# The GNSS/Acoustic One-Step Positioning Model with Attitude Parameters 

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#### Abstract

Compared with the current multi-step method to solve the coordinate of the transponder, the combined solution of GNSS observation data and acoustic observation data is more consistent with the actual operation state, but if the error of attitude angle cannot be eliminated in the process of coordinate transformation, the accuracy of the transponder positioning will be seriously affected. Aiming at this shortcoming, based on the proposed GNSS/acoustic joint positioning model in this paper, the attitude angle is used as the parameter to be estimated to participate in the filtering solution, and the filtering effect of the new model is verified by simulation and experimental data in this paper. The results show that: after the attitude parameters' adding into the filtering model for estimation, the results of the three-dimensional coordinate error of the transponder are improved in varying degrees, and when the range of attitude error is larger, the improvement effect is more obvious.


Keywords: GNSS/acoustic • Coordinate change • The attitude error • Transponder position $\cdot$ Filtering effect

## 1 Introduction

To plan the ocean, safeguard maritime rights and interests, and achieve the goal of building a maritime power requires the support of the marine geodetic control network, one of the important basic work is to determine the precise position of the Marine control point. At present, the most effective method is to provide high-precision sea surface datum for underwater positioning with the help of the rapidly developing global satellite positioning technology [1-4]. The method can be expressed as follows: on the sea surface, the shipborne antenna is used to receive satellite observation data, and the acoustic pulse response can be established between the transducer mounted on the bottom of the vessel and the transponder mounted on the bottom of the vessel. In this way, the absolute coordinates of the seabed control point can be determined by realtime position of the vessel. This method is of low cost and simple implementation [5].

In the data processing stage, limited by the underwater positioning technology with unbreakable accuracy, the dynamic positioning on the sea is carried out in the traditional method firstly, then the solved position coordinates, regarded as the known intermediate quantity, participate in the underwater positioning. However, the above
process is not consistent with the actual operation state. In addition, on the one hand, due to the influence of Marine environmental factors, the vessel is not moving smoothly on the sea surface, and the vessel itself, as well as the onboard GNSS antenna, transducer and other sensors have instantaneous attitude changes; On the other hand, there is a position offset between the transducer coordinates and the GNSS antenna central geographic coordinates obtained by dynamic positioning. When the attitude angle error exists and cannot be ignored, the datum on the sea surface will have a large displacement, which will affect the position of the transponder.

Different from the traditional method of independent solution on the sea surface and underwater, coordinate transformation is required before the processing of underwater data. In this paper, the original GNSS data and acoustic data are put together for position, and the attitude angle is taken as the parameter to be estimated to participate in the filtering solution, so that the GNSS/acoustic joint positioning process can be described more accurately.

## 2 Change of Hull Attitude

### 2.1 Definition of Attitude Angle

The attitude change of hull mainly includes roll, pitch and heading changes.


Fig. 1. Three hull attitude angles. $\mathrm{X}, \mathrm{Y}$ and Z denote the axes of three directions in the hull's coordinate system. $\alpha$ in the left panel denotes the roll angle. $\beta$ in the middle panel denotes the pitch angle. $\lambda$ in the right panel denotes the heading angle

From the three panels of Fig. 1, the three attitude changes can be defined as follows:

Roll: the hull's sway from side to side. It is agreed that when the port deck of the hull tilts downward, it is positive; when the starboard deck tilts downward, it is negative.

Pitch: the pitch is generally expressed as the rising and sinking of the hull. It is agreed that the pitch is positive when the bow sinks and negative when the bow rises.

Heading: the horizontal deviation from the hull's direction. It is agreed that when the direction of the bow is on the right side of the heading line, it is positive, otherwise, it is negative.

