Evaluation of the Aircraft Distribution in Satellite Spotbeams

Christoph Petersen, Maciej Mühleisen, and Andreas Timm-Giel

Institute of Communication Networks
Schwarzenbergstr. 95, 21073 Hamburg, Germany
{chr.petersen,maciej.muehleisen,timm-giel}@tuhh.de
<http://www.tuhh.de/comnets>

Abstract. Upcoming inflight entertainment and other onboard data services request broadband data connections during long- and short-haul flights all over the world. To cope with this amount of traffic, especially over oceanic areas where no ground infrastructure is available, satellite systems offer broadband satellite links for avionic purpose.

This work analyzes the distribution of aircraft traffic for selected satellite spotbeams for intercontinental, rural and urban scenarios. For this, the statistic properties of the number of arriving and departing aircraft within a time interval are analyzed. Our results show, that the law of small number and therefore a Poisson distribution is only applicable to areas with low air traffic. Aircraft arrivals to- and departures from satellite spotbeams can be modeled using the negative binomial distribution, as proven by presented results.

1 Introduction

Aircraft are fast and popular conveyances today. For safety and passenger comfort, broadband data communication links are often installed. In some regions, but only over terrain, direct air to ground links are maintained, e.g. Aircell GoGo in the United States of America. Other avionic data links as VHF or HF do not offer high bandwidth, are highly congested and very expensive, so they are not used for services requesting high data rates. Therefore, the most widely used worldwide available avionic broadband data communication are satellite links. Fields of application range from inflight entertainment to maintenance or even flight recorder data transmission. To assess the performance of current satellite systems and estimate the capacity demands, reliable information about the number of aircraft inside specific areas are required.

In order to estimate the aggregated capacity demands, the actual offered data traffic load needs to be analyzed. It is feasible to assume the offered data traffic load being proportional to the number of aircraft in that area. Although traces of aircraft densities are available, analytical models allow more general conclusions about offered traffic loads and help to become independent from large data sets of aircraft positions. Also conclusions about areas where no aircraft position information are available may be drawn. These models are also suitable to

extrapolate prospective situations or determine system performance for extraordinary situations, e.g. aviation distress. Furthermore, an analytical model can be used for systems simulation.

The same methods were applied to derive realistic vehicular traffic models. Exemplarily in [1], a model for car arrivals on highways around the city of Madrid, Spain, is developed. The analysis of the arrival process identifies two different types of vehicles to be further investigated: Cars that drive together in a bursty way and isolated ones without any dependencies e.g. in traveling speed. Therefore, a Gaussian-exponential mixture model is proposed to handle bursty traffic with Gaussian distributed interarrival times and isolated traffic with exponential distributed interarrival times. For sparse vehicular traffic, [2] comes to the same result of exponential distributed vehicular interarrival times and therefore Poisson distributed vehicle arrival rates.

In [3] the German Aerospace Center (Deutsches Zentrum fuer Luft- und Raumfahrt - DLR) developed an aircraft distribution model to evaluate aircraft-to-aircraft communication. In [4] the feasibility of that approach is shown, based on data from the airline flight schedule database published by the International Air Transport Association (IATA) and therefore, assuming all aircraft being equipped with ad-hoc communication equipment. The analysis presented in the following is based on more realistic aircraft position data derived from Flightradar24 [5].

Fig. 1. Two weeks of captured Flightradar24 data

2 Captured Data Set and scope

Flightradar24 captures and visualizes Automatic Dependant Surveillance Broadcast (ADS-B) messages from aircrafts, received by the Federal Aviation Administration (FAA) and a community of people privately operating ADS-B receivers. The available aircraft position data does not cover the whole globe. In fact, oceanic areas are very rarely covered as visible in Figure 1, showing all observed

aircraft position during a two week period. Conclusions regarding aircraft densities over the ocean can be derived from aircraft position data close to the ocean shore. This is visible in Figure 2, representing the observed aircraft with scope over the United States of America.

Fig. 2. Spotbeam over Newfoundland

3 Aircraft Arrival Process

In order to derive worst case capacity demands, the busy hour of each day is analyzed. The area of interest for this model is a satellite spotbeam with a satellite system specific radius. Considering one particular day, the quadruple $d = \{lat, lon, t, id\}$ represents one data record with the unique aircraft ID, the aircraft position in geographic coordinates lat, lon at time instance t . The set $\mathbb{D}_t$ contains all data records worldwide at time t , whereas the subset $\mathbb{D}_{t,A} \subset \mathbb{D}$ is reduced to the records inside the satellite spotbeam area A , defined by a radius r around its center point lat_C, lon_C . Furthermore, the subset $\mathbb{D}_{t,A,H} \subset \mathbb{D}_{t,A}$ describes all data records of $\mathbb{D}_{t,A}$ within the given busy hour of an arbitrary day in this area. To identify the aircraft arrivals and departures, the set $\mathbb{X}_{t,A,H}$ is created containing all aircraft ID records from $\mathbb{D}_{t,A,H}$ of time instance t . The update interval of the captured flight data is limited to the update interval of Flightradar24 being $\Delta t = 10s$. Therefore, a time-discrete model describing the aircraft arrivals and departures during one time instance Δt is developed. With $\mathbb{X}_{t,A,H}$ and $\mathbb{X}_{t+\Delta t,A,H}$ as the set of aircraft for the next time instance,

$$a(t) = |\mathbb{X}_{t+\Delta t,A,H} \setminus \mathbb{X}_{t,A,H}| \quad (1)$$

is the number of aircraft (IDs) being in that area at time instance $t + \Delta t$ but not at time instance t . Hence, $a(t)$ is the number of aircraft arrivals in area A during that update interval. Analogously, the number of departures from the area is defined as

$$d(t) = |\mathbb{X}_{t,A,H} \setminus \mathbb{X}_{t+\Delta t,A,H}| \quad (2)$$

The data for a and d is collected and analyzed for several weeks. Figure 3 shows the distribution of aircraft arrivals inside a $200km$ radius of Newfoundland. It allows the hypothesis that the number of aircraft arrivals follow a Poisson distribution. This hypothesis is verified using the Chi Square test.

