Merging Faces: A New Orthogonal Simplification of Solid Models

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Abstract. A new approach to simplify orthogonal pseudo-polyhedra (OPP) and binary volumes is presented. The method is incremental and produces a level-of-detail (LOD) sequence of OPP. Any object of this sequence contains the previous objects and, therefore, it is a bounding orthogonal approximation of them. The sequence finishes with the minimum axis-aligned bounding box (AABB). OPP are represented by the Extreme Vertices Model, a complete model that stores a subset of their vertices and performs fast Boolean operations. Simplification is achieved using a new approach called *merging faces*, which relies on the application of 2D Boolean operations. We also present a technique, based on the model continuity, for a better shape preservation. The method has been tested with several datasets and compared with two similar methods.

Keywords: Simplification, LOD, Bounding Volumes, Orthogonal Polyhedra, Binary volumes.

1 Introduction

The large size and complexity of the models often affects the computation speed-up of their characteristics and their rendering efficiency. Simplification techniques can diminish these problems. Moreover, in some situations it is advantageous to exchange an exact geometric representation of an object for an approximated one, which can be processed more efficiently. Bounding structures are used for model simplification to accelerate tasks such as collision detection or distance computation. The most used bounding structures are AABB, spheres, oriented boxes or convex polyhedra. In this paper we present an approach to simplify OPP. The method computes a LOD sequence of bounding volumes (BV), that are also OPP, denoted as bounding OPP (BOPP). BOPP satisfy the two following properties: (1) any BOPP contains the previous one and (2) all the BOPP, as well as the original object, have the same AABB. Fig. 1 depicts a 3D model and the obtained BOPP sequence, with the AABB for each one. We use the Extreme Vertices Model (EVM) to represent OPP. The presented simplification approach, called *merging faces*, relies on the application of 2D Boolean operations, which are fast using EVM, over the OPP faces. The presented method deals with general 3D orthogonal objects with any number of shells, cavities and through

Fig. 1. A level-of-detail sequence of orthogonal pseudo-polyhedra (OPP) generated by our approach. From left to right: Original model, OPP with 19.7%, 9.4%, 4.2% and 0.8% of extreme vertices (EV). In the last one, the 1st and 5th objects are put together.

holes. We also develop a technique for a better shape preservation that avoids abrupt changes. The method has been tested with several datasets and compared with similar methods, showing satisfactory results.

2 Related Work

Model simplification has been extensively applied to triangular meshes [5] and extended to tetrahedral meshes evaluating the approximation error and the quality of the obtained mesh [4]. Methods for LOD sequences of triangular and tetrahedral meshes can also be found extensively in the literature [16] as well as methods to simplify quadrilateral meshes [11,21]. In contrast to these methods, that rely on geometric operations as edge-collapse or clustering, simplification can follow other strategies. Morphological operators as filleting and rounding can be used to simplify 2D binary images as well as 3D triangular meshes [26]. A carving strategy is applied to an octree model [23] as well as to a tetrahedral mesh [12] to simplify the topology. Simplification strategies have also been developed for B-Rep models [19] by removing connected sets of faces.

Several applications as collision detection [14], ray tracing [25] and volume of interest computation [9] use approximated shapes that are BV. Simple spheres [10] and AABB [20] are used as well as more sophisticated shapes as convex [15] or oriented [6] polytopes. Orthogonal polyhedra have also been proposed as BV [8] and as geometric bounds for CSG [1]. Orthogonally convex polygons are computed as orthogonal hulls for 2D images [3]. An orthogonal polygon is orthogonally convex if any axis-parallel line intersects it in at most one line segment. This problem has been extended to orthogonally convex polyhedra [2].

A sequence of BV can be obtained using alternative representations as octrees [18,23] resulting in a simplified geometry and topology, or BSP [13] obtaining a LOD sequence with a decreasing number of nodes. OP simplification has been carried out with a *moving faces* strategy that performs face displacements (but fails for objects with holes or more than one connected component) and the *rectangle pairs* strategy based on a box partition of the OP [8].