According to the definition of three attitude changes, the specific form of attitude rotation matrix can be given as follows:

$$
\left\{\begin{array}{c}
\boldsymbol{R}_{\text {heading }}=\left[\begin{array}{ccc}
\cos (-\lambda) & \sin (-\lambda) & 0 \\
-\sin (-\lambda) & \cos (-\lambda) & 0 \\
0 & 0 & 1
\end{array}\right]  \tag{1}\\
\boldsymbol{R}_{\text {pitch }}= \\
\boldsymbol{R}_{\text {roll }}=\left[\begin{array}{ccc}
\cos (\beta) & 0 & -\sin (\beta) \\
0 & 1 & 0 \\
\sin (\beta) & 0 & \cos (\beta)
\end{array}\right] \\
{\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos (-\alpha) & \sin (-\alpha) \\
0 & -\sin (-\alpha) & \cos (-\alpha)
\end{array}\right]}
\end{array}\right.
$$

### 2.2 The Effect of Attitude Error on the Instantaneous Coordinate of Transducer

Effect of roll angle error: the instantaneous roll angle is $\alpha$, the roll angle error is assumed to be $\Delta \alpha$, the transducer actually rotates at an angle of $\alpha+\Delta \alpha$ around the Y axis in XOZ plane, and the effect on the coordinate of the transducer during attitude correction is:

$$
\begin{align*}
& {\left[\begin{array}{c}
X_{t} \\
Y_{t} \\
Z_{t}
\end{array}\right]=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos (\alpha+\Delta \alpha) & -\sin (\alpha+\Delta \alpha) \\
0 & \sin (\alpha+\Delta \alpha) & \cos (\alpha+\Delta \alpha)
\end{array}\right]\left[\begin{array}{l}
X \\
Y \\
Z
\end{array}\right]} \\
& \Rightarrow\left[\begin{array}{c}
\Delta X_{t} \\
\Delta Y_{t} \\
\Delta Z_{t}
\end{array}\right]_{\text {roll }}=\left[\begin{array}{c}
0 \\
-(Y \sin \alpha+Z \cos \alpha) \Delta \alpha \\
(Y \cos \alpha-Z \sin \alpha) \Delta \alpha
\end{array}\right] \tag{2}
\end{align*}
$$

According to Eq. (2), the roll angle error has an impact on the coordinate of the transducer in the Y and Z directions, but not in the X direction.

Effect of pitch angle error: the instantaneous pitch angle is $\beta$, because of the pitch angle error $\Delta \beta$, the transducer actually rotates around the X axis at the Angle $\beta+\Delta \beta$ in the YOZ plane, and the effect on the coordinate of the transducer during attitude correction is:

$$
\begin{align*}
& {\left[\begin{array}{l}
X_{t} \\
Y_{t} \\
Z_{t}
\end{array}\right]=\left[\begin{array}{ccc}
\cos (\beta+\Delta \beta) & 0 & -\sin (\beta+\Delta \beta) \\
0 & 1 & 0 \\
\sin (\beta+\Delta \beta) & 0 & \cos (\beta+\Delta \beta)
\end{array}\right]\left[\begin{array}{l}
X \\
Y \\
Z
\end{array}\right]} \\
& \Rightarrow\left[\begin{array}{l}
\Delta X_{t} \\
\Delta Y_{t} \\
\Delta Z_{t}
\end{array}\right]_{\text {pitch }}=\left[\begin{array}{c}
-(X \sin \beta+Z \cos \beta) \Delta \beta \\
0 \\
(X \cos \beta-Z \sin \beta) \Delta \beta
\end{array}\right] \tag{3}
\end{align*}
$$

According to Eq. (3), pitch angle error has an effect on the coordinates of the transducer in the X and Z directions, but not in the Y direction.

