will then reveal their common source. Interestingly, Weil gives no compelling reasons why this should always be the case. It is a view which he presents as a kind of common knowledge that all mathematicians will somehow recognise as the truth. He illustrates this common knowledge with the history of some mathematical concepts that now stand on solid foundations. He describes the intuitive notion of a group that Lagrange tried to come to grips with as a screen which he could only touch with the tip of his fingers. Then Galois brought the concept to light. Here the notion of analogy is understood in a very wide sense and includes all the various intuitive precursors of our modern concepts. Weil also gives a beautiful description of the analogy between number fields and function fields as a 'texte trilingue', although the Rosetta stone is still to be found.

It is surprising how deeply rooted this conviction is. Somehow mathematicians tend to attribute to their analogies the same kind of 'objectivity' as mathematical objects. The statement that 'X is the analogue of Y' is of course no rigorous mathematical statement but it can be so useful to mathematicians that it acquires quite often a degree of reliability that ressembles that of a mathematical theorem or concept. Such statements are considered to be the theorems and concepts of the future, couched in the metaphysics which is implicit in the word 'analogue'.

Can the discovery of analogies be considered as a mathematical discovery? Weil mentions the care taken by Hilbert to remove all traces of such discoveries from his collected works. He writes:

> Les lois non écrites de la mathématiques moderne interdisent en effet de publier des vues métaphysiques de cette espece.

Weil regretes that Hilbert's valuable ideas, notably those on the analogy between number fields and function fields, remained almost unpublished. Weil wrote his essay in the sixties. Probably it is fair to say that these 'unwritten laws of mathematics' have become out of fashion in modern mathematics. One can see this very clearly in subjects were the pursuit of an analogy has become to some extent an end in itself; Metaphysical discoveries of this kind are then published without any precise indication how these can be put to direct use. This can be seen in areas of mathematics where 'the mysteries of the infinite prime' are at the centre. No one would criticize these habits since this part of mathematics looks so promising (and has fullfilled some of these promises). Some people think it will lead to a proof of the Riemann hypothesis.

Ironically, Weil's own view on analogy as a hidden concept is in itself a very interesting metaphysical conviction. The reasoning behind it carries a subtle element of deceit. It is based on the observation that so many modern concepts of mathematics have 'grown out of analogies'. One could also say that these concepts were at some point in history 'less clear'. Reformulated in this way, the observation has almost become a tautological statement. Apart from the natural numbers, which Kronecker identified as a divine gift, all concepts have emerged over a certain period of time. Somehow this is implicit in the idea of 'concept'. As a tautological statement it can not be used as an argument. It certainly can't be used to demonstrate that concepts are hidden behind the analogies of today.

What could be an alternative for this belief? Perhaps a comparison with physics can be illuminating. Here, the mysterious relation between nature and mathematics may help to conceive o an alternative paradigm. It is the miracle described by Wigner as the unreasonable effectiveness of mathematics in physics. Yet, it is not considered to be a problem which physicists will ever adress as a problem of physics.

Analogy arising in number theory may well be considered to be a 'natural phenomenon'. Here, it appears to be the natural means of achieving the isolation of aspects which in other parts of mathematics is achieved by abstraction and generalisation. Only the future of mathematics can decide if this apparent difference is fundamental.

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Liebe MathematikerInnen,
ich bin in der 6. Klasse und wir haben gerade periodische Dezimalbrüche durchgenommen. Wir haben gelernt: $1/9 = 0.111\ldots$, $3/9 = 0.333$ usw.
Was aber ist dann $0.999\ldots$? Unsere Lehrerin hat gesagt, das wäre $9/9$. Das kann aber doch nicht sein. Das wäre doch 1 und $0.999\ldots$ ist doch ein Unendlichstel kleiner als 1. Gibt es $0.999\ldots$ überhaupt? Aber eine Zahl, die ich mir ausdenken kann, muss es doch geben. Wie kommt man an $0.999\ldots$?
Ich würde mich über eine Antwort freuen.

Lina

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