3 Extreme Vertices Model (EVM)

OP are two-manifold polyhedra with all their faces oriented in the three main axes. OPP are regular OP with a possible non-manifold boundary. General OPP are OPP with vertex coordinates having any value in $\mathbb{R}^3$. Polycubes are a subset of OPP all of whose vertices have integer coordinates, formed by joining one or more equal cubes (voxels) face to face. A 3D binary digital image represents an object as the union of its foreground voxels and its continuous analog is an OPP. In this paper we work with all these kind of objects represented with EVM. Let Q be a finite set of points in $\mathbb{R}^3$, the ABC -sorted set of Q is the set resulting from sorting Q according to A -coordinate, then to B -coordinate, and then to C -coordinate. Let P be an OPP, a *brink* is the maximal uninterrupted segment built out of a sequence of collinear and contiguous two-manifold edges of P and its ending vertices are called *extreme vertices* (EV). An OPP can be represented in a concise way with the ABC -sorted set of its EV and such representation scheme is called EVM. EVM is a complete solid model [24].

Let P be an OPP and Π_c a plane whose normal is parallel, without loss of generality, to the X axis, intersecting it at $x = c$, where c ranges from $-\infty$ to ∞ . Then, this plane sweeps the whole space as c varies within its range, intersecting P at some intervals. Let us assume that this intersection changes at $c = c_1, \dots, c_n$. More formally, $P \cap \Pi_{c_i-\delta} \neq P \cap \Pi_{c_i+\delta}$, $i = 1, \dots, n$, where δ is an arbitrarily small quantity. Then, $C_i(P) = P \cap \Pi_{c_i}$ is called a *cut* of P and $S_i(P) = P \cap \Pi_{c_s}$, $c_i < c_s < c_{i+1}$, is called a *section* of P . Two cuts bounding a section C_i and C_{i+1} are called *consecutive cuts*. See Fig. 2. Sections can be computed from cuts and vice versa:

$$\overline{S_0(P)} = \overline{S_n(P)} = \emptyset, \quad \overline{S_i(P)} = \overline{S_{i-1}(P)} \otimes^* \overline{C_i(P)}, i = 1 \dots n - 1 \quad (1)$$

$$\overline{C_i(P)} = \overline{S_{i-1}(P)} \otimes^* \overline{S_i(P)}, i = 1 \dots n \quad (2)$$

where n is the number of cuts and $\otimes$ denotes the xor operation. Overline symbolizes the project operator, that projects a d -dimensional set of vertices lying on an orthogonal plane, like a cut or a section, onto the corresponding main plain, discarding their d th coordinate. The star exponent $*$ denotes a regularized Boolean operation. A regularized set is defined as the closure of its interior. Regularized Boolean operations are needed to ensure 3D homogeneity [22].

Eq. 2 can be rewritten by expressing the $\otimes^*$ operation as the union of differences: $\overline{C_i(P)} = (\overline{S_{i-1}(P)} -^* \overline{S_i(P)}) \cup^* (\overline{S_i(P)} -^* \overline{S_{i-1}(P)})$, and any cut can be decomposed into its *forward difference* (FD) and *backward difference* (BD):

$$\overline{FD_i(P)} = \overline{S_{i-1}(P)} -^* \overline{S_i(P)}, \quad \overline{BD_i(P)} = \overline{S_i(P)} -^* \overline{S_{i-1}(P)}, i = 1 \dots n \quad (3)$$

$FD_i(P)$ is the set of $C_i(P)$ faces whose normal vector points to the positive side of the coordinate axis perpendicular to $C_i(P)$ and $BD_i(P)$ is the set of faces whose normal vector points to the negative side (see Fig. 2). This property guarantees the correct orientation of faces and the computation of the non-EV. EVM Boolean operations are computed by applying recursively (in nD) the same Boolean operation over the $(n-1)D$ OPP sections. The base case performs this

Fig. 2. An EVM-encoded (ABC -sorted) OPP. Left: EV marked with dots, cuts decomposed in FD (blue) and BD (orange), normal vectors represented with arrows; a vertical brink from vertex a to c is marked showing that these vertices are both EV while the vertex b is a non-EV. Right: The sections of the object highlighted in yellow.

operation in 1D. The $\otimes^*$ is even faster, $EVM(P \otimes^* Q) = EVM(P) \otimes^* EVM(Q)$, i.e, it is a simple point-wise xor, without section computation. For more details concerning EVM see [1] and [17].

4 Algorithm Overview

Let $B(P)$ be the bounding orthogonal pseudo-polyhedron (BOPP) of an OPP P and let ϕ_0 be an initial OPP. A finite sequence $\phi_1, \phi_2, \dots, \phi_p$ of OPP is generated that fulfill the following properties:

1. $\phi_{i+1} = B(\phi_i)$, $i = 0 \dots p - 1$
2. $\phi_i \subseteq B(\phi_i)$, $i = 0 \dots p - 1$, and, therefore, $\phi_i \subseteq \phi_{i+1}$, $i = 0 \dots p - 1$
3. $\phi_p = AAB B(\phi_i)$, $i = 0 \dots p$

The first property indicates that the approach is incremental. The second one, called subset property, is intrinsic in bounding structures. The last property states that the sequence is finite and that ends with the AAB B that is shared by all the OPP of the sequence.