4 Scenarios

Several scenarios for the Iridium [6],[7] low earth orbit satellite systems are further investigated. The aircraft arrival process for one spotbeam is analyzed. Depending on its position inside the overall satellite footprint, the area of the spotbeam equates to a circle becoming oval towards the border of the footprint. In this work, a circle with radius r and according area A is assumed. The system specific radius is $r_{Iridium} = 200km$ for the Iridium system.

Fig. 3. Observed arrivals over Newfoundland against Poisson hypothesis, $r = 200km$

The parameters for the model differ depending on the flight scenario and e.g. the used communication system. As flight scenarios, data for urban, rural or intercontinental areas is analyzed. The transition between these scenarios is smooth, but as typical representatives a spotbeam over New York City has been chosen for the urban scenario, over Omaha in Nebraska for the rural scenario and over the ocean shore of Newfoundland as intercontinental scenario.

5 Results

Figure 3 shows the distribution of aircraft arrivals over Newfoundland for the intercontinental scenario as well as the theoretical distribution according to a

Poisson distribution. For Chi Square testing, k classes are defined and the class number, except for the last class k , represents the number of arrivals during a time instance. The empirical rate parameter $\hat{\lambda}$ is set to the average number of arrivals within the considered timeframe and busy hours. The k^{th} class summarizes the occurrences of even more or equal than $k - 1$ arrivals to avoid classes with too few arrivals.

As visible in the figure, only a few aircraft pass this area. The result is based on two weeks of flight data in December 2012. The Chi Square test supports the hypothesis of aircraft arrivals being Poisson distributed with 95% confidence.

Fig. 4. Observed arrivals over New York City against Poisson hypothesis, $r = 200km$

Aircraft arrivals in the urban scenario over New York City are not following a Poisson distribution according to the Chi Square test. The empiric distribution and the Poisson distribution with the same mean value is shown in Figure 4. Both distributions are obviously different. The Chi Square test value exceeds the critical value by far, as shown in Table 1. As the hypothesis of arrivals and departures following a Poisson distribution must be rejected, a new one is formulated. Further suitable distributions are examined with focus on generalization of the Poisson distribution. The negative binomial distribution is a generalization of the Poisson distribution with a second parameter allowing to independently select the mean and variance. The Probability Density Function (PDF) of a negative binomial distribution is stated in Equation 3

$$P(X = x) = \begin{cases} p^n (1-p)^x \binom{n+x-1}{n-1} & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (3)$$

Therefore, the hypothesis for the Chi Square test is changed and the data set is tested against the negative binomial distribution. As shown in [8], the negative binomial distribution is a mixture of Poisson distributions. Whereas for the Poisson distribution hypothesis only the mean value has to be estimated, for the negative binomial hypothesis an additional estimation of the variance is needed. Hence, the degree of freedom for the Chi Square test is increased by one. As visible in Figures 5, 6 and Table 2, the aircraft arrivals follow a negative binomial distribution.

Table 1. Overview: Chi Square test results against Poisson hypothesis

Scenario	$r_{Spotbeam}$ [km]	Mean arrivals / Interval	Chi Square test value	Critical value	Test result
Intercontinental	200 km	0,1275	3,0836	3,84	pass
Rural	200 km	0,4052	367,82	7,82	fail
Urban	200 km	2,49	5960	19,68	fail

Fig. 5. Observed arrivals over Omaha against negative binomial hypothesis, $r = 200km$

The results show that inside the observed areas, the aircraft arrivals and departures follow a negative binomial distribution. It is therefore suitable to assume a negative binomial for aircraft arrivals and departures in those areas. As the three investigated areas cover representative flight scenarios, it can be assumed that this model can also be applied for air traffic in other areas. Even the first assumption of poissonian aircraft arrivals remains valid since the Poisson distribution is a special case of the negative binomial distribution.

Fig. 6. Observed arrivals over New York City against negative binomial hypothesis, $r = 200\text{km}$

Table 2. Overview: Chi Square test results against negative binomial hypothesis

Scenario	$r_{Spotbeam}$ [km]	Mean arrivals / Interval	Variance	Chi Square test value	Critical value	Test result
Intercontinental	200 km	0,1275	0.1339	0.3044	3,84	pass
Rural	200 km	0,4052	0.4494	7.5867	7,82	pass
Urban	200 km	2,49	4.4252	16,505	19,68	pass

6 Conclusion

Aircraft position data has been collected and analyzed regarding aircraft arrivals and departures within time intervals of $\Delta t = 10\text{s}$. The investigations focussed on an intercontinental, rural, and urban area. In all areas, the stochastic aircraft arrival process is very similar to the departure process and follows a negative binomial distribution.

7 Future Work

In this work, we found out that the aircraft arrivals and departures inside a satellite spotbeam follow a negative binomial distribution. Future work should further clarify the reasons for this. Inside one spotbeam, especially in urban areas, arrival and departure rates of different aircraft types occur. While long range flights often just pass by, the impact of short haul flights e.g. by take off or landing is higher inside an urban spotbeam. This would also be an explanation for the varying results of the Chi Square test against the Poisson hypothesis.

For further validation of the hypothesis more regions have to be investigated. Thereby, generally valid distribution parameter (mean value and variance) can be determined.

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