Shrinktech: A Tool for the Robustness Analysis of Timed Automata^{*}

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Abstract. We present a tool for the robustness analysis of timed automata that can check whether a given time-abstract behaviour of a timed automaton is still present when the guards are perturbed. The perturbation model we consider is *shrinking*, which corresponds to increasing lower bounds and decreasing upper bounds in the clock guards by parameters. The tool synthesizes these parameters for which the given behaviour is preserved in the new automaton if possible, and generates a counter-example otherwise. This can be used for 1) robustness analysis, and for 2) deriving implementations under imprecisions.

1 Introduction

Timed Automata and Robustness. Timed automata [3] are a well-established formal model for real-time systems. They can be used to model systems as finite automata, while using, in addition, a finite number of clocks to impose timing constraints on the transitions. Timed automata are, however, abstract models, and therefore make unrealistic assumptions on timings, such as perfect continuity of clocks, infinite-precision time measures and instantaneous reaction times. An important amount of work has been done in the timed automata literature to endow timed automata with a realistic semantics. The works [14] and [10] showed that perturbations on clocks, either imprecisions or clock drifts, and regardless of how small they are, may yield additional qualitative behaviours in some timed systems. On the other hand, assuming bounds on the reaction times can disable desired behaviours [8,1]. These observations mean that there is a need for checking the *robustness* of timed automata models, that is, whether the behaviour of a given timed automaton is preserved in presence of small perturbations. Robustness is an important property of critical embedded systems [12], since it requires that the system will behave correctly when the environment's behaviour deviates slightly from the assumptions.

Clock Imprecisions and Shrinking. A prominent approach to model imprecisions in timed automata, initiated in [11], consists in introducing imprecisions in the model by syntactically *enlarging* all guards, that is, turning a guard $x \in [a, b]$ into $x \in [a - \Delta, b + \Delta]$ for some parameter $\Delta > 0$. Model-checking algorithms on

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timed automata have been revisited in order to take into account such imprecisions (see *e.g.* [10,6]). These algorithms check whether any really new behaviour appears when timing constraints are relaxed by a small (parameterized) amount.

Recently we studied the dual notion of robustness, which consists in checking whether any behaviour is lost when the guards are *shrunk*, that is tightened by a small (parameterized) amount. More precisely, shrinking means converting a guard $x \in [a, b]$ into $x \in [a + \delta, b - \delta']$ for some $\delta, \delta' > 0$. In [15], we showed that one can decide whether all guards can be shrunk –by possibly different amounts, so that the resulting timed automaton can still time-abstract simulate the original automaton. In this case, one can also synthesize these shrinking parameters for each atomic guard. By checking shrinkability of timed automata, one ensures that the behaviour of the automaton does not depend on exact timings, or on its ability to take the transitions on the boundaries of the guards. A shrinkable timed automaton preserves all its behaviours when, for instance, task execution times are shorter than the worst-case, and waiting times are longer than the best-case. One can also detect unrealistic runs, including Zeno runs [15]. We believe that shrinkability complements the robustness approach based on guard enlargement of [11,10].

Shrinkability can also be used for deriving implementations with imprecise clocks. In fact, if the guard $x \in [a, b]$ is shrunk into $x \in [a + \delta, b - \delta]$, then under imprecisions modelled by guard enlargement (as in [11]), this guard becomes $x \in [a + \delta - \Delta, b - \delta + \Delta] \subseteq [a, b]$, where the inclusion holds whenever $\Delta < \delta$. Hence, the behaviours of a shrunk timed automaton with bounded imprecisions (*i.e.* guard enlargement) are entirely included in the behaviours of the initial timed automaton. Further, using shrinkability, one can synthesize parameters δ for each guard, so that the resulting automaton still contains some useful time-abstract behaviour.

Related Work. Existing verification tools for timed automata may be used for *non-parameterized* robustness checking by modeling explicitly the imprecisions, although this increases the size of the models [2]. The semi-algorithm of HyTech was used to synthesize guard enlargement parameters in timed automata in [11]. An extension of Uppaal for robustness against guard enlargement was used in [13]; this feature is no longer available in Uppaal. Note that shrinkability cannot be solved by existing model-checkers for timed automata since we are interested in parameter synthesis so as to ensure time-abstract simulation. Other similar work includes (the undecidable problem of) parameter synthesis in timed automata, where guards are written using parameters, and one tries to find the valuations for which the system satisfies some specification, *e.g.* [4]. This is difficult to realize due to the large number of parameters (upto millions) and the time-abstract simulation condition we consider. Robustness against large decreases in task execution times using simulation was considered in [1].

