Line Segment Detection with Hough Transform Based on Minimum Entropy

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Abstract. The Hough transform is a popular technique used in the field of image processing. In this paper, fitting and interpolation techniques are employed to compute high-accuracy peak parameters by considering peak spreading. The entropy is selected to measure the scatter-degree of voting. The voting in each column is considered as a random variable and voting values are considered as a probabilistic distribution. The corresponding entropies are computed and used to estimate the peak parameters. Endpoint coordinates of a line segment are computed by fitting a sine curve with more cells. It is more accurate and robust compared to solving directly two equations. The proposed method is tested on simulated and real images.

Keywords: Hough transform, peak detection, entropy, endpoints.

1 Introduction

The *Hough transform* (HT) [1,2,3] is a common technique for line segment extracting. However, image noise and parameter quantization [4,5,6] cause peak spreading problems in Hough space.

In order to deal with peak spreading and to achieve accurate peaks, researchers have proposed variations and extensions to the HT.

Some papers are focused on peak enhancement [7] in accumulator space by modifying the HT voting scheme. By assigning different weights [8] or utilizing a voting kernel [9,10], peaks in Hough space become more distinct and peak finding becomes easier. Other papers aim at obtaining high accuracy peak parameters, given an existing peak. Two accurate peak detection methods are presented in [11,12]; a smoothing window and interpolation are employed to achieve accurate peak parameters. Another high accuracy HT [13] is based on the theoretical symmetry of the butterfly.

The information-theoretic measure of entropy [14,15,16] is often applied in image analysis, such as for clustering or pattern recognition [17], distortion correction [18], image alignment [19], or image thresholding [20,21]. The peak localization method presented here is motivated by information entropy. The peak is located at θ where voting is clustered and entropy is minimal.

The HT can also be used for extracting complete parameters of a line segment in an image although the HT provides only distance ρ and the angle θ .

Some methods provide a complete line-segment description using the HT. One kind of methods is based on image space [22,23,24]. After applying the standard HT, the line is broken into disjoint segments in image space. The endpoints of a line segment are determined by taking the projection of the feature points on either the x - or y -axis. Two thresholds are needed for the length of line segment and gaps between line segments. Another kind of methods is based on the Hough space. By analysing the butterfly distribution [25,26] around a peak in the accumulator array, butterfly features are used to discover parameters of a line segment. The first and last non-zero values are obtained and used to compute the endpoints of the line segment by solving two sets of simultaneous equations [27,28,29,30]. However, how to find the first and the last non-zero voting value is a difficult problem.

This paper focuses on improving accuracy and robustness of peak detection. By considering various uncertainties, the voting in each column is considered as a random variable and voting values are considered as a probabilistic distribution. Corresponding entropies are computed and used to estimate the peak parameter. Endpoint coordinates of a line are computed by fitting a sine curve with upside and downside cells in columns around a peak rather than solving two equations generated from just two cells.

The rest of the paper is organized as follows. Section 2 describes our peak detection method based on minimum entropy. Section 3 outlines endpoint detection by fitting sine curves. Section 4 compares methods by providing experimental results. Section 5 concludes.

2 Peak Detection

After voting all pixels in an image for all possible cells into Hough space, the next step is peak detection. Considering the peak spreading, we locate the peak where the voting is the most clustered.

2.1 Voting Distribution around a Peak

Under an ideal situation, voting distribution around a peak is symmetrical and convergent into a focal cell in the middle column. In each column around a peak, the sum of voting values is a constant; see Fig. 1. A peak can be found and computed easily in this case.

However, due to various uncertainties, like image noise, quantization noise, or line thickness, cells around an actual peak are usually not symmetrical and there is not a focal cell. In each column around a peak, the sum of voting values is constant only approximately; see Fig. 2. Voting is scattered in ρ direction in some θ columns. The degree of voting scattered or clustered is different in the columns. The final peak is located where the voting is the ‘most clustered’.