The simplification strategy, *merging faces*, works with pairs of consecutive cuts of P . For each cut of a pair, a displacement to its faces in the direction of their corresponding normal vector is applied and, then, the displaced faces are merged with those faces of the other cut with the same normal vector. The process is controlled by the displacement parameter, d , that indicates the maximum displacement allowed, in such a way that only pairs of consecutive cuts that are at a distance $\leq d$ are actually merged. If P is represented as an ABC -sorted EVM, the application of this process only will coarsen P in the A -coordinate. Therefore to obtain $B(P)$ the process is repeated for the other two main directions. The result can be slightly different depending on the ordering in which the three main directions are selected. In order to speed up the computation, the best first candidate would be the ABC -ordering with A -axis having less number of

cuts, but as EVM does not report this value, it can be approximated by $c_n - c_1$, c_1 and c_n being respectively the coordinates of the first and last cut in this direction. We can compute the whole sequence of BOPP or ask for a BOPP with a maximum number of EV.

The input of the whole method is an EVM represented OPP, P , which corresponds to the initial object ϕ_0 and the desired maximum number of EV, nv , in the simplified model, and returns an EVM represented OPP, Q , corresponding to the object ϕ_k of the sequence, which has no more than nv EV. As the method is incremental, it actually computes all the objects between ϕ_1 and ϕ_k . It performs k times the *merging faces* process for successive values of d , $d = d_i, i = 1, \dots, k$, obtaining the corresponding ϕ_i . Displacements d_i can be in any units and incremented in any quantity. For OPP corresponding to digital images the basic unit is one voxel, i.e. $d_i = i$. For general OPP, with float coordinate values, d_i can take any values ranging from the minimum distance between cuts and the AABB size. Note that if the increment of d is too large, far apart consecutive cuts can be merged in early iterations causing abrupt changes in the simplified object. The next pseudocode shows the iterative algorithm and concerning d considers digital images. *Get_nev()* returns the number of EV of the given object P , *mergingFaces()* receives the object P and a distance d , and returns the object $Q = B(P)$, merging pairs of consecutive cuts at a distance $\leq d$. To compute the whole LOD sequence of BOPP, nv must be 8.

```
EVM Simplification(EVM P, int nv) {
    EVM Q = P;
    for(int d=1; Q->Get_nev() > nv; d++){
        Q->SetSorting(ABC);    Q = mergingFaces(Q, d);
        Q->SetSorting(BAC);    Q = mergingFaces(Q, d);
        Q->SetSorting(CAB);    Q = mergingFaces(Q, d); }
    return Q; }
```

5 Merging Faces Approach

In this section we first explain the basic process, then the treatment of the void space and finally, we discuss the way to select pairs of cuts. Let P be an OPP and let C_A and C_B be two consecutive cuts of P with FD_A, BD_A, FD_B, BD_B as their corresponding forward and backward differences (see Eq. 3). To obtain a coarsened OPP, the *merging faces* process displaces BD_B to the position of BD_A , and FD_A to the position of FD_B . Then the new cuts $newC_A$ and $newC_B$ that will replace C_A and C_B , respectively, in the input model, are computed as:

$$\overline{newC_A} = \overline{BD_A} \cup^* \overline{BD_B}, \quad \overline{newC_B} = \overline{FD_A} \cup^* \overline{FD_B} \quad (4)$$

This process fulfills the properties stated in Sec. 4. Face displacements are done in the direction of their respective normal vector, i.e. outward of the object. Then, for any P , $P \subseteq B(P)$ (subset property). The property concerning AABB can be proved by considering that the AABB of an OPP can be defined as

Fig. 3. 2D example: (a) Faces merged in a first step. (b) Result after applying Eq. 4 and end for XY-sorting. (c) Faces merged by YX-sorting. (d) Resulting BOPP.

the intersection of the six planes corresponding to the first and last cut in the three main directions, and the fact that the object will never extend beyond the first and last cut of the initial object because the displacements are bounded by these cuts. Moreover, in the last iteration, the first cut of ϕ_p (the AABB), will correspond to the union of all BD of all cuts, and likewise the last cut to the union of all FD . Fig. 3 shows a 2D example where the process is applied.