2 Shrinkability

Let us define shrinkability more formally. We assume that the reader is familiar with the syntax and semantics of Alur-Dill timed automata, and refer to [3] for

Fig. 1. A timed automaton $\mathcal{A}$ (above) and its shrinking $\mathcal{A}_{-k\delta}$ (below). Timed automaton $\mathcal{A}_{-k\delta}$ can time-abstract simulate $\mathcal{A}$ for all $\delta \in [0, \frac{1}{6}]$ ([15]).

details. We only need the following definitions. Given a finite clock set $\mathcal{C}$, an *atomic guard* is an expression of the form $x \leq k \mid x \geq k \mid x - y \leq k$, where $x, y \in \mathcal{C}$ and $k \in \mathbb{Z}$. A *guard* is a conjunction of atomic guards. The *shrinking* of an atomic guard g by δ , denoted by $\langle g \rangle_{-\delta}$ is defined as $\langle x \leq k \rangle_{-\delta} = x \leq k - \delta$, $\langle x \geq k \rangle_{-\delta} = x \geq k + \delta$, and $\langle x - y \leq k \rangle_{-\delta} = x - y \leq k - \delta$.

Note that the only variables that appear in timed automata are clocks. Discrete variables with bounded domains can be considered, but we assume these are encoded in the locations.

Let $\mathcal{A}$ be a timed automaton, and let I be the vector of the atomic guards of $\mathcal{A}$. Given a vector δ of nonnegative rational numbers indexed by I , we denote by $\mathcal{A}_{-\delta}$, the automaton obtained from $\mathcal{A}$ by shrinking each atomic guard by the corresponding element of δ . We are going to write the vector δ as $k\delta$ for an integer vector k and rational δ . This is always possible since we are interested in rational parameters. Figure 1 is an example of shrinking.

We are interested in shrinking the atomic guards of a given timed automaton by positive values, while preserving *some* of the behaviours. We only consider timed automata with non-strict guards; in fact, using strict guards makes little sense when one is interested in shrinking (or enlarging) the guards [10]. We also assume that the edges have distinct labels, since we are interested in comparing two timed automata that have the same underlying structure. The problem is formulated as follows:

Definition 1 (Shrinkability). *Given a timed automaton $\mathcal{A}$, and a finite automaton $\mathcal{F}$ such that $\mathcal{F} \sqsubseteq_{ta} \mathcal{A}$, decide whether for some $\delta > 0$, $\mathcal{F} \sqsubseteq_{ta} \mathcal{A}_{-\delta}$.*

In this definition, $\sqsubseteq_{ta}$ denotes *time-abstract delay simulation*. We say that $\mathcal{A}$ is *shrinkable w.r.t. $\mathcal{F}$* if the above condition is satisfied. Thus, shrinkability requires that some behaviour $\mathcal{F}$, that is included in the initial model $\mathcal{A}$, should be still possible in the shrunk automaton. When $\mathcal{F}$ is the region graph of $\mathcal{A}$ [3], or a time-abstract bisimulation quotient [16], shrinkability implies that the shrunk automaton can time-abstractly simulate the original automaton. In this case, we say that $\mathcal{A}$ is simply *shrinkable*. Shrinkability w.r.t. $\mathcal{F}$ is decidable in polynomial time if $\mathcal{F}$ is part of the input; shrinkability is decidable in exponential time, if the time-abstract bisimulation graph is not given. The vector δ can be computed in the same time complexity [15].

Fig. 2. Overview of **shrinktech**

The present tool builds on the theoretical results presented in [15]. There, we show that given a finite automaton $\mathcal{F}$, the shrinking parameter of each atomic guard can be expressed as a function of the other parameters using only maximization and sum. The problem is then reduced, in polynomial time, to solving nonlinear fixpoint equations in the max-plus algebra. We gave in [15] graph-based algorithms to solve these equations.

Partial shrinkability. Although we defined shrinkability by requiring that *all* atomic guards should be shrunk by a positive amount, one can relax this condition and shrink only some of the guards; our algorithms are valid also in this case. In our experiments, we shrunk all the guards but equality constraints.

3 The Tool **shrinktech**

We present the tool **shrinktech** that analyzes the shrinkability of timed automata and synthesizes shrinking parameters. Given a network of timed automata, the tool either finds a counter-example to shrinkability, such as a path or a cycle that cannot be executed by any shrinking of the automaton, whatever the value of δ 's are, or outputs a shrinking of the timed automata that witnesses the shrinkability.

Figure 3 shows an overview of the tool. To check the shrinkability of a timed automaton, the user can either provide a finite automaton $\mathcal{F}$, or let **shrinktech** compute the full finite bisimilarity graph using Kronos¹. Note that if the full bisimilarity graph is too big, one can also try to shrink with respect to a portion of it, or with respect to a randomly generated trace. This is to be compared with bounded model-checking, which is useful for detecting bugs, but also for “partially” proving the correctness of a system. The tool comes with scripts to compute the bisimilarity graph, extract some (random) portion of it, and generate random executions.