Effect of heading Angle error: the instantaneous heading angle is $\lambda$, due to the existence of heading angle error $\Delta \lambda$, the transducer actually rotates at the angle of $\lambda+\Delta \lambda$ around Z axis in XOY plane, and the effect of attitude correction on the coordinate of the transducer is:

$$
\begin{align*}
& {\left[\begin{array}{c}
X_{t} \\
Y_{t} \\
Z_{t}
\end{array}\right]=\left[\begin{array}{ccc}
\cos (\lambda+\Delta \lambda) & -\sin (\lambda+\Delta \lambda) & 0 \\
\sin (\lambda+\Delta \lambda) & \cos (\lambda+\Delta \lambda) & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
X \\
Y \\
Z
\end{array}\right]} \\
& \Rightarrow\left[\begin{array}{c}
\Delta X_{t} \\
\Delta Y_{t} \\
\Delta Z_{t}
\end{array}\right]_{\text {heading }}=\left[\begin{array}{c}
-(X \sin \lambda+Y \cos \lambda) \Delta \lambda \\
(X \cos \lambda-Y \sin \lambda) \Delta \lambda \\
0
\end{array}\right] \tag{4}
\end{align*}
$$

According to Eq. (4), pitch angle error has an influence on the coordinates of the transducer in the X and Y directions, but not in the Z direction.

In the actual motion of the vessel, several errors usually exist together, and the combined effects on the coordinate of the transducer is as follows:

$$
\begin{align*}
& {\left[\begin{array}{l}
X_{t} \\
Y_{t} \\
Z_{t}
\end{array}\right]=\boldsymbol{R}_{\text {heading }}(\lambda+\Delta \lambda) \cdot \boldsymbol{R}_{\text {pitch }}(\beta+\Delta \beta) \cdot \boldsymbol{R}_{\text {roll }}(\alpha+\Delta \alpha)\left[\begin{array}{l}
X \\
Y \\
Z
\end{array}\right]} \\
& {\left[\begin{array}{l}
\Delta X_{t} \\
\Delta Y_{t} \\
\Delta Z_{t}
\end{array}\right]=\left[\begin{array}{l}
\frac{\partial X_{t}}{\partial \alpha} \Delta \alpha+\frac{\partial X_{t}}{\partial \beta} \Delta \beta+\frac{\partial X_{t}}{\partial \lambda} \Delta \lambda \\
\frac{\partial Y_{t}}{\partial \alpha} \Delta \alpha+\frac{\partial Y_{t}}{\partial \beta} \Delta \beta+\frac{\partial Y_{t}}{\partial \lambda} \Delta \lambda \\
\frac{\partial Z_{t}}{\partial \alpha} \Delta \alpha+\frac{\partial Z_{t}}{\partial \beta} \Delta \beta+\frac{\partial Z_{t}}{\partial \lambda} \Delta \lambda
\end{array}\right]} \tag{5}
\end{align*}
$$

In order to reflect the effects of three main attitude errors on the coordinates of the transducer more directly, the deviation is simulated by using the measured data of Lingshan island. The dynamic PPP module of Bernese software is used for the preliminary calculation of GNSS data, and the high-precision positioning result is obtained as the GNSS antenna central coordinate of each epoch. The three kinds of measured attitude angle are combined with random errors artificially, the errors range in $\left[-0.1^{\circ}\right.$, $0.1^{\circ}$ ]. According to the Eqs. (2), (3) and (4), the effect of attitude errors on the coordinates of transducers in $\mathrm{X}, \mathrm{Y}$ and Z direction is analyzed:


Fig. 2. The effect of attitude angle errors on the transducers coordinates in three directions (m). Blue curves in the left panels denote the effect in X direction. Orange curves in the middle panels denote the effect in Y direction. Red curves in the right panels denote the effect in Z direction.

The following conclusions can be drawn from the analysis of Fig. 2:
(1) When the three attitude errors exist alone, the deviation on the transducer coordinate are within 1 m , while under the combined effects of all attitude errors, the coordinate deviation of the transducer is close to 2 m .
(2) If only a single error is considered, the roll angle error has no effect in the X direction, and the pitch angle error has no effect in the Y direction. In both directions, the heading angle error has a greater impact on coordinates and the error sequence fluctuates more violently. In the Z direction, the roll angle error has the most significant influence on coordinates, while heading angle error has no effect. The longitudinal comparison results show that the roll angle error has a greater impact on the Z-direction coordinate than the Y-direction coordinate, the pitch angle error has a greater impact on the X -direction coordinate than the Z-direction coordinate, and the heading angle error has a greater impact on the X-direction coordinate than the Y-direction coordinate.
(3) In the actual operation, the three kinds of attitude error often affect transducer coordinate together. It can be found that the error interval in the simulation has resulted in the nonnegligible deviation of the converted coordinates. If such coordinates are used in the subsequent underwater solution, the accuracy of the result will be greatly reduced, so it is necessary to take the appropriate method to reduce the attitude error effect.

