k$)	JENDL-3.3 (% $\Delta k/k$)	C/E	ENDF/B-VI.2 (% $\Delta k/k$)	C/E
1	0.79 ± 0.06	0.75 ± 0.02	0.95	0.82 ± 0.02	1.04
2	0.68 ± 0.05	0.68 ± 0.02	1.01	0.67 ± 0.02	0.99
3	0.89 ± 0.06	0.98 ± 0.02	1.09	0.91 ± 0.02	1.02
4	0.70 ± 0.05	0.76 ± 0.02	1.09	0.73 ± 0.02	1.04
5	1.76 ± 0.12	1.71 ± 0.02	0.97	1.72 ± 0.02	0.97

Table 3.3 Comparison of measured subcriticalities obtained by the source multiplication method and calculated subcriticalities by MCNP (Ref. [2])

Case	Experiment (% $\Delta k/k$)	JENDL-3.3 (% $\Delta k/k$)	C/E	ENDF/B-VI.2 (% $\Delta k/k$)	C/E
11	0.72 ± 0.05	0.69 ± 0.02	0.96	0.69 ± 0.02	0.97
12	2.72 ± 0.19	2.88 ± 0.02	1.06	2.88 ± 0.02	1.06
15	5.96 ± 0.42	6.44 ± 0.02	1.08	6.50 ± 0.02	1.09
18	8.66 ± 0.61	10.46 ± 0.02	1.21	10.54 ± 0.02	1.22

region, respectively. Nonetheless, the results agree with the validation of subcriticality measurement techniques by the rod drop and the positive period methods for such cores down to about 2 % $\Delta k/k$.

Measurements by the source multiplication method (Table 3.3) show the results of subcriticalities in the experiments and the calculations for the neutron-guided core (Fig. 3.1f) with zero to three extracted SV assemblies. The measured subcriticalities were obtained by the source multiplication method down to about 9 % $\Delta k/k$. Nonetheless, a reliable relative discrepancy within 10% C/E between experiments and calculations was confirmed only down to 6 % $\Delta k/k$. Therefore, as a measurement methodology for the KUCA core with the neutron guide, this source multiplication method proved convenient and complementary to the rod drop and the calibration curve methods and reliable on a relatively large subcriticality range of up to 6 % $\Delta k/k$.

3.1.3.2 Kinetic Parameters

Adjustment of the fuel density in the calculation was first considered for the neutron-guided core, leading to a correcting factor of 5%. Only for cases of very small reactivity did the relative difference appear within the order of calculation precision. For consistency, the following results are those obtained by applying the same correction of 5% to all calculations.

A representative selection of the calculation results of the subcriticalities is shown in Table 3.4 with the C/E values for each detector. Although better for small subcriticality, an overall 10% in relative error was taken into account for a comparison of the subcriticalities. For a comparison between the detectors, remarkably Fiber #1

Table 3.4 Comparison of measured subcriticalities obtained by the area ratio method and calculated subcriticalities by MCNP (JENDL-3.3) (Ref. [2])

Case	MCNP (% $\Delta k/k$)	Fiber #1 (% $\Delta k/k$)	C/E	Fiber #2 (% $\Delta k/k$)	C/E	Fiber #3 (% $\Delta k/k$)	C/E
10	0.97 ± 0.03	0.99 ± 0.01	0.98	0.96 ± 0.01	1.01	0.99 ± 0.01	1.03
11	1.83 ± 0.03	1.88 ± 0.02	0.97	2.15 ± 0.02	0.85	1.78 ± 0.02	1.03
12	2.55 ± 0.03	2.55 ± 0.03	1.00	3.12 ± 0.03	0.82	2.42 ± 0.02	1.05
13	3.45 ± 0.03	3.40 ± 0.03	1.02	3.25 ± 0.03	1.06	3.63 ± 0.04	0.95
14	4.15 ± 0.03	4.49 ± 0.04	0.93	4.00 ± 0.04	1.04	4.60 ± 0.05	0.90
15	6.24 ± 0.03	5.89 ± 0.06	1.06	6.54 ± 0.07	0.95	6.87 ± 0.07	0.91
16	6.76 ± 0.03	6.59 ± 0.07	1.03	10.01 ± 0.10	0.67	7.56 ± 0.08	0.89
17	7.41 ± 0.03	7.55 ± 0.08	0.98	8.18 ± 0.08	0.91	8.64 ± 0.09	0.86
18	10.38 ± 0.03	10.24 ± 0.10	1.01	12.28 ± 0.12	0.85	11.93 ± 0.12	0.87

appeared little affected by the increase in the subcriticality, with the discrepancy being within 7% even for the largest subcriticality, while Fibers #2 and #3 reached about 30%. This tendency can be clearly seen in Fig. 3.6.

The results allowed some estimation of the relative effects among the various parameters suspected to influence the measurement of the subcriticality by the neutron pulsed technique and the prompt neutron decay constant. Although the same value of the delayed neutron fraction was used for the area ratio method (Table 3.5), the results from Fiber #2 allowed the evaluation of the variation in the delayed neutron fraction to be in the smallest order of the uncertainty over the subcriticality. The measurements from Fiber #2 (Fig. 3.7), although underestimating the subcriticality (Fig. 3.6), tended to give a relatively good evaluation of the prompt neutron decay constant compared with those from Fiber #3. Fiber #1, however, gave a marked underestimation of the prompt neutron decay constant of less than 7%. Finally, Fiber

Fig. 3.6 Measured and calculated results of subcriticalities in KUCA kinetic experiments (Ref. [2])

Table 3.5 Comparison of C/E values in measured and calculated neutron decay constants (Ref. [2])