Fig. 1. An ideal peak region

Fig. 2. An actual peak region

2.2 Entropy of Voting

In information theory [14], if X is a discrete random variable with distribution $P(X = x_k) = p_k$ $k = 1, 2, \dots$ then the entropy of X is defined as follows:

$$E(X) = - \sum p_k \log p_k \quad (1)$$

The entropy is used to measure the uncertainty of the probability distribution. The smaller the entropy E is, the larger the cluster-degree is. Voting in each column is considered to be a random variable; entropy can be used to measure the voting scattering. In θ_i column in the peak region, the probability of voting for a cell $H_{ij}(\theta, \rho)$ is computed as $p_{ij} = H_{ij}(\theta, \rho) / \sum H_{ij}(\theta, \rho)$ where j belongs to a peak region. The entropy of the θ_i column is then

$$E_i = - \sum p_{ij} \log p_{ij} \quad (2)$$

Because voting distributions differ in different column, the E_i -values for columns also differ. The more clustered a voting is, the smaller the entropy. The more scattered a voting is, the bigger the entropy. A lower entropy distribution means that voting is clustered and can be considered as a peak. By minimizing information entropy of voting, an optimal θ peak is detected.

2.3 Peak θ Computation

Around a peak region, we perform the following operations: (1) Compute the entropy E_i of each θ_i column according to Equ. (2). Fig. 3 shows the entropy E of each θ column. (2) Fitting a curved function f .

$$f : E = f(\theta)$$

(3) Compute θ_{peak} where the fitted function has a minimum $\theta_{peak} = \theta | f'(\theta) = 0$.

2.4 Peak ρ Computation

Around a peak region, we also do: (1) Compute means

$$m_i = \sum [H_{ij}(\rho, \theta) \cdot \rho_j] / \sum H_{ij}(\rho, \theta) \quad (3)$$

of each θ column. (2) A sinusoidal curve fitting. Sine curve is a nonlinear function. Fitting directly a sine curve is difficult, so we fit it by taking advantage of a linear function. In the equation of standard HT $\rho = x \cdot \cos \theta + y \cdot \sin \theta$, the relation between ρ and θ is not linear. It can be linear by using the following transform:

$$\rho / \cos \theta = y \cdot \tan \theta + x$$

$\rho / \cos \theta$ and $\tan \theta$ are in linear relation. With all these $(m_i / \cos \theta_i, \tan \theta_i)$, a linear function is fitted. The RANSAC technique is employed by considering various uncertainties:

$$g : \rho / \cos \theta = g(\tan \theta) \triangleq b_1 \tan \theta + b_0$$

(3) Interpolation to compute ρ_{peak} corresponding to ρ_{peak} :

$$\rho_{peak} = b_0 \cos(\theta_{peak}) + b_1 \sin(\theta_{peak}) \quad (4)$$

Fig. 3. Entropy of each column around a peak

2.5 Multiple Line Detection

The existence of multiple lines in the given image leads to overlays of multiple peaks in Hough space. If two peaks are far away, peak detection is only a little influenced because the final peak parameters are fitted and interpolated. If a peak region overlaps partly with other peak regions, the voting values and entropies in the overlapped columns become bigger. These columns are neglected for fitting and interpolation.

After a peak $(\rho_{peak}, \theta_{peak})$ is detected, the voting values corresponding to this peak are removed in order not to preclude the detection of another peak. First, with $(\rho_{peak}, \theta_{peak})$, those pixels contributing to this peak are selected in the image space. Second, the votes of those pixels are inverted by decreasing value of corresponding cells by 1.

3 Endpoint Detection

We find the endpoint coordinates of a line segment, called left endpoint $p(x_l, y_l)$ and right endpoint $p(x_r, y_r)$. Left and right is defined by the origin (i.e. the perpendicular point). See Fig. 4. According to the equation of the HT, if we have two cells (θ_1, ρ_1) and (θ_2, ρ_2) where a left endpoint votes for, the coordinates of the left endpoint can be computed directly by solving two simultaneous equations

$$\begin{aligned}\rho_1 &= x \cdot \cos \theta_1 + y \cdot \sin \theta_1 \\ \rho_2 &= x \cdot \cos \theta_2 + y \cdot \sin \theta_2\end{aligned}$$

By considering various uncertainties, rather than solving directing two equations, we compute the coordinates of endpoints by fitting more cells.

3.1 Valid Cell Identification

Due to parameter quantization and image noise, a peak region is not distinct in Hough space. It is usually as in Fig. 5. Cells corresponding to endpoints can not be determined simply by identifying zero and non-zero voting values, which is used in many methods for extracting endpoints.