The tool **shrinktech** can be used for several kinds of systems modelled by timed automata. We believe it can be used mainly for two purposes:

1. Robustness analysis, to find out whether the behaviour of the system is preserved when the time bounds are disturbed (shrunk). This analysis complements the robustness checking by enlarging the guards as in [11,2]. This

¹ Kronos is a model-checker for timed automata [7] that can *minimize* the region graph of a timed automaton as described in [16]. It is available at <http://www-verimag.imag.fr/DIST-TOOLS/TEMPO/kronos/>

can for instance help to detect unrealistic executions such as Zeno or other convergence phenomena, but also timing anomalies in scheduling problems (see [5]).

2. Deriving implementations from timed automata. As explained above, the behaviour of a shrunk timed automaton is included in that of the initial model in presence of imprecisions. So lower and upper bounds on the delays can be “shrunk” in the implementation to guarantee that these will be respected despite imprecisions. In fact, we proved in [15] that shrinkable timed automata can be implemented in a concrete semantics with imprecise clocks and reaction times.

Implementation details and availability. The tool is implemented in C++ and the source code has about 5Klocs. It uses the Uppaal DBM library², and implements a parameterized extension of this data structure, introduced in [15]. The input formats are (networks of) timed automata in the Kronos format, and finite automata in the Aldebaran format³. The tool Kronos can be plugged in the tool-chain in order to compute the finite time-abstract bisimilarity graph of a given timed automaton, to be used as the finite automaton $\mathcal{F}$. **Shrinktech** is open source software and is distributed under GNU General Public Licence 3.0. It is freely available at: <http://www.lsv.ens-cachan.fr/Software/shrinktech>

4 Experimental Results

We used **shrinktech** on several case studies found in the literature. The table 1 summarizes the results. The Lip Synchronization Protocol has been the subject of robustness analysis (by guard enlargement) before [13]. This is an algorithm that synchronizes video and sound streams that arrive in different frequencies. The model is not shrinkable neither for video frames arriving in exact frequency, nor for those arriving within a bounded interval. Observe that the model is shrinkable w.r.t. a small subgraph with 501 nodes, but it is not shrinkable w.r.t. the whole graph, which has 4484 nodes. Shrinkable models include Philips Audio Retransmission protocol [9], and some asynchronous circuit models. We were able to analyze Fischer’s Mutual Exclusion Protocol upto 4 agents; while for 5 agents we could only partially analyze w.r.t. a randomly generated trace. The non-shrinkability of most models is due to equality constraints. In fact, although we only shrink non-punctual guards, some behaviours may still disappear immediately, however small the shrinking parameter is.

Note that some of these models were designed at a level of abstraction where imprecisions were not taken into account. So, our results do not necessarily imply that these systems are not robust, but rather that the present models are not good for direct implementation. This is best illustrated in the Latch Circuit models, where the exact model that extensively uses equalities is not shrinkable,

² <http://people.cs.aau.dk/~adavid/UDBM/>

³ This is a graph description format of the CADP tool suite, also used by Kronos. See <http://www.inrialpes.fr/vasy/cadp/>

Table 1. The column `sim-graph` is the number of states and the number of transitions of the finite automaton $\mathcal{F}$ w.r.t. which the shrinkability is checked. An asterisk indicates bounded shrinkability, where only a subgraph of the time-abstract bisimulation graph (given by a BFS) or a random trace was used. The tests were performed on an Intel Xeon 2.67 GHz. All models are available on the tool’s website.

Model	states	trans	clocks	sim-graph	time	shrinkable
Lip-Sync Prot. (Exact)	230	680	5	4000/8350* (subgraph)	9s	No
Lip-Sync Prot. (Interval)	230	680	5	501/1282* (subgraph)	9s	Yes*
Lip-Sync Prot. (Interval)	230	680	5	4484/48049	28s	No
Philips Audio Prot.	446	2097	2	437/2734	46s	Yes
Root Contention Prot.	65	138	6	500/3455* (subgraph)	7s	No
Train Gate Controller	68	199	11	952/8540	34s	No
Fischer’s Protocol 3	152	464	3	472/4321	20s	Yes
Fischer’s Protocol 4	752	2864	4	4382/65821	310min	Yes
Fischer’s Protocol 5	3552	16192	5	10000/10000* (trace)	42s	Yes*
And-Or Circuit	12	20	4	80/497	1.3s	Yes
Flip-Flop Circuit	22	34	5	30/64	0.9s	Yes
Latch Circuit (Interval)	32	77	7	105/364	1.6s	Yes
Latch Circuit (Exact)	32	77	7	100/331	0.6s	No

but its relaxation to intervals is. Notice also how most of the circuit models which define bounds on stabilization times are shrinkable.

Our approach depends on the computation of the finite automaton $\mathcal{F}$, thus it is limited by the feasibility of this computation. To deal with this problem, one could consider adapting the algorithm of [16] to compute on-the-fly some bounded part of the graph. To be able to treat even larger systems, we will consider extending the theoretical results of [15], in order to use under- and over-approximations of the automaton $\mathcal{F}$, and refine these by counter-examples.

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