Case	Fiber #1	Fiber #2	Fiber #3
10	1.02 ± 0.02	1.12 ± 0.03	1.07 ± 0.02
11	1.16 ± 0.02	1.11 ± 0.03	1.18 ± 0.03
12	1.05 ± 0.02	1.07 ± 0.03	1.09 ± 0.02
13	1.04 ± 0.03	1.07 ± 0.04	1.12 ± 0.04
14	1.07 ± 0.03	1.04 ± 0.04	1.14 ± 0.04
15	1.16 ± 0.03	1.18 ± 0.05	1.21 ± 0.04
16	1.11 ± 0.04	1.03 ± 0.05	1.06 ± 0.04
17	0.88 ± 0.03	0.96 ± 0.05	0.83 ± 0.03
18	1.13 ± 0.03	1.06 ± 0.03	1.12 ± 0.03

Fig. 3.7 Measured and calculated results of neutron decay constants in KUCA kinetic experiments (Ref. [2])

#2 showed a good evaluation of the prompt neutron decay constant, considering the detector position dependency of prompt neutron decay constant measurements.

3.2 Pulsed-Neutron Source Method

3.2.1 Experimental Settings

3.2.1.1 Core Configurations

The ADS experiments with 100 MeV protons (Pb–Bi target) were carried out in the A-core (Fig. A2.11) [10] comprising a highly-enriched uranium (HEU) fuel, a polyethylene moderator, and Pb–Bi reflector rods. The fuel assembly “F” (3/8”p36EU) is composed of 36 unit cells, and upper and lower polyethylene blocks are about 25” and 20” long, respectively, in an aluminum (Al) sheath $2.1'' \times 2.1'' \times 60''$, as shown in Fig. A2.12. A special fuel assembly “f” (1/8” p5EUEU <1/8”Pb-Bi30EUEU>

1/8"p15EUEU) shown in Fig. 2A.13 is composed of a total of 60 unit cells: 30 unit cells with HEU plate 1/8" thick and Pb–Bi plate 3.426 mm thick, and 30 unit cells with HEU plate 1/8" thick and a polyethylene plate 1/8" thick. As shown in Fig. 2A.14, numeral 16" (3/8"p16EU) corresponds to the number of fuel plates in the partial fuel assembly used for reaching critical mass. The neutron spectra were numerically obtained by the MCNP calculations, when 100 MeV protons are injected onto the Pb–Bi target, as shown in Fig. 3.8a, b.

Subcriticality was attained by full insertion of control and safety rods, and the substitution of fuel assemblies for polyethylene ones, as shown in Table 2A.13 and Fig. 2A.16: an insertion (Cases II-1, II-2, and II-3 ranged between 1160 and 2483 pcm in subcriticality) of control and safety rods, and the substitution (Cases II-4, II-5, and II-6 ranged between 4812 and 11556 pcm) of fuel assemblies for polyethylene moderators. In Cases II-1, II-2, and II-3, the subcriticality was deduced experimentally with the combination of the worth of control (C1, C2, and C3) and safety (S4, S5,

Fig. 3.8 Comparison between neutron spectra at Pb–Bi target, Fibers #1 and #2 (Ref. [10])

(a) Locations of Pb–Bi target and Fiber #2

(b) Location of Fiber #1

and S6) rods by the rod drop method and its calibration curve by the positive period method. Furthermore, in Cases II-4, II-5, and II-6, the subcriticality was numerically obtained by the MCNP6.1 [11] code with the JENDL-4.0 [12] library, because the reactivities of control and safety rods were varied by the substitution of fuel assembly rods for polyethylene ones.

The Pb–Bi target was located inside the core at the location (15, L) shown in Fig. 2A.11 on the basis of the characteristics of the location of the target outside the core, as discussed in the previous study [13]. Note that the location of the original target is not easily moved to the center of the core, because control and safety rods are fixed in the core to function as the control driving system at KUCA. The Pb–Bi target was 50 mm in diameter and 18 mm thick. The main characteristics of proton beams were 100 MeV energy, 0.7 nA intensity, 40 mm beam spot, 20 Hz beam repetition, 100 ns beam width, and $1.0 \times 10^7 \text{ s}^{-1}$ neutron yield.

3.2.1.2 Measurements

During the injection of 100 MeV protons onto the Pb–Bi target located at (15, L) shown in Fig. A2.11, the time evolution of prompt and delayed neutron behavior was examined by the optical fiber detectors [14] set at three locations: Fiber #1 at (12-11, T-R) in Fig. A2.16 between the polyethylene moderator rods, Fiber #2 at (14-13, P-O) in Fig. A2.16 outside, and Fiber #3 at (15-14, O-M) inside the ^{235}U -fueled and Pb–Bi-zoned core. The optical fiber was shaped with a mixture of lithium-6-enriched LiF and ZnS (Ag) scintillator pasted at its 1 mm diameter tip.

From the results of neutron signals shown in Fig. 3.9, prompt neutron decay constant α was deduced from the exponential function fitting of the PNS measurements in the region of prompt neutron behavior as follows:

$$N = C_{\text{PNS}} \cdot \exp(-\alpha t) + B_{\text{PNS}}, \quad (3.1)$$

where N indicates the counting rate of the neutron signal, and C_{PNS} and B_{PNS} the constant values obtained by the least-squares fitting. Additionally, subcriticality $\rho_{\text{\$}}$ in dollar units was deduced by the PNS method, on the basis of the following theoretical background: in the area ratio method [15], subcriticality $\rho_{\text{\$}}$ in dollar units was determined by the ratio of two prompt and delayed components in the decay of neutron density as follows:

$$\rho_{\text{\$}} = \frac{\rho}{\beta_{\text{eff}}} = -\frac{A_p}{A_d}, \quad (3.2)$$

where ρ indicates the subcriticality in pcm units, β_{eff} the effective delayed neutron fraction, A_p the area of the decay curve by prompt neutrons, and A_d the area of delayed neutrons. For reducing the spatial higher mode components of neutron flux, the extrapolated area ratio method [5] was introduced into the measurement of subcriticality as follows:

Fig. 3.9 Time evolution by the PNS method (Ref. [10])

(a) Case 1 (1160 pcm)

(b) Case 5 (9895 pcm)

$$\rho_{\S} = \frac{\rho}{\beta_{\text{eff}}} = -\exp(\alpha t_w) \frac{\int_{t_w}^T A_p(t) dt}{\int_0^T A_d(t) dt}, \quad (3.3)$$

where T indicates the measurement time, and t_w the waiting time for reducing the higher mode components of neutron flux.

