Ordered Solution Generation for Implicit AND/OR Search Spaces

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Abstract. In this paper we address the problem of generating alternative solutions for implicit AND/OR DAGs which are described using a start state and a set of transformation rules. Unlike the explicit representation of AND/OR graph, for implicitly specified AND/OR graphs, the transformation rules are used to dynamically make a portion of an AND/OR graph explicit as per requirement. We propose an algorithm, ASGAO*, for generating alternative solutions for implicitly specified AND/OR graphs. Experimental results in several domains show that, on an average, our proposed algorithm performs hundred times better with respect to running time and fifty times better with respect to space than the existing algorithm. In view of the renewed research interest towards applying AND/OR graphs in domains like graphical model, constraint satisfaction, and service composition, it is not hard to see useful applications of ASGAO*.

1 Introduction

Search methods in artificial intelligence are being increasingly used to solve a wide variety of optimization problems from diverse domains like scheduling, routing, constraint satisfaction, CAD for VLSI, etc. In terms of the structure of the state space, OR-graphs, AND/OR graphs and game trees are among the most well studied, among which, AND/OR graphs enable the modeling of alternative choices in conjunctive decompositions of the problem and its subproblems. Recently there has been a renewed research interest towards applying AND/OR structures in a several areas like planning with uncertainty [1], graphical models [2], web service composition [3], etc. In view of these recent research, the problem of generating ordered solutions becomes important for AND/OR graphs. Some of important applications include – (a) developing useful variants of the AO* algorithm, (b) graphical models, service composition [4], etc. The existing algorithms [5,4] work on explicit AND/OR graphs – thus the entire search space of the problem has to be made explicit before starting the algorithm. However, making the entire search space explicit is often not required for generating a reasonable number of ordered solutions. For several domains good heuristic functions are available and these estimates can be used to restrict the portion of the search space that is made explicit. We present a new algorithm, ASGAO*, which uses the heuristic estimates to dynamically make the search space explicit like AO* while computing the ordered solutions. Moreover, for problems like matrix chain multiplication, finding secondary structure of RNA, multi-peg tower of Hanoi, etc, whose search spaces have AND/OR structures, the optimal cost of subproblems can be computed efficiently. This exact

value of the cost information can be used to further reduce the portions of AND/OR graph that is made explicit. Experimental results on domains like, including matrix chain multiplication, secondary structure of RNA, etc., reported in Section 4, demonstrate hundred times improvement in running time as well as significant improvement in space requirement over the existing algorithm on an average.

2 Background and Existing Approach

$G_{\alpha\beta} = \langle V, E \rangle$ denotes an AND/OR directed acyclic graph where V is the set of nodes, and E is the set of edges. The nodes of $G_{\alpha\beta}$ with no successors are called *terminal* nodes. The *non-terminal* nodes of $G_{\alpha\beta}$ are of two types – i) OR nodes and ii) AND nodes. The start (or root) node of $G_{\alpha\beta}$ is denoted by v_R . *OR edges* and *AND edges* are the edges that emanate from OR nodes and AND nodes respectively. We use the standard notion of *solution graph*, $S(v_q)$, rooted at any node $v_q \in V$ [6]. By a solution graph S of $G_{\alpha\beta}$ we mean a solution graph with root v_R . The notion of cost is as follows. In $G_{\alpha\beta}$, every edge $e_{qr} \in E$ from node v_q to node v_r has a finite non-negative cost $c_e(\langle v_q, v_r \rangle)$ or $c_e(e_{qr})$. Similarly every node v_q has a finite non-negative cost denoted by $c_v(v_q)$. We use the notion of additive costs for given *solution graph* S [6]. For every node v_q in S , $C(S, v_q)$ denotes the cost of the solution graph, $S(v_q)$. The cost of a solution S is $C(S, v_R)$ which is also denoted by $C(S)$. We denote the optimal solution below every node v_q as $opt(v_q)$. Therefore, the optimal solution of the entire AND/OR DAG $G_{\alpha\beta}$, denoted by S_{opt} , is $opt(v_R)$. The cost of the optimal solution rooted at every node v_q in $G_{\alpha\beta}$ is $C_{opt}(v_q)$. The cost of the optimal solution S_{opt} of $G_{\alpha\beta}$ is denoted by $C_{opt}(v_R)$ or, alternatively, by $C_{opt}(S_{opt})$. The *aggregated cost*, c_a , for an edge e_{ij} from node v_i to node v_j , is defined as : $c_a(e_{ij}) = c_e(e_{ij}) + C_{opt}(v_j)$

