Synchronous Property—Key Fact on Quantum Interferences

Particle Simulation on Double Path Experiment

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Abstract Double-slit experiment plays a key role in Quantum Theory to distinct particle and wave interactions according to Feynman's claims. In this chapter, double path model and variant logic principle are applied to establish a simulation system for exhaustive testing targets. Using Einstein quanta interaction, different measure quaternion structures are investigated. Under Symmetry/Anti-symmetry and Synchronous/Asynchronous interaction conditions, eight groups of statistical results are generated as eight histograms to show their distributions. From this set of simulation results, it can be recognized that the synchronous condition is the key fact to generate quantum wave interference patterns and, in addition, the asynchronous condition is the key fact to make classic particle distributions. Sample results are illustrated and explanations are discussed.

Keywords Double path · Interaction · Probability · Statistics · Simulation

1 Introduction

Feynman explored quantum measurement puzzles deeply [1, 2] and emphasized: "The entire mystery of quantum mechanics is in the double-slit experiment." This experiment directly illustrates both classical and quantum interactive results. Under single and double slit conditions, dual visual distributions are shown in particle and wave statistical distributions linked to von Neumann's measure theory [3].

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From the 1970s, piloted by CHSH [4], Aspect used experiments to test Bell inequalities [5–7]. After 40 years of development, many accurate experiments [8–10] have been performed successfully worldwide using Laser, NMRI, large molecular, quantum coding, and quantum communication approaches [5–8, 11–26].

In this chapter, a double path model is established using the Mach–Zehnder interferometer. Different approaches of quantum measures: Einstein, CHSH, and Aspect are investigated by quaternion structures. Under multiple-variable logic functions and variant principle, logic functions can be transferred into variant logic expression as variant measures. Under such conditions, a variant simulation model is proposed. A given logic function f can be represented as two meta-logic functions f_+ and f_- to simulate single and double path conditions. N bits of input vectors are exhausted by 2^N states for measured data, recursive data are organized into eight histograms. Results are determined by symmetry/anti-symmetry properties evident in these histograms. Both results are obtained consistently from this model on synchronous/asynchronous conditions. Based on this set of simulation results, synchronous condition shows significant relationship linked to interference properties.

2 Double Path Model and Their Measures

2.1 Mach–Zehnder Interferometer Model

The Mach–Zehnder interferometer is the most popular device [6, 20] to support Young’s double-slit experiment.

In Fig. 1a, a double path interferometer is shown. An input signal X under control function f causes Laser LS to emit the output signal ρ under BP (Bi-polarized filter) operation output a pair of signals: ρ^+ and ρ^- . Both signals are processed by SW output ρ_L^+ and ρ_R^- , and then IM to generate output signals $IM(\rho_L^+, \rho_R^-)$. In Fig. 1b, a representation model has been described with the same signals being used.

Fig. 1 Double path model **a** Mach–Zehnder double path model, **b** Description model

2.2 Emission and Absorption Measures of Quantum Interaction

Einstein established a model to describe atomic interaction [27–30] with radiation in 1916. For two-state systems, let a system have two energy states: the ground state E_1 and the excited state E_2 . Let N_1 and N_2 be the average numbers of atoms in the ground and excited states, respectively. The numbers of states are changed from an emission state E_2 to E_1 with a rate $\frac{dN_{21}}{dt}$, in the same time; the numbers of ground states are determined by absorbed energies from E_1 to E_2 with a rate $\frac{dN_{12}}{dt}$, respectively. Let N_{12} be the number of atoms from E_1 to E_2 and N_{21} be the numbers from E_2 to E_1 . In Einstein's model, a measurement quaternion is $\langle N_1, N_2, N_{12}, N_{21} \rangle$.

CHSH proposed spin measures testing Bell inequalities [4, 6]. They applied $\perp \rightarrow +$ and $\parallel \rightarrow -$ to establish a measurement quaternion

$$\langle N_{++}(a, b), N_{+-}(a, b), N_{-+}(a, b), N_{--}(a, b) \rangle.$$

Experimental testing of Bell inequalities was performed by Aspect [5] in 1982. Four parameters are measured: transmission rate N_t , reflection rate N_r , correspondent rate N_c , and the total number N_ω in ω time period. This set of measures is a quaternion $\langle N_t, N_r, N_c, N_\omega \rangle$. Among these, N_c is a new data type not in Einstein and CHSH methods, this parameter could be an extension of synchronous/asynchronous time-measurement.