Another approach to the α value was attempted by the Feynman- α method [16] with the use of neutron noise data shown in Fig. 3.10. The α value was deduced from the least-squares fitting for the Y value with gate width t_g , on the basis of the theoretical background by taking delayed neutron effects into account [17], as follows:

$$Y = \frac{C_{\text{Noise}}}{\alpha^2} \left(1 - \frac{1 - \exp(-\alpha t_g)}{\alpha t_g} \right)$$

Fig. 3.10 Neutron noise data by the Feynman- α method (Ref. [10])

$$+ \frac{B_{\text{Noise}}}{t_g} \sum_{n=1}^{\infty} \left(\frac{1}{n^4 \alpha^2 T_0^2 + 4n^6 \pi^2} \sin^2 \left(\frac{n\pi}{T_0} W \right) \sin^2 \left(\frac{n\pi}{T_0} t_g \right) \right), \quad (3.4)$$

where C_{Noise} and B_{Noise} indicate the constant values obtained from the noise data, and W and T_0 the pulsed width as a fitting parameter and the pulsed frequency obtained from proton beam characteristics (20 Hz), respectively. In Eq. (3.4), the maximum value of n of the second term was set as 1000 leaving a margin from the saturation of the fitting results by setting about 300 in the maximum value. Moreover, the α value of the second term in Eq. (3.4) was acquired as the fitting parameter from experimental noise data by the Feynman- α method.

Table 3.6 Comparison between measured and calculated (MCNP6.1 with JENDL-4.0) reactivities [pcm] of excess and control rods (C1, C2, and C3) (Ref. [10])

Reactivity	Calculation	Experiment	C/E ^a
Excess	149 ± 11	149 ± 3	1.00 ± 0.08
C1	567 ± 11	549 ± 3	1.03 ± 0.02
C2	189 ± 11	194 ± 1	0.98 ± 0.06
C3	498 ± 11	483 ± 2	1.03 ± 0.02

^aCalculation/Experiment

3.2.1.3 Numerical Simulations

Numerical calculations were performed by the Monte Carlo transport code, MCNP6.1 together with JENDL-4.0 for transport and with JENDL/HE-2007 [18] for high-energy protons and spallation neutrons. Here, in MCNP6.1, since the effects of reactivity by neutron detectors (optical fiber, FC, and UIC detectors) and control (safety) rods are not negligible, neutron detectors and control (safety) rods were included in the simulated geometry and transport calculations. The precision of numerical reactivities of excess and control rods (C1, C2, and C3) in pcm units was attained by the eigenvalue calculations within a relative difference of 3% between the experiment and the calculation, as shown in Table 3.6, with a total number of 1×10^8 histories and a statistical error of less than 5 pcm. Note that the relative difference of 3% in C/E values was attributable to numerical reactivity by MCNP with 20 pcm at most in the KUCA core.

3.2.2 Results and Discussion

3.2.2.1 Kinetics Parameters

The prompt neutron decay constant was attained by the PNS and the Feynman- α methods shown in Eqs. (3.1) and (3.2), respectively, with the use of neutron signals obtained from the three optical fibers at the locations in the core shown in Fig. 2A.11. As shown in Table 3.7, well-known findings were observed with a small difference between the results of Fibers #1 and #2, from the viewpoint of three issues: the neutron spectrum (Figs. 4.5a, b) and the subcriticality measurement methods (PNS and Feynman- α methods), on the subcriticality ranging between 1160 and 2483 pcm (Cases II-1 to II-3). A small difference between Fibers #1 and #2 was found in the measurements, because one-point reactor approximation is assumed to be valid in the shallow level of subcriticality. Conversely, on the subcriticality ranging between 4812 and 11556 pcm (Cases II-4 to II-6), a notable difference was observed in the experimental results between two methods, although the detector position dependency and neutron spectrum were small on the prompt neutron decay constant,

Table 3.7 Measured prompt neutron decay constants α [s^{-1}] in Cases II-1 to II-6 (Ref. [10])

Case	PNS method			Feynman- α method		
	Fiber #1	Fiber #2	Fiber #3	Fiber #1	Fiber #2	Fiber #3
II-1	398 ± 3	327 ± 3	495 ± 10	401 ± 9	367 ± 6	554 ± 17
II-2	507 ± 3	453 ± 3	685 ± 11	498 ± 5	464 ± 4	700 ± 7
II-3	673 ± 4	632 ± 2	1020 ± 8	655 ± 2	620 ± 2	877 ± 5
II-4	983 ± 4	971 ± 3	1378 ± 18	815 ± 6	822 ± 5	1029 ± 12
II-5	1665 ± 9	1681 ± 5	1828 ± 23	1365 ± 9	1400 ± 7	1669 ± 17
II-6	1911 ± 7	1931 ± 3	2061 ± 38	1556 ± 13	1636 ± 10	1917 ± 22

as compared with Fibers #1 and #2. For Fiber #3, a strong influence of spallation neutrons at the location of the Pb–Bi target was observed in the results of the prompt neutron decay constant by both PNS and Feynman- α methods, as compared with those of Fibers #1 and #2.