## 3 The GNSS/Acoustic One-Step Positioning Model with Attitude Parameters

### 3.1 Traditional Data Processing Methods

At present, when the absolute coordinates of the seafloor transponder are obtained through data processing, it is generally believed that the observation accuracy of GNSS on the sea is much higher than that of underwater signals. Therefore, the two parts are often separated and adjusted independently. (1) Using the observation data between the satellite and the shipborne GNSS antenna on the sea to implement the dynamic positioning, then the absolute coordinates of the shipborne antenna in the WGS84 coordinate system can be obtained; (2) Based on the attitude parameters provided by the shipborne attitude and direction finding sensor, the conversion matrix is constructed to transform the positioning coordinate from the GNSS antenna center to the transducer center; (3) By using the transducer coordinates and the observation signal between shipborne transducer and seafloor transponder underwater, the adjustment is carried out to obtain the absolute coordinates of the seafloor transponder. In this paper, this method is called "step-by-step method". With the development of measuring equipment and technical means, underwater acoustic positioning has been able to achieve the accuracy of decimeter or higher [6, 7]. The decrease of accuracy difference between the two positioning means provides a new way to calculate the seafloor point. On this basis, this paper proposes an "one-step method" to solve the positioning of seafloor points, that is, GNSS positioning and underwater positioning are combined to solve the problem as a whole.

### 3.2 The Function Model of "One-Step Method"

According to the principle of positioning on the sea and underwater, corresponding observation equations can be given. The original data received on the sea includes GNSS pseudo-range data and carrier phase data, both observations are combined to eliminate ionospheric delays, as shown in the following equation:

$$
\left\{\begin{array}{c}
P=\sqrt{\left(x_{\text {sat }}-x\right)^{2}+\left(y_{s a t}-y\right)^{2}+\left(z_{s a t}-z\right)^{2}}+c d T+M_{w} z p d_{w}+\varepsilon_{P}  \tag{6}\\
\Phi=\sqrt{\left(x_{\text {sat }}-x\right)^{2}+\left(y_{s a t}-y\right)^{2}+\left(z_{\text {sat }}-z\right)^{2}}+c d T+M_{w} z p d_{w}+N \lambda+\varepsilon_{\Phi} \\
\rho=\sqrt{\left(x_{t}-x_{\rho}\right)^{2}+\left(y_{t}-y_{\rho}\right)^{2}+\left(z_{t}-z_{\rho}\right)^{2}}+\delta \rho_{d}+\delta \rho_{v}+\varepsilon
\end{array}\right.
$$

Where, $P, \Phi$ and $\rho$ respectively represent the pseudo-range observation, carrier phase observation and the calculated distance of acoustic signal from transducer to transponder; $\left(x_{\text {sat }}, y_{\text {sat }}, z_{\text {sat }}\right)$ is the coordinate of GNSS satellite, $(x, y, z)$ is the coordinate of shipborne antenna, $\left(x_{\rho}, y_{\rho}, z_{\rho}\right)$ is the coordinate of responder and $\left(x_{t}, y_{t}, z_{t}\right)$ is the coordinate of transducer; $c d T$ and $z p d_{w} M_{w}$ respectively represent the influence of clock offset on the receiving device of the vessel and the influence of tropospheric zenith path delay, $\lambda N$ represent the influence of the integer ambiguity, $\delta \rho_{d}$ represent the systematic error caused by the receiving time delay of the transducer, and $\delta \rho_{\mathrm{v}}$ represent
the systematic error caused by the space-time change of the sound velocity structure. $\varepsilon_{P}, \varepsilon_{\Phi}$ and $\varepsilon$ respectively represent the unmodeled errors of the two parts.