3 Simulation Systems

3.1 Simulation Model

Using variant principle described in the next subsections, a N bit 0-1 vector X and a given logic function f , all N bit vectors are exhausted, variant measures generate two groups of histograms. This variant simulation system is composed of three components: Pre-process, Interaction, and Post-process, respectively, and shown in Fig. 2.

In Fig. 2a, three components of the variant simulation model are presented. At the pre-process, a N bit 0-1 vector X and a function f feed in to output a signal ρ . After interactive component process, two groups of signals are the output: u for symmetry group and v for anti-symmetry group. In the post-process, all N bit vectors are processed by pre-processing and interactive components until all of the 2^N data set has been processed to transform symmetry and anti-symmetry signals into eight histograms: four for symmetry distributions and another four for anti-symmetry distributions.

Fig. 2 Variant simulation system; **a** Variant simulation system; **b** interactive component

In Fig. 2b, only the interaction component is selected, input signal ρ processed by BP to generate two signals $\{\rho_-, \rho_+\}$. SW output triple signals $\{\rho_-, 1 - \rho_-, \rho_+\}$ though IM to generate two groups of signals u and v .

3.2 Variant Principle

The variant principle is based on n -variable logic functions [31–33]. For any n -variables, $x = x_{n-1} \dots x_i \dots x_0$, $0 \leq i < n, x_i \in \{0, 1\} = B_2$. Let a position j be the selected bit $0 \leq j < n$, $x_j \in B_2$ be the selected variable. Let output variable y and n -variable function f , $y = f(x)$, $y \in B_2$, $x \in B_2^n$. For all states of x , a set $S(n)$ composed of the 2^n states can be divided into two sets: $S_0^j(n)$ and $S_1^j(n)$.

$$\begin{cases} S_0^j(n) = \{x | x_j = 0, \forall x \in B_2^n\} \\ S_1^j(n) = \{x | x_j = 1, \forall x \in B_2^n\} \\ S(n) = \{S_0^j(n), S_1^j(n)\} \end{cases}$$

For a given logic function f , there are input and output pair relationships to define four meta-logic functions $\{f_\perp, f_+, f_-, f_T\}$:

$$\begin{cases} f_\perp(x) = \{f(x) | x \in S_0^j(n), y = 0\} \\ f_+(x) = \{f(x) | x \in S_0^j(n), y = 1\} \\ f_-(x) = \{f(x) | x \in S_1^j(n), y = 0\} \\ f_T(x) = \{f(x) | x \in S_1^j(n), y = 1\} \end{cases}$$

Two logic canonical expressions: AND-OR form is selected by $\{f_+(x), f_T(x)\}$ as $y=1$ items, and OR-AND form is selected from $\{f_-(x), f_\perp(x)\}$ as $y=0$ items. Considering $\{f_T(x), f_\perp(x)\}$, $x_j = y$ items, they are invariant themselves.

To select $\{f_+(x), f_-(x)\}$; $x_j \neq y$ forming variant logic expression. Let $f(x) = \langle f_+|x|f_- \rangle$ be a variant logic expression. Any logic function can be expressed as a variant logic form. In $\langle f_+|x|f_- \rangle$ structure, f_+ selected 1 item in $S_0^j(n)$ as same as the AND-OR standard expression, and f_- selecting relevant parts as same as the OR-AND expression 0 items in $S_1^j(n)$. For a convenient understanding of variant representation, two-variable logic structures are illustrated for its 16 functions shown in Table 1.

For example, checking two functions $f = 3$ and $f = 12$:

$$\begin{aligned} \{f = 3 := \langle 0 | 3 \rangle, f_+ = 11 := \langle 0 | \phi \rangle, f_- = 2 := \langle \phi | 3 \rangle\} \\ \{f = 12 := \langle 2 | 1 \rangle, f_+ = 14 := \langle 2 | \phi \rangle, f_- = 8 := \langle \phi | 1 \rangle\} \end{aligned}$$

3.3 Variant Measures

Let Δ be variant measure function [23, 33].

$$\Delta = \langle \Delta_\perp, \Delta_+, \Delta_-, \Delta_T \rangle$$

$$\begin{aligned} \Delta f(x) &= \langle \Delta_\perp f(x), \Delta_+ f(x), \Delta_- f(x), \Delta_T f(x) \rangle \\ &= \langle \Delta f_\perp(x), \Delta f_+(x), \Delta f_-(x), \Delta f_T(x) \rangle \end{aligned}$$

$$\Delta f_\alpha(x) = \begin{cases} 1, & \text{if } f(x) = f_\alpha(x), \alpha \in \{\perp, +, -, T\} \\ 0, & \text{others} \end{cases}$$

For any given n-variable state there is one position in $\Delta f(x)$ to be 1 and other three positions are 0.