Subcriticality in dollar units was deduced experimentally by the extrapolated area ratio method with the use of prompt and delayed neutron components and by the α -fitting method [19] with the α value (the Feynman- α method). Nonetheless, attention was paid to the conversion coefficient β_{eff} of subcriticality in dollar units to one in pcm units, where β_{eff} was obtained by MCNP6.1 with JENDL-4.0 in each subcritical state of the core shown in Fig. 2A.16.

As shown in Table 3.8, the measured subcriticality by the PNS and the Feynman- α methods showed good agreement with the calculated one, with an error around 10% in the C/E (calculation/experiment) value, in the subcriticality ranging between 1160 and 2483 pcm, at the locations of Fiber #2. Additionally, in a comparison between Fibers #1 and #2, the detector position dependency by both the PNS and the Feynman- α methods was found to be attributable to placing the fiber detectors at different locations. At the locations of Fibers #1 and #2, with subcriticality ranging between 4812 and 11556 pcm, the deeper the subcriticality, the less accurate were the experimental results of measurements by both the PNS and the Feynman- α methods. The reason for this tendency was considered to be the effect of spallation neutrons on the neutron flux becoming greater with the deep subcriticality. As a consequence, the discrepancy between the experiments and the calculations is identified as the limitation of the applicability of the measurement method to a deep subcriticality level of over or about 10000 pcm. For Fiber #3, a large influence on spallation neutrons, as well as on the measurements of prompt neutron decay constant, was observed with the two measurement methods, although the Feynman- α method showed very good agreement in Cases II-5 and II-6. Neutron noise data are assumed to be dominant over the Poisson distribution, demonstrating that the effect of spallation neutrons becomes stronger with the deep subcriticality. In the case of the deep subcriticality, kinetic parameters of ρ and Λ shown in Tables 3.8 and 3.9, respectively, were numerically obtained by the MCNP eigenvalue calculations. Subsequently, values of kinetic parameters should be exactly acquired by the time-dependent MCNP calculations

Table 3.8 Comparison between subcriticality ρ [pcm] of reference, PNS, and Feynman- α methods (Ref. [10])

		PNS method		
Case	ρ [pcm]	Fiber #1	Fiber #2	Fiber #3
II-1	1160 ± 5	1224 ± 10 (0.95 ± 0.01)	1153 ± 8 (1.01 ± 0.01)	2108 ± 59 (0.55 ± 0.02)
II-2	1684 ± 6	1880 ± 15 (0.90 ± 0.01)	1604 ± 11 (1.05 ± 0.01)	4490 ± 109 (0.38 ± 0.01)
II-3	2483 ± 6	2824 ± 22 (0.88 ± 0.01)	2250 ± 15 (1.10 ± 0.01)	10347 ± 116 (0.24 ± 0.01)
II-4	4812 ± 6	5656 ± 62 (0.85 ± 0.01)	4177 ± 31 (1.15 ± 0.01)	22939 ± 1279 (0.21 ± 0.01)
II-5	9895 ± 6	12819 ± 137 (0.77 ± 0.01)	8187 ± 54 (1.21 ± 0.01)	14985 ± 1136 (0.66 ± 0.05)
II-6	11556 ± 6	16312 ± 160 (0.71 ± 0.01)	9738 ± 64 (1.19 ± 0.01)	19109 ± 2563 (0.60 ± 0.08)
		Feynman- α method		
Case	ρ [pcm]	Fiber #1	Fiber #2	Fiber #3
II-1	1160 ± 5	1491 ± 57 (0.87 ± 0.03)	1283 ± 36 (0.90 ± 0.03)	2445 ± 105 (0.47 ± 0.02)
II-2	1684 ± 6	2024 ± 28 (0.83 ± 0.01)	1814 ± 24 (0.93 ± 0.01)	3250 ± 41 (0.52 ± 0.01)
II-3	2483 ± 6	2881 ± 15 (0.86 ± 0.01)	2677 ± 11 (0.93 ± 0.01)	4203 ± 29 (0.59 ± 0.01)
II-4	4812 ± 6	4336 ± 38 (1.11 ± 0.01)	4384 ± 35 (1.10 ± 0.01)	5740 ± 77 (0.84 ± 0.01)
II-5	9895 ± 6	8087 ± 61 (1.22 ± 0.01)	8316 ± 50 (1.19 ± 0.01)	10110 ± 113 (0.98 ± 0.01)
II-6	11556 ± 6	9633 ± 88 (1.20 ± 0.01)	10175 ± 68 (1.14 ± 0.01)	12096 ± 150 (0.96 ± 0.01)

Bracket means the C/E (calculation/experiment) value

Table 3.9 Comparison between $\rho_{\$}$ [\$], β_{eff} [pcm], and Λ [$\times 10^{-5}$ s] by the PNS method and MCNP calculations (Ref. [10])

	$\rho_{\$}$ [\$]	β_{eff} [pcm]	Λ [$\times 10^{-5}$ s]	
Case	Fiber #2 (PNS)	MCNP6.1	MCNP6.1	Fiber #2 (PNS)
II-1	1.47 ± 0.01	785 ± 4	4.88 ± 0.01	5.93 ± 0.08
II-2	2.04 ± 0.01	785 ± 4	4.77 ± 0.01	5.28 ± 0.09
II-3	2.87 ± 0.01	783 ± 5	4.64 ± 0.01	4.80 ± 0.05
II-4	5.30 ± 0.02	788 ± 5	5.16 ± 0.01	5.11 ± 0.06
II-5	10.03 ± 0.03	816 ± 5	5.43 ± 0.01	5.53 ± 0.05
II-6	12.08 ± 0.03	806 ± 5	5.50 ± 0.01	5.46 ± 0.05

in the previous study [20], to get a better understanding of the time behavior of the kinetic parameters.