An OPP may have any number of rings on faces, through holes and shells. In these cases, Eq. 4 could not give the desirable result, for instance, the object in Fig. 4(a), remains the same after the application of Eq. 4. To deal with this issue, we first detect and then close void spaces. Given C_A and C_B , BD_A and BD_B are sets of faces whose normal vectors point to the opposite direction of FD_A and FD_B , but FD_A and BD_B define a void space as $vSpace(C_A, C_B) = \overline{FD_A} \cap^* \overline{BD_B}$. Removing $vSpace$ from both $newC_A$ and $newC_B$, closes the void space between C_A and C_B . Then, Eq. 4 is extended as:

$$\begin{aligned} \overline{newC_A} &= (\overline{BD_A} \cup^* \overline{BD_B}) -^* (\overline{FD_A} \cap^* \overline{BD_B}) \\ \overline{newC_B} &= (\overline{FD_A} \cup^* \overline{FD_B}) -^* (\overline{FD_A} \cap^* \overline{BD_B}) \end{aligned} \quad (5)$$

Observe that when $vSpace = \emptyset$, Eq. 5 and 4 are equivalent. The subset and the AABB properties are also guaranteed as we remove interior void spaces. Fig. 4(a) shows an example with a single hole, here $\overline{FD_A} = \overline{BD_B}$ and $vSpace(C_A, C_B) = \overline{FD_A} = \overline{BD_B}$. Then, applying Eq. 5, $newC_A = \emptyset$ and the hole is closed (Fig. 4(b)). In Eq. 5, basic merging and void space removal are performed jointly. Some void spaces, as the simple concavity in Fig. 4(c), are solved with Eq. 4. However, general void spaces, as those in Fig. 4(d-f), require the application of Eq. 5. Some void spaces can be detected in all three directions, as the cavity depicted in Fig. 4(f) but some others are only detected in one or two directions. Fig. 4(g-j) show a working example in an object with through holes.

Two EVM properties, that state that for two special cases union and difference consist in simple point-wise xor operations [1], together with two EVM theorems, permit to rewrite Eq. 5 in a way faster to compute.

- *Property 1:* Let P and Q be two OPP such that $P \cap^* Q = \emptyset$, having $EVM(P)$ and $EVM(Q)$ as their models, then $EVM(P \cup^* Q) = EVM(P) \otimes^* EVM(Q)$.
- *Property 2:* Let P and Q be two OPP such that $P \supseteq Q$, with $EVM(P)$ and $EVM(Q)$ as their models, then $EVM(P -^* Q) = EVM(P) \otimes^* EVM(Q)$.

Fig. 4. *Merging faces and treatment of void space*

Theorem 1. *The projection of FD and BD of two consecutive cuts C_A and C_B are quasi-disjoint sets respectively, i.e. $\overline{FD_A} \cap^* \overline{FD_B} = \emptyset$ and $\overline{BD_A} \cap^* \overline{BD_B} = \emptyset$.*

Proof. The proof is based on the Jordan theorem and the fact that any ray crossing the boundary of the polyhedron, alternatively goes from outside to inside and vice versa. Therefore, in any OPP, assuming that $\overline{FD_A} \cap^* \overline{FD_B} \neq \emptyset$, would mean that a ray could cross FD_A going outside and then cross FD_B going outside again, which is a contradiction. The same reasoning applies to BD . $\square$

Theorem 2. $vSpace(C_A, C_B) \subseteq (\overline{BD_A} \cup^* \overline{BD_B})$

Proof. $vSpace(C_A, C_B) = \overline{FD_A} \cap^* \overline{BD_B}$. Without loss of generality:

$$(\overline{FD_A} \cap^* \overline{BD_B}) \subseteq \overline{BD_B} \subseteq (\overline{BD_A} \cup^* \overline{BD_B}), \text{ and thus:}$$

$$(\overline{FD_A} \cap^* \overline{BD_B}) \subseteq (\overline{BD_A} \cup^* \overline{BD_B})$$

In a similar way $vSpace(C_A, C_B) \subseteq (\overline{FD_A} \cup^* \overline{FD_B})$ can be proved. $\square$

According to these theorems and the EVM properties, Eq. 5 is rewritten as:

$$\begin{aligned} \overline{newC_A} &= \overline{BD_A} \otimes^* \overline{BD_B} \otimes^* (\overline{FD_A} \cap^* \overline{BD_B}) \\ \overline{newC_B} &= \overline{FD_A} \otimes^* \overline{FD_B} \otimes^* (\overline{FD_A} \cap^* \overline{BD_B}) \end{aligned} \quad (6)$$