We briefly present the overview of the existing ASG algorithm for AND/OR trees. More details can be found in [4]. ASG algorithm works iteratively – starts by computing the optimal solution and subsequently generates the next best solution over iterations from the solutions that are already generated. In the context of AND/OR trees, e_q is used to denote the edge that points to the vertex v_q . ASG algorithm has been developed on the notion of – (a) marking of an OR edge, (b) swap option and swap operation and (b) the implicit representation of a solution. Consider a solution, S_{cur} , containing an OR edge $e_i = (v_q, v_i)$, where $S_{cur}(v_i)$ is actually $S_{opt}(v_i)$. Now, e_i is marked with the cost increment which will be incurred to construct the next best solution from S_{cur} by choosing another child of v_q , say v_j , and this difference is succinctly represented by a swap option $\sigma_{ij} = \langle e_i, e_j, \delta_{ij} \rangle$, where $\delta_{ij} = c_a(e_j) - c_a(e_i)$.

Swap operation basically denotes the application of a swap option to a solution. ASG use a swap option based compact representation, named *signature*, for storing the solutions. Intuitively, any alternative solution can be described as a set of swap operations performed on the optimal solution S_{opt} . The minimal sequence of swap options corresponding to a solution, S_m , is defined as the *signature*, $Sig(S_m)$, of that solution. The *swap list* corresponding to a solution S_m , $\mathcal{L}(S_m)$, is the list of swap options that are applicable to S_m . Let $Sig(S_m) = \{\sigma_1, \dots, \sigma_m\}$ and $\forall i, 1 \leq i \leq m$, each swap option σ_i belongs to node v_{p_i} . The application of all other swap options that belong to the OR edges in the path from the root node to v_{p_i} is invalidated in the solution S_m .

Only the remaining swap options that are not invalidated in S_m can be applied to S_m for constructing the successor solutions of S_m . The set of successors of a solution S_m , denoted by $Succ(S_m)$, is the set $\{S'_m\}$ such that S'_m can be constructed from S_m by applying a swap option in $\mathcal{L}(S_m)$. These concepts are discussed in detail in [4].

ASG algorithm maintains a list, Open, which initially contains only the optimal solution S_{opt} . At any point of time Open contains a set of candidate solutions from which the next best solution in the non-decreasing order of cost is selected. At each iteration the minimum cost solution (S_{min}) in Open is removed from Open and added to another list, named, Closed which contains the set of ordered solutions generated so far. Then $Succ(S_{min})$ is constructed and any successor solution which is not currently present in Open as well as is not already added to Closed is inserted to Open.

3 Proposed Algorithm for Implicit AND/OR Graphs : ASGAO*

We use the underlying strategy of node expansion and cost revision of AO* repetitively in order to make the relevant portion of the AND/OR graph explicit. In the first step, ASGAO* computes the optimal solution using AO*. At this point of time portions of the AND/OR graph have been made partially explicit, i.e., some of the tip nodes of the AND/OR graph are terminal nodes, whereas the others are yet to expand.

Algorithm 1. Alternative solution generation for implicit AND/OR Graphs

input : $G_{\alpha\beta}$ containing only the root node v_R and the state transformation rules
output : Ordered set of solutions

- 1 Run AO* and compute the optimal solution of $G_{\alpha\beta}$;
- 2 Create two lists, Open and Closed, that are initially empty;
- 3 Compute the swap list, $\mathcal{L}(S_{opt})$, of S_{opt} ;
- 4 Construct $Succ(S_{opt})$ using $\mathcal{L}(S_{opt})$;
- 5 **foreach** $P_m \in Succ(S_{opt})$ **do** Add P_m to Open;
- 6 **while** Open is not empty **do**
- 7 **if** P_{min} is a solution **then**
- 8 P_{min} represents the next best solution S_{min} ;
- 9 Add S_{min} to the Closed list and construct $Succ(S_{min})$;
- 10 **foreach** $P' \in Succ(S_{min})$ **do**
- 11 **if** $P' \notin$ Open **and** $P' \notin$ Closed **then** Add P_m to Open;
- 12 **end**
- 13 **else**
- 14 Expand a tip node in P_{min} using the state transformation rules;
- 15 Start bottom-up cost revision and adjust the swap options;
- 16 Insert P_{min} back to Open;
- 17 **end**
- 18 Report the solutions in Closed;