After linearization of Eq. (6), the error equation can be written as follows:

$$
\left[\begin{array}{l}
\boldsymbol{V}_{P}  \tag{7}\\
\boldsymbol{V}_{\Phi} \\
\boldsymbol{V}_{\rho}
\end{array}\right]=\left[\begin{array}{l}
\boldsymbol{A}_{P} \\
\boldsymbol{A}_{\Phi} \\
\boldsymbol{A}_{\rho}
\end{array}\right] \delta \boldsymbol{X}-\left[\begin{array}{l}
\boldsymbol{L}_{P} \\
\boldsymbol{L}_{\Phi} \\
\boldsymbol{L}_{\rho}
\end{array}\right]
$$

This is the error equation of GNSS/acoustic one-step method calculation model. Where $\boldsymbol{V}=\left[\begin{array}{lll}\boldsymbol{V}_{P} & \boldsymbol{V}_{\Phi} & \boldsymbol{V}_{\rho}\end{array}\right]^{\mathrm{T}}, \boldsymbol{L}=\left[\begin{array}{lll}\boldsymbol{L}_{P} & \boldsymbol{L}_{\Phi} & \boldsymbol{L}_{\rho}\end{array}\right]^{\mathrm{T}}$ respectively represent residual and free terms of the error equation, and the specific form of the parameters to be estimated is $\delta \boldsymbol{X}=\left[\begin{array}{lllllllll}\delta x & \delta y & \delta z & \delta t_{r} & \delta z p d_{w} & \delta x_{\rho} & \delta y_{\rho} & \delta z_{\rho} & \delta N_{(i \in[1, n s a t])}^{i}\end{array}\right]^{\mathrm{T}}$.

### 3.3 Introduction of Attitude Parameters

In the actual data processing, GNSS positioning can only obtain the GNSS antenna central geographic coordinates, which cannot be directly used as the surface datum for underwater positioning. In the traditional "step-by-step method", coordinate transformation is carried out before underwater positioning. In order to describe the "one-step method" function model more accurately, this paper proposes to take attitude parameters as estimated parameters to participate in the joint adjustment.

To get the error equation with attitude parameters, we can start from the process of coordinate transformation. As shown in Fig. 3, GNSS dynamic positioning can obtain the GNSS antenna central geographic coordinates in the WGS-84 coordinate system. After the coordinate transformation, the final need is the transducer geographic coordinates in the WGS-84 coordinate system. This process is also accompanied by the coordinate offset caused by the attitude changes of the hull at any time (mainly referring to the influence of roll, pitch and heading angle).


Fig. 3. Coordinate transformation

From the analysis above, coordinate transformation includes two steps [8]. First, it is necessary to solve the problem of coordinate transformation of the same point in different coordinate systems. To make better use of the obtained GNSS antenna coordinates, the subsequent hull coordinate system will be defined with the GNSS
antenna center point as the ordinate origin instead of the gravity point of the hull. After some axis rotation (including the opposite of the Y-axis, a rotation around the reverse Y-axis and the Z-axis), the transducer coordinates in the hull coordinate system can be transformed into the WGS-84 coordinate system. However, considering the attitude change, there are also coordinate offsets in the inner hull system. To solve this problem, an un-deformable "connecting rod" is assumed to connect the GNSS antenna center and the transducer center. When the initial relative positions of various sensors are calibrated before departure, its lengths can be obtained. Then, when the hull's attitude changes, the "connecting rod" can play a transmission role. The coordinate offset of GNSS antenna center is reflected by the coordinate offset of the transducer. Formula (8) can be used to summarize the above coordinate transformation process:

$$
\left[\begin{array}{l}
x_{t}  \tag{8}\\
y_{t} \\
z_{t}
\end{array}\right]=\boldsymbol{R}_{L} \cdot \boldsymbol{R}_{B} \cdot\left[\begin{array}{lll}
1 & & \\
& -1 & \\
& & 1
\end{array}\right] \cdot \boldsymbol{R}_{\text {heading }} \cdot \boldsymbol{R}_{\text {pitch }} \cdot \boldsymbol{R}_{\text {roll }} \cdot\left[\begin{array}{l}
0 \\
0 \\
l
\end{array}\right]+\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]
$$

where $L$ is longitude, $B$ is latitude, head is heading angle, pitch is pitch angle, roll is roll angle, $l$ is the assumed length of "connecting rod".