Furthermore, most of the C/E values demonstrated a notably greater tendency of the calculation results to the deep subcriticality attributable to the effect of the spallation neutrons on neutron flux, whereas the value of Fiber #1 by the PNS method showed a decreasing tendency.

3.2.2.2 Evaluation of $\beta_{\text{eff}}/\Lambda$

For the α -fitting method, prompt neutron decay constant α is easily deduced by combining subcriticality ρ in pcm units, effective delayed neutron fraction β_{eff} , and neutron generation time Λ as follows:

$$\alpha = \frac{\beta_{\text{eff}} - \rho}{\Lambda}. \quad (3.5)$$

Using the relationship between ρ and $\rho_{\text{\$}}$ in pcm and dollar units shown in Eq. (3.2), respectively, $\rho_{\text{\$}}$ can be expressed as follows:

$$\rho_{\text{\$}} = -\frac{\Lambda}{\beta_{\text{eff}}} \left(\alpha - \frac{\beta_{\text{eff}}}{\Lambda} \right). \quad (3.6)$$

Assuming that the experimental results of α and $\rho_{\text{\$}}$ validate the accuracy of kinetics parameters, the value of Λ was deduced by Eq. (3.6), as shown in Table 3.9, with the combined use of the measured α and $\rho_{\text{\$}}$, and the calculated β_{eff} . Through the experimental analyses of the ADS with spallation neutrons in KUCA, a previous study [21] has clearly demonstrated that the value of β_{eff} has a small effect on the evaluation of subcriticality in pcm units converted from that in dollar units, as compared with the results of numerical subcriticality in pcm units. Here, considering the characteristics of β_{eff} , attention was directed to another kinetic parameter, Λ , obtained with the combined use of α and $\rho_{\text{\$}}$ by varying the subcriticality. The deduction of Λ in Fiber #2 was based on Eq. (3.6), by using the value of β_{eff} obtained from the MCNP6.1 calculations, since the experimental results of Fiber #2 by the PNS method showed a relatively good agreement with the numerical simulations, as described in Sect. 3.2.2.1. As shown in Table 3.9, a comparison between the experimental and the numerical values of Λ demonstrated the same tendency as the acceptable relative difference between MCNP6.1 and Fiber #2 by varying the subcriticality. Furthermore, using the experimental results of α and $\rho_{\text{\$}}$ of Fibers #1 and #2 by the PNS method, the fitting lines (Fiber #1: $\rho_{\text{\$}} = -12.27\text{E-}03\alpha + 4.10$; Fiber #2: $\rho_{\text{\$}} = -6.82\text{E-}03\alpha + 1.27$) by Eq. (3.5) were obtained with the values of the gradient (Fiber #1: $-12.27\text{E-}03$; Fiber #2: $-6.82\text{E-}03$) of $\Lambda/\beta_{\text{eff}}$ as shown in Fig. 3.11, as compared with the results of MCNP6.1, Fibers #1 and #2 shown in Table 3.10. The experimental fitting of Fiber #2 demonstrated good agreement with the MCNP6.1 calculations by varying the subcriticality, although the relative difference between the experiments

Fig. 3.11 Linearity between prompt neutron decay constant α [s^{-1}] and subcriticality ρ_s [\$] (Ref. [10])

Table 3.10 Comparison between $\Lambda/\beta_{\text{eff}} [\times 10^{-3}]$ values of MCNP6.1, Fibers #1 and #2 (Fitting in Fig. 3.11) by the PNS method (Ref. [10])

Case	MCNP6.1	Fiber #1	Fiber #2
II-1	6.22 ± 0.01	12.27 ± 0.08	6.82 ± 0.05
II-2	6.08 ± 0.01		
II-3	5.93 ± 0.01		
II-4	6.55 ± 0.01		
II-5	6.65 ± 0.01		
II-6	6.82 ± 0.01		

and the calculations was 15% at most. From the results in Fig. 3.11, and Tables 3.9 and 3.10, the kinetic parameters were easily deduced by the combination of experiments and calculations, and verified by the PNS and the α -fitting methods.

From the results in Tables 3.6, 3.7, 3.8, 3.9 and 3.10, these experimental benchmarks are expected to play an important role in the study of weight functions related to the physical interpretation and correction factors of the experimental results, from the theoretical and the numerical aspects, respectively, as well as of detector position dependency, neutron spectrum, and subcriticality measurement methods on kinetic parameters.

3.3 Inverse Kinetic Method

3.3.1 Theoretical Background

In the extended Kalman filter (EKF), the state space $\mathbf{x}(k)$ in time step k (1, 2, ...) and the observation equation $\mathbf{y}(k)$ are expressed as follows:

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k)) + \mathbf{b}v(k), \quad (3.7)$$

$$y(k) = h(\mathbf{x}(k)) + w(k), \quad (3.8)$$

where $\mathbf{f}$ is the matrix expressing state space, $\mathbf{b}$ the vector distributing system noise, v the (white) system noise (average = 0.0, dispersion = σ_v^2), h the observable matrix, and w the observation noise (average = 0.0, dispersion = σ_w^2). EKF is applicable to nonlinearity with first-order approximation by considering derivative $\mathbf{A}$ and $\mathbf{c}^T$ as follows:

$$\mathbf{A}(k) = \left. \frac{\partial \mathbf{f}(\mathbf{x}(k))}{\partial \mathbf{x}} \right|_{\mathbf{x}=\hat{\mathbf{x}}^-(k)} \quad (3.9)$$

$$\mathbf{c}^T(k) = \left. \frac{\partial h(\mathbf{x}(k))}{\partial \mathbf{x}} \right|_{\mathbf{x}=\hat{\mathbf{x}}^-(k)}, \quad (3.10)$$

where $\hat{\mathbf{x}}^-$ is the priori estimate (prediction estimation of $\mathbf{x}$ in time step k based on collected experience until time $k-1$). The procedure of the general Kalman filter technique is divided into a prediction step and a filtering step. For the prediction step, the priori estimate is evaluated with the use of state estimate $\hat{\mathbf{x}}$ in a previous time step as follows:

$$\hat{\mathbf{x}}^-(k) = \mathbf{f}(\hat{\mathbf{x}}(k-1)). \quad (3.11)$$

Next, the priori error covariance matrix $\mathbf{P}^-$ is evaluated as follows:

$$\mathbf{P}^-(k) = \mathbf{A}(k)\mathbf{P}(k-1)\mathbf{A}^T(k) + \sigma_v^2\mathbf{b}\mathbf{b}^T, \quad (3.12)$$

where $\mathbf{P}$ is a posteriori error covariance matrix. Here, in the first step, initial values of $\hat{\mathbf{x}}^-$ and $\mathbf{P}^-$ are requisite to perform EKF. For the filtering step, the Kalman gain $\mathbf{g}$ is determined as follows:

$$\mathbf{g}^-(k) = \frac{\mathbf{P}^-(k)\mathbf{C}(k)}{\mathbf{C}^T(k)\mathbf{P}^-(k)\mathbf{C}(k) + \sigma_w^2}. \quad (3.13)$$

The state estimate is evaluated with the use of observation results and the priori state estimate by the most likelihood parameter $\mathbf{g}$ as follows:

$$\hat{\mathbf{x}}(k) = \hat{\mathbf{x}}^-(k) + \mathbf{g}(k)(y(k) - h(\hat{\mathbf{x}}^-(k))). \quad (3.14)$$

Finally, in the next step, the posteriori error covariance matrix is prepared as follows:

$$\mathbf{P}(k) = (\mathbf{I} - \mathbf{g}(k) - \mathbf{C}^T(k))\mathbf{P}^-(k). \quad (3.15)$$

In this study, initial values of Eqs. (3.11) and (3.12) were prepared by the inverse kinetic method based on one-point kinetic equations as follows:

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta_{\text{eff},i}}{\Lambda} n(t) + \sum_{i=1}^6 \lambda_i C_i(t), \quad (3.16)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_{\text{eff}}}{\Lambda} n(t) + \lambda_i C_i(t), \quad (3.17)$$

where n is the neutron density, ρ the reactivity, $\beta_{\text{eff},i}$ the effective delayed neutron fraction of the i -th group, λ_i the delayed neutron decay constant of the i -th group, and C_i the density of delayed neutron precursor. For obtaining reactivity in Eq. (3.16) at every time step, the time variation of C_i in Eq. (3.17) is expressed with a backward difference as follows:

$$\frac{C_i(k) - C_i(k-1)}{\Delta k} = \frac{\beta_{\text{eff},i}}{\Lambda} n(k) + \lambda_i C_i(k) \quad (k = 2, 3, \dots), \quad (3.18)$$

ρ is obtained by substituting $C_i(k)$ in Eq. (3.18) for Eq. (3.16) as follows:

$$\begin{aligned} \rho(t) = & \left. \frac{dn(t)}{dt} \right|_{t=1} \frac{\Lambda}{n(k)} + \beta_{\text{eff}} \\ & - \frac{\Lambda}{n(k)} \sum_{i=1}^6 \frac{\lambda_i}{1 + \lambda_i \Delta k} \left(\frac{\beta_{\text{eff},i}}{\Lambda} n(k) \Delta k C_i(k-1) \right). \end{aligned} \quad (3.19)$$

Here, when an experiment is assumed to have started at a critical state, the initial value (at $k = 1$) of C_i is estimated as follows:

$$C_i(1) = \frac{\beta_{\text{eff},i}}{\lambda_i \Lambda} n(1). \quad (3.20)$$

When monitoring the subcriticality by the EKF technique in the critical core, the result of Eq. (3.20) was used as the initial values.

In ADS experiments with a stable external neutron source, the one-point kinetic equation on neutron derivative is changed as follows:

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta_{\text{eff}}}{\Lambda} n(t) + \sum_{i=1}^6 \lambda_i C_i(t) + S_{\text{eff}}, \quad (3.21)$$

where S_{eff} is the effective strength of the stable neutron source. By substituting C_i in Eq. (3.18) for Eq. (3.21), as the same procedure in Eq. (3.19), ρ is expressed in ADS experiments as follows:

$$\rho(t) = \left. \frac{dn(t)}{dt} \right|_{t=1} \frac{\Lambda}{n(k)} + \beta_{\text{eff}} - \frac{\Lambda}{n(k)} \sum_{i=1}^6 \frac{\lambda_i}{1 + \lambda_i \Delta k} \left(\frac{\beta_{\text{eff},i}}{\Lambda} n(k) \Delta k C_i (k - 1) \right) - \frac{\Lambda}{n(k)} S_{\text{eff}}. \quad (3.22)$$

Here, assuming that the external neutron source is stable, S_{eff} was determined as follows:

$$S_{\text{eff}} = -\frac{\rho(1)}{\Lambda} n(1). \quad (3.23)$$

When monitoring subcriticality by the EKF technique with the external neutron source, $\rho(1)$, C_i in Eq. (3.20) and S_{eff} in Eq. (3.23) were used as the initial values.