We use $G_{\alpha\beta}$ to denote a *partial explicit AND/OR graph* which initially consists of the only the root node v_R . The notion of cost for tip node is as follows. The cost of a tip node is returned by a heuristic function, denoted $h_{\alpha\beta}$. If a tip-node is not a terminal node, $h_{\alpha\beta}$ returns an estimate of the cost of solving the problem represented by that tip-node. On the contrary, when a tip-node is a terminal node, $h_{\alpha\beta}$ returns the cost of

the atomic subproblem represented by that terminal node. In this paper, we assume that the heuristic function is admissible and consistent. Unlike the ASG, ASGAO* works with potential solution graphs (psgs) [7] of $G_{\alpha\beta}$. In a potential solution graph, every node with no child is a tip node. Swap options, swap lists, etc. can be computed on the partial explicit AND/OR graphs in likewise manner by considering the heuristic costs of the tip nodes. For partial explicit AND/OR graphs there are two types of swap options. Consider a swap option $\sigma_{pqr} = \langle e_{pq}, e_{pr}, \delta_{pqr} \rangle$ belonging to node v_p whose original edge is e_{pq} and the swapped edge is e_{pr} . If both of the optimal potential solution graphs rooted at v_q and v_r , do not have any tip node, then swap option σ_{pqr} is a *fixed* swap option, otherwise σ_{pqr} is a *changeable* swap option.

The first step of ASGAO* is to run AO* until it generates the optimal solution and after the computation of S_{opt} , the swap options are computed on the partial explicit graph, $G_{\alpha\beta}$. AO* works over a number of iterations performing (i) computation of the current best psg, and (ii) expansion and cost update, till the optimal solution is found. Then the set of successor psgs of $Succ(S_{opt})$ are computed and added to Open. It is possible that some of the psgs can actually be a solution depending on whether all the tip nodes are terminal nodes. Next, ASGAO* performs the following over a sequence of iterations. First, the minimum cost psg, P_{min} , is removed from Open and there are the two possibilities. On the one hand, if P_{min} is a solution, it represents the next best solution S_{min} . ASGAO* adds S_{min} to the Closed list and constructs the successor set of psgs of S_{min} . Among the successor psgs, each psg which is not currently present in Open as well as is not already added to Closed is inserted to Open. On the other hand, if P_{min} is not an actual solution, P_{min} will contain some tip nodes which are not terminal nodes and one of them (chosen arbitrarily), say v_t , is selected and expanded using state transformation rules. If the cost of v_t is changed or its label is changed to solved due to the expansion, a bottom up cost revision step similar to AO* is triggered. In this cost revision step, apart from revising the cost, the swap options are also updated accordingly. After that, P_{min} is added back to Open. At any point of time, the Closed list contains the set of ordered solutions generated so far.

Correctness of ASGAO* : We present the key lemma and theorems to establish the correctness of ASGAO* under consistent and admissible heuristics. The details of the proofs are skipped for the sake of brevity.

Property 1. At any point of time during the execution of ASGAO*, the following holds.

1. For every solution S_{min} present in Closed, $Sig(S_m)$ contains no changeable swap option;
2. For every psg P_m present in Open, at most one changeable swap option can be present in $Sig(P_m)$;
3. While updating the swap options, the original edge of each changeable swap option which belongs to the signature of a psg, does not change. $\square$

Lemma 1. For an initial partial explicit AND/OR graph $G_{\alpha\beta}$, for every solution of the corresponding fully explicit graph $G_{\alpha\beta}^c$, ASGAO* generates that solution. $\square$

Lemma 2. ASGSO* adds solutions to Closed in non-decreasing order of cost. $\square$

Table 1. Comparison of time required (in seconds) for AND/OR DAGs corresponding to the search spaces of *matrix-chain multiplication* with different number of matrices

#mat	ASG				ASGAO*					
	#n	10	20	30	#n	10	#n	20	#n	30
20	1521	0.026	0.029	0.032	245	0.003	286	0.005	314	0.006
30	4931	0.179	0.186	0.194	506	0.006	517	0.009	539	0.012
40	11441	0.786	0.799	0.812	866	0.012	879	0.016	892	0.019
50	22051	2.590	2.612	2.637	1331	0.021	1344	0.026	1357	0.031
60	37761	6.847	6.878	6.940	1893	0.035	1900	0.041	1902	0.048
70	59571	16.003	16.104	16.212	5765	0.234	5945	0.275	5970	0.317
80	88481	33.968	34.187	34.408	4683	0.167	6368	0.322	6404	0.384
90	125491	66.220	66.601	66.989	5691	0.233	6915	0.380	7604	0.511
100	171601	120.326	121.042	121.605	6656	0.300	7880	0.464	8569	0.620