According to Eq. (8), the underwater observation equation can be rewritten as:

$$
\begin{equation*}
\rho=\sqrt{\left.[x+\Delta x)-x_{\rho}\right]^{2}+\left[(y+\Delta y)-y_{\rho}\right]^{2}+\left[(z+\Delta z)-z_{\rho}\right]^{2}}+\delta \rho_{d}+\delta \rho_{v}+\varepsilon \tag{9}
\end{equation*}
$$

$\Delta x($ roll, pitch, heading $), \Delta y$ (roll, pitch, heading) and $\Delta z($ roll, pitch, heading $)$ are coordinates increments determined by three attitude parameters jointly. Based on Eq. (7), three attitude parameters are added to the estimated parameters. The results of partial derivatives of attitude parameters are obtained according to Eq. (9), and the corresponding terms of each attitude parameter are added to the coefficient matrix. Then the GNSS/acoustic "one-step" positioning model with attitude parameters can be obtained.

## 4 Experiments and Results Analysis

In order to evaluate the effect of attitude error on the results of seafloor transponder positioning, and the application effect of GNSS/acoustic joint filtering with attitude parameters, a group of measured data are used in the subsequent experiments based on the software GNSSer under the different conditions of error ranges.

The data was measured in the waters of Lingshan island on December 1, 2017. Carrying sensors such as Applanix's POS-MV positioning system, the sound velocity profiler, etc., the ship made about 75 min of observations on the sea. The depth in the area is about 25 m , and the sampling interval of all the observed data is preprocessed to 2 s . The surface sound velocity of the transducer is obtained by gradient formula each epoch, and the initial incident angle is obtained by iteration using the method in literature [9]. In advance, Bernese software was used for the preliminary calculation of the GNSS observations. In addition, the coordinate of the transponder is determined by
the equal-gradient sound velocity tracking algorithm in the same layer [10], which is used as the reference value for the comparison of experimental results

In subsequent experiments, the three kinds of measured attitude angles are added with random errors in each epoch, the errors range in $\left[-0.02^{\circ}, 0.02^{\circ}\right],\left[-0.03^{\circ}, 0.03^{\circ}\right]$, $\left[-0.05^{\circ}, 0.05^{\circ}\right],\left[-0.08^{\circ}, 0.08^{\circ}\right]$ and $\left[-0.1^{\circ}, 0.1^{\circ}\right]$, corresponding to scheme $1,2,3,4$ and 5 respectively. The 3D coordinate errors of the experimental results are obtained by using the formula $3 D=\sqrt{\Delta X^{2}+\Delta Y^{2}+\Delta Z^{2}}$. The comparison results are shown as follows:


Fig. 4. The comparison results of the three-dimensional (3D) positioning errors under different conditions of attitude error ranges. Green curves in the left panels denote the effect of attitude errors on 3D positioning results, which is the deviation sequence between the situation with attitude errors and the original positioning results. Red curves in the right panels denote the improvement of positioning effect after the introduction of attitude parameters, which is the deviation sequence between the new model's positioning results and the situation with attitude errors.

## As shown in Fig. 4:

(1) With the addition of random attitude errors, the three-dimensional coordinate errors of the transponder in each scheme are significantly increased. When the change of the attitude error range within $\left[-0.03^{\circ}, 0.03^{\circ}\right]$, the amplified errors compared with the results without attitude errors has exceeded 1.5 m in some epochs. This is consistent with the conclusions in Sect. 2.2.
(2) After the attitude parameters are added to the filtering model for estimation, the 3D transponder coordinate error results of each scheme are improved to varying degrees due to the more realistic description of the inner function relations and the more accurate description of the motion state. and the greater the variation interval of the attitude error is, the more obvious the improvement effect is. Besides, when
the range of attitude error is larger, the improvement effect is more obvious. When the change of the attitude error range within $\left[-0.1^{\circ}, 0.1^{\circ}\right.$ ], the three-dimensional point position accuracy of the transponder can be improved by nearly 2 m in some epochs.