3.3.2 Experimental Settings

3.3.2.1 Critical Core

Transient experiments [22] were carried out in the uranium-polyethylene (EE1) core at KUCA shown in Fig. A4.1. The core was constituted by fuel rods (1/8"p60EUEU), made of an HEU (2" × 2" × 1/16") and a polyethylene moderator (p; 2" × 2" × 1/8") in an aluminum sheath 2.1" × 2.1" × 60", as shown in Fig. A4.2a. The core spectrum was hard in the polyethylene-moderated core at KUCA (an H/U: hydrogen/uranium ratio of approximately 50 in the thermal reactor).

Time evolution of neutron signals was obtained by an optical fiber detector (Eu:LiCaAlF₆ scintillator) [23] set at the core center for monitoring reactivity based on one-point kinetic approximation (to prevent measuring higher mode components in neutron flux and variation of detector efficiency).

The transient experiments were conducted after attaining a critical state with the C2 control rod (with all the other control and safety rods withdrawn); C1 control rod (Fig. 4.1) was then dropped from the fully withdrawal position to the fully inserted position. The neutron counts were obtained every 1 s.

3.3.2.2 Subcritical Core

The ADS transient experiment was carried out with the subcritical core at $k_{\text{eff}} = 0.97$ (target range of the subcriticality monitoring in ADS) shown in Fig. 3.12. The 14 MeV neutrons were generated by the injection of deuteron beam (intensity 0.15 mA, pulsed width 90 μs, and pulsed frequency 100 Hz) onto a tritium target located at (14–15, Y; Fig. A4.1).

Fig. 3.12 Description of the subcritical core with 14 MeV neutrons at KUCA (Ref. [22])

	11	12	13	14	15	16	17	18	19
I	○	p	p	p	p	p	p	p	p
J	BF ₃ detector					p	p	p	p
K	p	○ C3	p	F	F	F	p	○ S5	p
L	p	p	F	F	F	F	F	p	p
M	○	p	F	F	F	F	F	p	p
O	p	○ S4	F	F	F	F	F	○ C1	p
Q	p	p	p	F	F	F	p	p	p
R	p	○ C2	p	p	p	p	p	○ S6	p

In the preparation of the transient experiment, all control and safety rods were withdrawn, and 14 MeV neutrons were then injected into the subcritical core. After 500 s, the C1 control rod was (slowly) inserted by an actuator-driven mechanism from the fully withdrawal position to the fully inserted position. The BF₃ detector used in this experiment was placed at (11, M; Fig. 3.12). Time evolution of the neutron count was obtained every 1 s.

3.3.3 Transient Analyses

3.3.3.1 Calibration in Critical Core

To perform the EKF technique [24, 25] in a time-variable system, setting the variance values of system noise and observation noise is requisite to be set. Furthermore, an initial priori error covariance matrix is needed for the calibration of the filter. The validity of the initial conditions in EKF parameters was evaluated by comparing the results by the rod drop method with those of the EKF technique, and those of the inverse kinetic method in the transient experiment (C1 rod drop) with the critical core.

Numerical analyses of kinetics parameters were conducted with the use of MCNP6.1 together with ENDF/B-VII.1 [26] (total histories were $5\text{E} + 08$ ($5\text{E} + 05$ history per cycle and $1\text{E} + 03$ active cycle)). The variance value of system noise was set only for the reactivity of $8\text{E}-08$ and the observation noise was set as for the neutron count obtained at the time step for the variance value. Furthermore, the error covariance matrix was set zero except for the reactivity ($1\text{E}-07$) as the EKF parameters.

Here, the variable on the state-space model was set as follows:

$$\mathbf{x}(k) = {}^t(n(k) \ C_1(k) \ C_2(k) \ C_3(k) \ C_4(k) \ C_5(k) \ C_6(k) \ \rho(k)), \quad (3.24)$$

where t indicates transposition of the matrix. Furthermore, function $f(\mathbf{x}(k))$ in Eq. (3.7) is described as follows:

$$\mathbf{f}(\mathbf{x}(k)) = \begin{pmatrix} n(k) + T \left(\frac{\rho(k) - \beta_{eff}}{\Lambda} n(k) + \sum_{i=1}^6 \lambda_i C_i(k) \right) \\ C_1(k) + T \left(\frac{\beta_{eff,1}}{\Lambda} n(k) - \lambda_1 C_1(k) \right) \\ C_2(k) + T \left(\frac{\beta_{eff,2}}{\Lambda} n(k) - \lambda_2 C_2(k) \right) \\ C_3(k) + T \left(\frac{\beta_{eff,3}}{\Lambda} n(k) - \lambda_3 C_3(k) \right) \\ C_4(k) + T \left(\frac{\beta_{eff,4}}{\Lambda} n(k) - \lambda_4 C_4(k) \right) \\ C_5(k) + T \left(\frac{\beta_{eff,5}}{\Lambda} n(k) - \lambda_5 C_5(k) \right) \\ C_6(k) + T \left(\frac{\beta_{eff,6}}{\Lambda} n(k) - \lambda_6 C_6(k) \right) \\ \rho(k) \end{pmatrix}, \quad (3.25)$$

where T is the time resolution according to the forward difference: 1.0 in this study. Finally, function $\mathbf{A}(k)$ is expressed as follows:

$$\mathbf{A}(k) = \begin{pmatrix} 1 + T \left(\frac{\rho(k) - \beta_{eff}}{\Lambda} \right) & T\lambda_1 & T\lambda_2 & T\lambda_3 & T\lambda_4 & T\lambda_5 & T\lambda_6 & \frac{T h(\mathbf{x}(1))}{\Lambda} \\ T \frac{\beta_{eff,1}}{\Lambda} & 1 - T\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ T \frac{\beta_{eff,2}}{\Lambda} & 0 & 1 - T\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ T \frac{\beta_{eff,3}}{\Lambda} & 0 & 0 & 1 - T\lambda_3 & 0 & 0 & 0 & 0 \\ T \frac{\beta_{eff,4}}{\Lambda} & 0 & 0 & 0 & 1 - T\lambda_4 & 0 & 0 & 0 \\ T \frac{\beta_{eff,5}}{\Lambda} & 0 & 0 & 0 & 0 & 1 - T\lambda_5 & 0 & 0 \\ T \frac{\beta_{eff,6}}{\Lambda} & 0 & 0 & 0 & 0 & 0 & 1 - T\lambda_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (3.26)$$