Table 2. Comparison of time required (in seconds) for AND/OR DAGs corresponding to the search spaces of *secondary structure of RNA*

Test Case	ASG				ASGAO*					
	#n	10	20	30	#n	10	#n	20	#n	30
TC1	63237	23.999	24.022	24.127	1333	0.039	1386	0.057	1476	0.089
TC2	84274	42.919	43.289	43.253	941	0.025	1399	0.058	1581	0.100
TC3	56649	19.631	19.732	19.801	1367	0.034	1682	0.054	2224	0.108
TC4	96484	53.132	53.280	53.552	1288	0.035	1408	0.054	1805	0.098
TC5	76787	33.669	33.811	34.028	1470	0.043	2296	0.085	3029	0.150
TC6	72397	31.667	31.884	32.053	1186	0.038	1927	0.074	2403	0.133
TC7	43471	10.708	10.764	10.829	789	0.019	1062	0.041	1253	0.069
TC8	49874	15.755	15.868	15.930	1055	0.030	1157	0.043	1296	0.063
TC9	45699	12.771	12.860	12.920	693	0.022	854	0.041	1145	0.079
TC10	108929	70.853	71.403	74.326	1093	0.032	1409	0.059	1549	0.093
TC11	73381	31.425	31.804	32.052	1106	0.032	1152	0.051	1404	0.081
TC12	50686	15.722	15.882	16.038	1146	0.032	1426	0.062	1512	0.087
TC13	90603	50.570	50.973	51.223	1254	0.035	1453	0.054	1773	0.095
TC14	56547	18.694	18.880	19.047	1382	0.037	1605	0.057	1980	0.095

Table 3. Comparison of time required (in seconds) for AND/OR Trees corresponding to the search spaces of *5-peg tower of Hanoi problem* with different number of disks

#disks	ASG				ASGAO*					
	#n	10	20	30	#n	10	#n	20	#n	30
8	4941	0.019	0.020	0.021	101	0.004	148	0.007	157	0.009
9	17253	0.071	0.075	0.079	93	0.004	130	0.005	133	0.009
10	59049	0.227	0.246	0.261	99	0.005	102	0.006	197	0.010
11	199017	0.869	1.055	1.241	145	0.006	163	0.011	175	0.008
12	662661	3.101	3.767	4.436	215	0.009	288	0.016	302	0.018
13	2184813	10.742	12.764	14.949	267	0.011	410	0.018	436	0.021

Theorem 1. *For a given initial partial explicit AND/OR graph $G_{\alpha\beta}$, every solution of the corresponding fully explicit graph is generated by ASGSO* only once and in the non-decreasing order of costs.* $\square$

4 Experimental Results

To evaluate the performance of the ASGAO*, we have experimented on the following domains – (i) Matrix chain multiplication problem (ii) Secondary structure of RNA (iii) 5-peg tower of Hanoi problem and report the results in Table 1, Table 2 and Table 3 respectively. In these tables, $\#n$ denotes the number of nodes in the AND/OR graph and the time is reported for generating 10, 20, and 30 solutions. We also report the number of node present in the explicit graph used by ASG with the number of nodes expanded by ASGAO* for generating same number of ordered solutions. For all of these domains, we compute the exact optimal cost of solving the subproblem represented by any intermediate node using standard dynamic programming approach and use that value as heuristic. Due to the use of exact heuristic, ASGAO* does not invoke the cost revision step and significant improvement is achieved. The details of the test cases used for the secondary structure of RNA as well as the detailed description of each of the domains can be found in [4]. Table 1-3 show that when exact heuristic is used the size of the AND/OR DAG constructed by ASGAO* is significantly smaller than the size of the AND/OR DAG constructed by ASG. For matrix chain multiplication problem, we can observe around hundred times improvement in running time for bigger matrix chains and secondary structure of RNA. For generating 30 ordered solutions, the size of the explicit graphs are around five to twenty-five times larger compared to the size of the corresponding implicit graphs constructed by ASGAO* for matrix chain multiplication and secondary structure of RNA. For tower of Hanoi problem, the improvement is even more significant. Therefore, for domains like constraint satisfaction, planning etc., where good heuristics are available, ASGAO* can be used to generate ordered solutions efficiently, thereby opening up interesting research directions.

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