The collinear pixels and image noise all vote for the cells in a peak region. The voting values from collinear pixels are considered as foreground, while the voting

Fig. 4. Left endpoint and right endpoint

Fig. 5. Peak region before background separation

values from noise or other pixels are considered as background. A threshold is determined to divide cells in a peak region into background and foreground cells. In each column, a threshold is determined by a maximum inter-class variance method. Thus, background cells are separated. A peak region after background separation is shown in Fig. 6.

3.2 Endpoint Coordinate Computation

Consider foreground cells in the left column of the peak; upside cells are selected to compute the right endpoint and downside cells are selected to compute the

Fig. 6. Peak region after background separation

left endpoint. While in the right column of a peak, upside cells and downside cells are respectively used to compute right and left endpoint. Using all cell parameters (θ_i, ρ_i) for which the left endpoint votes, a sine curve is fitted.

As discussed above, $\rho/\cos\theta$ and $\tan\theta$ are in a linear relationship. With all these $(\rho_i/\cos\theta_i, \tan\theta_i)$, a linear function is fitted:

$$g : \rho/\cos\theta = g(\tan\theta) \triangleq b_1 \tan\theta + b_0 \quad (5)$$

Endpoint coordinates are then as follows: $x_l = b_0$ and $y_l = b_1$. Similarly, the right endpoint can be computed by fitting cell parameters (ρ_i, θ_i) for which the right votes.

4 Experimental Results

In this section, the proposed peak detection and endpoint extracting methods are applied to a set of simulated data to test the accuracy of line detection, and applied to real world image to verify the performance of line-segment detection.

4.1 Tests with Simulated Images

We generate $M \times N = 200 \times 200$ binary images that contain randomly generated noisy pixels. Endpoint coordinates of one line segment are also produced randomly, and line parameters (θ, ρ) are computed and recorded as ground truth.

We test the methods in term of different counts of noisy pixels, different quantization, or different line thickness. Fig. 7 shows the detection error under ideal conditions. Fig. 8 shows the detection error when parameter quantization, line thickness and image noise are comprehensively considered.

Fig. 7. Test under ideal conditions. Count of noise: 0, parameter quantization: 1, line thickness: 0.

Fig. 8. Comprehensive test. Count of noise: 500, parameter quantization: 2, line thickness: 2.

The detection results from the proposed peak detection method based on minimum entropy (*ME*) are compared with results of a method using smoothing and interpolating (*SI*) [12], and with results of least-squares fitting (*LSF*). 100 random line segments are tested in terms of the absolute error with respect to the ground truth. Detection results are shown in Fig. 9, where the count of noise is 500, parameter space is quantized as (2,2), and line thickness is 2.

Fig. 9. Comparison of detection errors

Fig. 10. Line segment detection in simulated images

With the proposed method based on minimum entropy, sub-cell accuracy is achieved. Detection results with the proposed method are close to the results of least-square fitting, even under coarse parameter quantization and image noise.

4.2 Endpoint Tests

A simulated image, which contain 500 random noise pixels and 20 random line segments, is used to test the proposed method. The parameter space is quantized with $(\Delta\theta, \Delta\rho) = (2, 2)$ and thickness of line segment is 2. Detection results are shown in Fig. 10. All line segments are detected in spit of image noise, coarse parameter quantization and line thickness.

4.3 Real World Tests

We use crystalline graphite fibre images [31] to test the proposed method for extracting line segments. A detection result is shown in Fig. 11; in this image only the 50 longest line segments are extracted. Although these fibre images are very complicated, many line segments are detected by identifying valid cells and fitting a sine curve with more cells parameters.

Fig. 11. Line segment detection for fibre image. *Left:* Original image. *Right:* Detected line segments.

5 Conclusions

A line segment detection method with Hough transform is proposed. The method for the distance and angle detection is based on minimum entropy and has been compared with other methods. Using fitting and interpolating techniques based the minimum entropy, the Hough transform can yield very accurate peak parameters that are close to results from least-squares fitting for simulated images. The method for endpoints extraction applies fitting a sine curve. It is more accurate and robust compared with methods that solve simultaneous equations.

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