In addition, the STD value and RMS value of positioning deviation in scheme 1 to scheme 5 are counted, and the results are shown in the following table:

Table 1. The STD and RMS values of the deviation results under different schemes in three directions. (1) Denotes the difference between the positioning results with the introduction of attitude errors and the reference value. (2) Denotes the statistical results of the deviation between the positioning results of the model proposed this paper and the reference value.

|  |  | STD/m |  |  | RMS/m |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\Delta X$ | $\Delta Y$ | $\Delta Z$ | $\Delta X$ | $\Delta Y$ | $\Delta Z$ |
| Scheme 1 | (1) | 0.9577 | 0.7153 | 1.5645 | 0.9719 | 0.7304 | 1.5698 |
|  | (2) | 0.8633 | 0.6952 | 1.4947 | 0.8777 | 0.7111 | 1.4998 |
| Scheme 2 | (1) | 1.0080 | 0.7051 | 1.5947 | 1.0291 | 0.7211 | 1.5977 |
|  | (2) | 0.8722 | 0.6960 | 1.5057 | 0.8872 | 0.7119 | 1.5108 |
| Scheme 3 | (1) | 1.0980 | 0.8489 | 1.8374 | 1.1208 | 0.8667 | 1.8427 |
|  | (2) | 0.8800 | 0.6879 | 1.5102 | 0.8959 | 0.7035 | 1.5152 |
| Scheme 4 | (1) | 1.1726 | 0.8215 | 1.8687 | 1.1992 | 0.8393 | 1.8716 |
|  | (2) | 0.8808 | 0.6699 | 1.5017 | 0.8972 | 0.6850 | 1.5068 |
| Scheme 5 | (1) | 1.1703 | 0.8757 | 1.9325 | 1.1962 | 0.8937 | 1.9374 |
|  | (2) | 0.8854 | 0.6672 | 1.5108 | 0.9017 | 0.6820 | 1.5161 |

By analyzing the results in Table 1, it can be seen that:
(1) With the expansion of attitude error range, the STD value and RMS value of GNSS/acoustic joint positioning results also show a trend of gradual increase. When the random errors range within $\left[-0.1^{\circ}, 0.1^{\circ}\right]$, the RMS value of deviation results of transponder coordinates in $\mathrm{X}, \mathrm{Y}$ and Z direction is close to $1.2 \mathrm{~m}, 0.9 \mathrm{~m}$ and 2 m respectively;
(2) After the introduction of attitude parameters, the STD value and RMS value in three directions are reduced to different degrees in each scheme. This indicates that the fluctuation and the obvious jumps of filtering result sequence decreases, the result is relatively more stable and the positioning effect is significantly improved. Among them, the improvement effect of scheme 5 is the most obvious. In terms of positioning stability, it has been improved by $24.34 \%, 23.81 \%$ and $21.82 \%$ in X, Y and Z direction respectively. In terms of positioning accuracy, it has been improved by $24.62 \%, 23.69 \%$ and $21.75 \%$ in $\mathrm{X}, \mathrm{Y}$ and Z direction respectively. The above results show that introducing attitude parameters into the estimation can make the function model more accurate and the result of transponder positioning more precise.

## 5 Conclusion

One of the important factors affecting the accuracy of dynamic positioning is the adequacy of the assumptions of the motion model. The GNSS/acoustic joint positioning model considering attitude parameters is more reasonable and realistic to describe the information of motion state, especially when there are attitude errors in the movement process. The model proposed in this paper can effectively reduce the influence of gross errors, obtain more stable filtering result sequence and higher accuracy of the transponder coordinate. Precision in X and Y direction is within 1 m , which can reach about 1.5 m in Z direction.

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