For any N bit 0-1 vector X ; $X = X_{N-1} \dots X_J \dots X_0$, $0 \leq J < N$, $X_J \in \beta_2$, $X \in \beta_2^N$ under n-variable function f , n bit 0-1 output vector Y , $Y = f(X) = \langle f_+|X|f_- \rangle$, $Y = Y_{N-1} \dots Y_J \dots Y_0$, $0 \leq J < N$, $Y_J \in \beta_2$, $Y \in \beta_2^N$.

For the J th position, be $x^J = [\dots X_J \dots] \in \beta_2^n$ to form $Y_J = f(x^J) = \langle f_+|x^J|f_- \rangle$, let N bit positions be cyclic linked. Variant measures of $f(X)$ can be decomposed

$$\Delta \langle X : Y \rangle = \Delta f(X) = \sum_{J=0}^{N-1} \Delta f(x^J) = \langle N_\perp, N_+, N_-, N_T \rangle$$

as a quaternion $\langle N_\perp, N_+, N_-, N_T \rangle$.

For example, $N = 10$, given f , $Y = f(X)$.

Table 1 Two variable logic functions and variable logic representation ($n = 2, j = 0$)

f No.	$f \in$ $S(2)$	3 11	2 10	1 01	0 00	$f_+ \in$ $S_0^0(2)$	3^0 11^0	2^1 10^1	1^0 01^0	0^1 00^1	$f_- \in$ $S_1^0(2)$
0	$\{\emptyset\}$	0	0	0	0	$\langle \emptyset $	1	0	1	0	$ 3,1\rangle$
1	$\{0\}$	0	0	0	1	$\langle 0 $	1	0	1	1	$ 3,1\rangle$
2	$\{1\}$	0	0	1	0	$\langle \emptyset $	1	0	0	0	$ 3\rangle$
3	$\{1,0\}$	0	0	1	1	$\langle 0 $	1	0	0	1	$ 3\rangle$
4	$\{2\}$	0	1	0	0	$\langle 2 $	1	1	1	0	$ 3,1\rangle$
5	$\{2,0\}$	0	1	0	1	$\langle 2,0 $	1	1	1	1	$ 3,1\rangle$
6	$\{2,1\}$	0	1	1	0	$\langle 2 $	1	1	0	0	$ 3\rangle$
7	$\{2,1,0\}$	0	1	1	1	$\langle 2,0 $	1	1	0	1	$ 3\rangle$
8	$\{3\}$	1	0	0	0	$\langle \emptyset $	0	0	1	0	$ 1\rangle$
9	$\{3,0\}$	1	0	0	1	$\langle 0 $	0	0	1	1	$ 1\rangle$
10	$\{3,1\}$	1	0	1	0	$\langle \emptyset $	0	0	0	0	$ \emptyset\rangle$
11	$\{3,1,0\}$	1	0	1	1	$\langle 0 $	0	0	0	1	$ \emptyset\rangle$
12	$\{3,2\}$	1	1	0	0	$\langle 2 $	0	1	1	0	$ 1\rangle$
13	$\{3,2,0\}$	1	1	0	1	$\langle 2,0 $	0	1	1	1	$ 1\rangle$
14	$\{3,2,1\}$	1	1	1	0	$\langle 2 $	0	1	0	0	$ \emptyset\rangle$
15	$\{3,2,1,0\}$	1	1	1	1	$\langle 2,0 $	0	1	0	1	$ \emptyset\rangle$

$$\begin{array}{rcll}
X & = & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
Y & = & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
\Delta(X : Y) & = & + & - & T & \perp & + & - & T & - & + & \perp
\end{array}$$

$$\Delta f(X) = \langle N_{\perp}, N_{+}, N_{-}, N_T \rangle = \langle 2, 3, 3, 2 \rangle, N = 10$$

Input and output pairs are 0-1 variables on the four combinations. For any given function f , the quantitative relationship of $\{\perp, +, -, T\}$ is determined directly from input/output sequences.

