MODELLING AND ALGORITHMS FOR LOCATION PROBLEMS

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This paper deals with several kinds of location problems, necessary optimality conditions and algorithms based on the optimality conditions.

We will study some generalizations of Fermat-Weber location problem, at first the following convex vector-valued L_p -Approximation problem:

$$(P1) \quad f(u) = \begin{pmatrix} \left\| \sum_{j=1}^n u_j f_j^1 - f_o^1 \right\| \\ \vdots \\ \left\| \sum_{j=1}^n u_j f_j^m - f_o^m \right\| \end{pmatrix} \rightarrow \underset{u \in U}{v\text{-min}}$$

with $f_j^i \in L_p(S)$, $f_o^i \in L_p(S)$, $1 \leq p \leq \infty$, $[i = 1, \dots, m]$, $S \subset \mathbb{R}^q$ compact, $U = \mathbb{R}^n$.

Since the vector optimization problem (P1) is convex, we can use a scalarization with linear continuous functionals. Applying a variational principle we get necessary conditions for approximately efficient points of (P1) by using the subdifferential calculus. These necessary conditions we will use in order to derive a proximal point algorithm for the vector optimization problem (P1).

Further, we will discuss a population model, which can be described as a nonconvex vector-valued location problem. So we can't apply the results of convex analysis. We will show relations between the efficient solutions of the vector optimization problem and a surrogate scalar optimization problem. By using the directional derivative of the objective functions and a variational principle we derive necessary conditions for approximately efficient solutions.