Merging faces takes pairs of consecutive cuts but we must establish the way in which they are selected. After analyzing several alternatives, we selected the one with best visual results. It consists in taking cuts two by two: $(C_A = C_i, C_B = C_{i+1})$, $i = 1, i \leq n - 1, i = i + 2$, i.e. first the pair (C_1, C_2) , then (C_3, C_4) , and so on. However, when C_A and C_B are such that $FD_A = BD_B = \emptyset$, *merging faces* has no effect (e.g. consider $C_A = C_1$ and $C_B = C_2$ in Fig. 4(h)). On the other hand,

only pairs of cuts at a distance $\leq d$ are processed. Then, given the pair of cuts $(C_A=C_i, C_B=C_{i+1})$, if Exp. 7 is fulfilled, this pair is processed and the method continues with the pair (C_{i+2}, C_{i+3}) . Otherwise C_i is copied to the resulting OPP and the method continues with the pair (C_{i+1}, C_{i+2}) . Moreover, for symmetry preservation purposes, the model is analyzed from both sides at a time.

$$(FD_A \neq \emptyset \text{ or } BD_B \neq \emptyset) \text{ and } distance(C_A, C_B) \leq d \quad (7)$$

6 Shape Preservation

Shape preservation is an important aspect in model simplification. Merging two cuts is performed in all their extension, but there are parts of these cuts that are isolated and merging them could result in too abrupt changes. Let (C_A, C_B) be the pair of cuts to be merged, we will refer as *isolated faces* those faces of C_A whose projections do not share either an edge or a vertex with others in C_B and vice versa. Removing these faces of the merging process results in a better approximation (see Fig. 5(d-f)). Let I_A and I_B be the *isolated faces* of C_A and C_B respectively, the new FD'_A, BD'_A, FD'_B and BD'_B are computed according to Eq. 8. These values are used in Eq. 6 to generate $newC'_A$ and $newC'_B$, and after that, the isolated faces can be reintegrated with a union operation. However, by definition I_A and C_A are disjoint sets and therefore, I_A and $newC'_A$ are also disjoint sets (the same applies to I_B, C_B and $newC'_B$). Then, according to *Property 1*, an xor operation is performed instead (See Eq. 9).

$$\overline{FD'_A} = \overline{FD_A} - * \overline{I_A}, \quad \overline{BD'_A} = \overline{BD_A} - * \overline{I_A} \quad (8)$$

$$\overline{FD'_B} = \overline{FD_B} - * \overline{I_B}, \quad \overline{BD'_B} = \overline{BD_B} - * \overline{I_B}$$

$$\overline{newC_A} = \overline{newC'_A} \otimes * \overline{I_A}, \quad \overline{newC_B} = \overline{newC'_B} \otimes * \overline{I_B} \quad (9)$$

Fig. 5(a-c) show an example where an isolated face (*if*) is depicted. Note that *if* remains in place, and only those faces that share either an edge or a vertex are taken into account throughout the *merging faces* process. The application of this technique does not affect the subset property (see Sec. 4). However, the property concerning finiteness cannot be guaranteed, as there may be consecutive

Fig. 5. Shape preservation technique. (a) A simple 3D model. (b) All cuts and EV; in red an isolated face. (c) Result of applying *merging faces* with the technique to the pair (C_A, C_B) . (d) Original DiskBrake model. (e) and (f) BOPP with and without the shape preservation technique respectively, both having less than 15% of EV.

cuts where all the faces are isolated and, in this case, we would not reach the AABB. Therefore, when all the faces are isolated, we do not apply the shape preservation technique and use directly Eq. 6. To detect isolated faces between C_A and C_B , we check each face of C_A against each face of C_B . For this process we use a decomposition model, the Compact Union of Disjoint Boxes (CUDB) [7], which is derived from EVM by splitting the orthogonal faces into a set of AB-sorted rectangles. We have implemented an algorithm for rectangle adjacency detection. As a 2D CUDB is a sorted set of rectangles, we apply a merging process, i.e., we perform a traversal of the CUDB models of each face in C_A and C_B at a time and check for rectangle intersection. Thus, if we have n faces with m_i boxes each one, the worst case time-complexity is $O(n \cdot M)$, $M = \sum_{i=1}^n m_i$.