3.4 Measurement Equations

Using variant quaternion, signals are calculated by following equations. For any N bit 0-1 vector X , function f , under Δ measurement: $\Delta f(x) = \langle N_{\perp}, N_{+}, N_{-}, N_T \rangle$, $N = N_{\perp} + N_{+} + N_{-} + N_T$ Signal ρ is defined by

$$\rho = \frac{\Delta f(x)}{N} = \langle \rho_{\perp}, \rho_{+}, \rho_{-}, \rho_T \rangle$$

$$\rho_{\alpha} = \frac{N_{\alpha}}{N}, \quad 0 \leq \rho_{\alpha} \leq 1, \quad \alpha \in \{\perp, +, -, T\}$$

Using $\{\rho_{+}, \rho_{-}\}$, a pair of signals $\{u, v\}$ are formulated:

$$\begin{cases} u = \langle u_0, u_{+}, u_{-}, u_1 \rangle = \{u_{\beta}\} \\ v = \langle v_0, v_{+}, v_{-}, v_1 \rangle = \{v_{\beta}\} \end{cases}$$

$$\beta \in \{0, +, -, 1\}$$

$$\begin{cases} u_0 = \rho_{-} \oplus \rho_{+} \\ v_0 = (1 - \rho_{-})/2 \oplus (1 + \rho_{+})/2 \\ u_{+} = \rho_{+} \\ v_{+} = (1 + \rho_{+})/2 \\ u_{-} = \rho_{-} \\ v_{-} = (1 - \rho_{-})/2 \\ u_1 = \rho_{-} + \rho_{+} \\ v_1 = (1 - \rho_{-} + \rho_{+})/2 \end{cases}$$

where $0 \leq u_\beta, v_\beta \leq 1, \beta \in \{0, +, -, 1\}$, $\oplus$: Asynchronous addition, $+$: Synchronous addition.

Using $\{u, v\}$ signals, each u_β (v_β) determines a fixed position in the relevant histogram to make vector X on a position. After complete 2^N data sequences, eight symmetry/anti-symmetry histograms of $\{H(u_\beta|f)\}(\{H(v_\beta|f)\})\beta \in \{0, +, -, 1\}$ are generated.

4 Simulation Results

The simulation provides a series of output results. In this section, two cases are selected: $N = \{12, 13\}$, $n = 2, j = 0, \{f = 3, f_+ = 11, f_- = 2\}$, and $\{f = 12, f_+ = 14, f_- = 8\}$. Corresponding to double path, right path, left path, symmetric and nonsymmetric conditions, respectively. For the convenience of comparison, sample cases are shown in Fig. 3a–c. In Fig. 3a, representation patterns are illustrated. Figure 3b represents $f = 3$ conditions and Fig. 3c represents $f = 12$ conditions, respectively. Eight histograms of $H(u_+|f) = H(u_-|f)$ are shown with results represented by symmetric meta-functions in four groups.

5 Analysis of Results

5.1 Visual Distributions

In $H(u_+|f) = H(u_-|f)$ conditions, $\{H(u_1|f), H(v_1|f)\}$ have significant interference patterns different from other conditions. Output results are balanced.

5.2 Particle Statistical Distributions

For all symmetric or nonsymmetric cases under $\oplus$ asynchronous addition operations, relevant values meet $0 \leq u_0, v_0, u_-, v_-, u_+, v_+ \leq 1$.

Checking $\{H(u_0|f), H(v_0|f)\}$ series, $\{H(u_+|f), H(u_-|f)\}$ and $\{H(v_+|f), H(v_-|f)\}$ satisfy the following equation:

$$\begin{cases} H(u_0|f) = H(u_-|f) + H(u_+|f) \\ H(v_0|f) = H(v_-|f) + H(v_+|f) \end{cases}$$

The equation is true even N and n in different values.

N	Left Path	Right Path	Double-Particle	Double-Wave	Conditions
12	$H(u_+ f)$	$H(u_- f)$	$H(u_0 f)$	$H(u_1 f)$	Symmetric Meta Distributions $H(u_+ f) = H(u_- f)$
13	$H(u_+ f)$	$H(u_- f)$	$H(u_0 f)$	$H(u_1 f)$	
12	$H(v_+ f)$	$H(v_- f)$	$H(v_0 f)$	$H(v_1 f)$	Anti-symmetric Meta Distributions $H(v_+ f) = H(1 - v_- f)$
13	$H(v_+ f)$	$H(v_- f)$	$H(v_0 f)$	$H(v_1 f)$	