Note that element (1, 8) in Eq. (3.26) was approximately $\frac{T h(\mathbf{x}(1))}{\Lambda}$ ($h(\mathbf{x}(1))$ indicates the initial neutron count in the experiment) instead of $\frac{T n(k)}{\Lambda}$. This value was introduced to take into account the strong nonlinearity in the model. In the estimate with the use of $\frac{T n(k)}{\Lambda}$, the results were unreliable and divergent.

The transient experiment was started at the critical state; C1 control rod was then dropped into the core, inducing a rapid decrease in the neutron counts shown in Fig. 3.13. Importantly, the EKF technique reproduced measured count distribution (Fig. 3.13). The results of subcriticality monitoring revealed fluctuation in the result by the inverse kinetic method shown in Fig. 3.14, demonstrating that slight variation in the neutron count in the region of the low count rate greatly affected the estimate of subcriticality. Conversely, the result of the EKF technique notably decreased the fluctuation notably even after the count rate reached almost zero and the estimated value asymptotically approached the reference value, although the overshoot was found when the variation in subcriticality stopped rapidly.

Fig. 3.13 Neutron count rate distribution during C1 drop transient (Ref. [22])

Fig. 3.14 Comparison of subcriticality monitoring between the inverse method and the EKF technique in C1 rod (worth: -888 pcm) drop experiment (Ref. [22])

By the transient experiment in the critical core, the validity of the transient analysis was confirmed with the superiority of the filtering technique in reducing the fluctuation of monitoring values.

3.3.3.2 Performance Evaluation

The objective transient experiment was conducted at the subcritical state in the presence of an external neutron source for evaluating the performance of subcriticality monitoring by the EKF technique, at the same initial condition described in Sect. 3.3.3.1. So as to consider the external neutron source in EKF, $T \times S_{\text{eff}}$ was appended to element (1) in Eq. (3.25)

The neutron count distribution was varied gradually compared to the rod drop experiment because of the insertion by actuator-drive, and kept at a constant value at the end of transient behavior in view of the presence of the external neutron source, as shown in Fig. 3.15. The estimate by the EKF technique of the neutron counts showed

Fig. 3.15 Neutron count rate distribution during C1 drop transient with external neutron source (Ref. [22])

almost the same distribution as of the experiment. In subcriticality monitoring by the inverse kinetic method shown in Fig. 3.16, the fluctuation in the monitoring values increased remarkably after the transient as the neutron count decreased. In EKF, the monitoring values importantly followed the subcriticality in the transient behavior without fluctuation. Moreover, the estimated subcriticality by the EKF technique was found to be significantly more accurate than that by the inverse kinetic method, demonstrating the validity of the estimate by EKF and the reliability of subcriticality monitoring.

In the transient experiment with the external neutron source, the S_{eff} values should be considered variable in addition to the detection efficiency since these values can be varied by the insertion of control rods. Thus, to improve monitoring accuracy, in

Fig. 3.16 Comparison of subcriticality monitoring between the inverse method and the EKF technique in ADS transient experiment (Ref. [22])

future studies, the modeling of the space-state equation has to be modified on the basis of S_{eff} and detection efficiency.

3.4 Conclusion

Kinetic experiments on ADS with 14 MeV neutrons were conducted at KUCA. The kinetic experimental and numerical analyses revealed the following: Measurement and calculation methods proven reliable for the evaluation of subcriticality effects down to 6 % $\Delta k/k$. Anticipating an ADS subcriticality level around 3 % $\Delta k/k$, the measurement methodology and the calculation precision were considered convenient for the study of ADS at KUCA. Moreover, optical fiber detectors were considered promising in the evaluation of the subcriticality and the prompt neutron decay constant evaluation at KUCA, although detector position dependency was observed in kinetic measurements by using the optical fiber detection system.

In the ADS experiments with spallation neutrons, the measurement of kinetics parameters was conducted by both the PNS and Feynman- α methods with the use of optical fiber detectors, under subcriticality ranging between 1160 and 11556 pcm. The results confirmed the validity of the prompt neutron decay constant and the subcriticality in dollar units through the deduction of kinetics parameters. The detector position dependency, neutron spectrum, and subcriticality measurement methods still remained, however, in these ADS experiments.

Transient experiments were carried out to evaluate the performance of subcriticality monitoring by the EKF technique in the presence of an external neutron source. To ensure the conditions in EKF, a basic transient experiment was conducted by dropping the control rod at the critical state. In a comparison of the subcriticality by the inverse kinetic method and the EKF technique, the advantage of EKF was indicated over the robustness of the estimate with good accuracy even at low counts. Additionally, the initial condition used in the filtering technique was confirmed as valid through comparison with measured subcriticality by the rod drop method in the basic transient experiment. In the transient experiments in the presence of an external neutron source, the EKF technique was applied to subcriticality monitoring (time resolution: 1 s), with the use of the initial condition in the basic experiments. The EKF technique significantly indicated a significant advantage in reducing fluctuation and estimating more accurately compared with the inverse method, demonstrating the applicability to the subcriticality monitor.

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