7 Results and Discussion

We have tested *merging faces* with and without shape preservation in several 3D datasets with different shape features, obtained from public repositories and our own collection. All of them have been converted to EVM using existing algorithms [17]. The presented algorithms have been written in C++ and executed on a PC Intel®Core i7 CPU 870 at 2.93GHz with 16Gb of RAM under Linux. The computation time of *merging faces* depends directly on the number of EV which is related to how well aligned is the object with respect to the three axes. This fact is shared by most of the EVM-based developed methods.

Table 1 shows the obtained results. Note that *merging faces* with shape preservation requires, in general, more time than without it. However, in some cases the time is almost the same (DiskBrake) or even less (Pegasus, Dragon). This is due to that in some datasets the application of shape preservation results in a significant reduction of the number of processed pairs and, consequently, in the execution time. Figures 1 and 6 depict some of these datasets with several BOPP of their LOD sequence using shape preservation. We consider EV as the basic geometric element and these figures show the number of EV and the percentage of reduction with respect to the number of EV the original dataset. We can observe that BOPPs with less than 25% of EV give a very good approximation of

Table 1. For each dataset: size in voxels and number of EV ($|EV|$); maximum distance d , number of processed pairs np and time T to compute the whole LOD sequence without shape preservation; d_{sp} , np_{sp} and T_{sp} : same values with shape preservation

Dataset	Size	$ EV $	d	np	$T(\text{sec.})$	d_{sp}	np_{sp}	$T_{sp}(\text{sec.})$
DoorPieces	267x394x72	11,784	46	894	0.35	46	953	0.47
Bunny	127x128x98	24,796	24	1,098	0.63	30	1,236	0.85
Foot	96x270x97	39,078	55	2,031	1.32	49	2,009	1.73
DiskBrake	299x300x43	55,246	32	2,335	1.85	35	2,173	1.95
Engine	140x197x108	92,922	19	2,189	3.08	22	2,761	4.95
Pegasus	382x512x367	290,916	103	20,782	37.85	137	9,228	19.55
Dragon	511x360x228	314,290	56	10,646	19.01	57	8,325	18.42

Fig. 6. Some results of the tested models, showing the number of extreme vertices

the objects, that BOPPs with less than 10% yield an acceptable approximation, and, although BOPPs with less than 1% look blocky, they still give an indication of the object shape. Also note how the holes progressively get closed, specially in Engine and DoorPieces and the connected components are joined in DoorPieces.

We have compared the presented method against two similar methods that compute bounding volumes (see Figs. 6 and 7). We compare the experimental results in terms of percentage of basic geometric elements reduction versus visual approximation. The first method (OCT) [18] is an octree-based approximation and compression method for 3D objects. Approximations are obtained by truncating the octree. The results of the method are presented in terms of number of blocks (octree nodes), which is the basic geometric element of the octree and the test model used is the Bunny dataset. The original model requires 29,007 blocks and 24,796 EV. Figures 7(left) show two objects generated by OCT. We can see that *merging faces* gives a better indication of the shape with less than 5.1% of elements than OCT, and even an object with 22.2% gives a better approximation than OCT with 52%. The second method (BSP)[13] is a progressive solid simplification of objects represented by a Binary Space Partition tree. It uses a volume bounded convex simplification and a plane collapse method to reduce

Fig. 7. Some results of OCT (Bunny dataset) and BSP (Foot dataset) methods, showing the number of elements, blocks and BSP nodes respectively. Figures have been taken from the references [18,13].

the BSP-tree depth. The basic geometric element is a plane (BSP node). We have used the most of the datasets presented in this work and converted them to EVM in such a way that the number of EV of the original model is approximately the same than the number of planes in the BSP tree. We report here the results of the Foot dataset. The original model has 39,078 EV and 38,535 planes. Figures 7(right) show two LOD-objects generated by BSP. We can observe that the shape of the fingers in *merging faces* is preserved until the 1.7% version while in the BSP method the fingers look wrapped since the 30% version. Similar results have been obtained with the other datasets.

8 Conclusions

We have proposed an approach to simplify orthogonal polyhedra and binary images, represented with the EVM. It generates a LOD sequence of BOPP, fulfilling the common subset and AABB properties of bounding structures. The approach is based on a merging strategy that involves pairs of consecutive cuts. We have showed that our approach can deal with objects with any number of holes and connected components, and presented a technique, based on the model continuity, for a better shape preservation. We have compared our method against two similar methods, and in general, our method gives better approximations with the same number of basic geometric elements. Directions for future work include the study of a lossless simplification approach based on the presented one.

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