(a) Statistical Histogram Patterns

(b) $N=\{12, 13\}$, $f = 3$, Histograms of Symmetric Meta Distributions

Fig. 3 Results of symmetric meta distributions

(c) $N=\{12, 13\}$, $f = 12$, Histograms of Symmetric Meta Distributions

Fig. 3 (continued)

5.3 Wave Interference Patterns

Different interference properties are observed clearly in $H(u_+|f) = H(u_-|f)$ and $H(v_+|f) = H(1 - v_-|f)$ conditions. Under synchronous addition operations, relevant values meet $0 \leq u_1, v_1, u_-, v_-, u_+, v_+ \leq 1$.

Checking $\{H(u_1|f), H(v_1|f)\}$ distributions especially in Fig. 3b-c $\{u_1, v_1\}$ cases extremely strong interferences appeared and compared with $\{H(u_+|f), H(u_-|f)\}$ and $\{H(v_+|f), H(v_-|f)\}$, there are significant differences. Spectra in different cases illustrate wave interference properties. From listed histogram distributions, they are all satisfied

$$\begin{cases} H(u_1|f) \neq H(u_-|f) + H(u_+|f) = H(u_0|f) \\ H(v_1|f) \neq H(v_-|f) + H(v_+|f) = H(v_0|f) \end{cases}$$

Single and double peaks are shown in interference patterns as classical double-slit distributions.

5.4 Quaternion Measures

It is interesting to see the relationship between the variant quaternion and other measures.

In the variant quaternion, $\Delta f(x) = \langle N_\perp, N_+, N_-, N_T \rangle$, $N = N_\perp + N_+ + N_- + N_T$.

In Einstein's two-state system of interaction $\langle N_1, N_2, N_{12}, N_{21} \rangle$ allows the following equations to be established:

$$\begin{cases} N_1 = N_\perp + N_+ \\ N_2 = N_- + N_T \\ N_{12} = N_+ \\ N_{21} = N_- \\ N = N_1 + N_2 \end{cases}$$

From the equations, the measured pair $\{N_{21}, N_{12}\}$ has a 1-1 correspondence to $\{N_-, N_+\}$.

Selecting $+\rightarrow 1, -\rightarrow 0$, CHSHs $N_{\pm,\mp}(a, b)$ measures meet

$$\begin{cases} N_{+,+}(a, b) \rightarrow N_T \\ N_{+,-}(a, b) \rightarrow N_- \\ N_{-,+}(a, b) \rightarrow N_+ \\ N_{-,-}(a, b) \rightarrow N_\perp \end{cases}$$

$$(N_{++}, N_{+-}, N_{-+}, N_{--}) \rightarrow (N_T, N_-, N_+, N_\perp),$$

Let $N = N_{++} + N_{+-} + N_{-+} + N_{--}$, CHSH quaternion is a permutation of the variant quaternion.

Aspect's quaternion $(N_t, N_r, N_c, N_\omega)$ have the following corresponding:

$$\begin{cases} N_t \rightarrow N_- \\ N_r \rightarrow N_+ \\ N_\omega \rightarrow N \end{cases}$$

There is no parameter in the variant quaternion for the parameter N_c . It indicates joined action numbers to distinguish single and double paths, corresponding to

$\{u_0, v_0\}$ and $\{u_1, v_1\}$ times. In an actual experiment, this parameter is significant. In a simulated system, the parameter is a control coefficient that separates two types of measured paths $\{u_0, v_0\}$ and $\{u_1, v_1\}$ in the integration of comparisons on real experiments.

6 Conclusions

Analyzing N bit 0-1 vector and its exhaustive sequences for variant measurement, this system simulates double path interference properties through different accurate distributions. Using this model, two groups of parameters $\{u_\beta\}$ and $\{v_\beta\}$ describe the left path, right path, double paths for particle, and double path for wave with distinguished symmetry and anti-symmetry properties.

Only synchronous conditions, double path system provides wave-like interference patterns different from classical ones.

Compared with the variant quaternion and other quaternion structures, it is helpful to understand possible properties of usages and limitations for variant simulation systems.

The complexity of n -variable function space has a size of 2^{2^n} . Whole simulation complexity is determined by $O(2^{2^n} \times 2^N)$ as ultra exponent productions. How to overcome the limitations imposed by such complexity and how best to compare and contrast such simulations with real-world experimentation will be key issues in future work.

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