Efficient Global Optimization Based 3D Carotid AB-LIB MRI Segmentation by Simultaneously Evolving Coupled Surfaces

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Abstract. Magnetic resonance (MR) imaging of carotid atherosclerosis biomarkers are increasingly being investigated for the risk assessment of vulnerable plaques. A fast and robust 3D segmentation of the carotid adventitia (AB) and lumen-intima (LIB) boundaries can greatly alleviate the measurement burden of generating quantitative imaging biomarkers in clinical research. In this paper, we propose a novel global optimization-based approach to segment the carotid AB and LIB from 3D T1-weighted black blood MR images, by simultaneously evolving two coupled surfaces with enforcement of anatomical consistency of the AB and LIB. We show that the evolution of two surfaces at each discrete time-frame can be optimized exactly and globally by means of convex relaxation. Our continuous max-flow based algorithm is implemented in GPUs to achieve high computational performance. The experiment results from 16 carotid MR images show that the algorithm obtained high agreement with manual segmentations and achieved high repeatability in segmentation.

Keywords: Carotid atherosclerosis, convex relaxation, continuous max-flow, image segmentation, GPGPU, coupled level sets.

1 Introduction

Stroke is the second leading cause of death worldwide and approximately 87% of the stroke cases are ischemic [1]. Atherosclerosis at the carotid bifurcation is a major cause of generation of thrombosis and subsequent cerebral emboli. Non-invasive, imaging-based biomarkers provide a direct measurement of plaque burden for monitoring plaque progression and regression in patients who undergo medical interventions [2]. MR imaging has shown promise in quantifying carotid measurements, such as vessel wall volume and thickness maps, plaque composition, and inflammation [2], for assessing the efficacy of medical treatment. A robust 3D segmentation of the carotid adventitia (AB) and lumen-intima (LIB) boundaries would greatly assist a comprehensive analysis of carotid atherosclerosis aiding in the translation of these measurements to clinical research.

Fig. 1. (a) Carotid MR image with overlaid manual segmentations; (b) Pictorial representation of image domain Ω and its sub-domains R_l , R_w , and R_b ; and (c) Contour evolution

Most existing techniques for the carotid AB and LIB MR segmentation are 2D slice-by-slice based methods and delineate the two boundaries separately and independently: Klooster *et al.* [3] used surface fitting for segmentation of the carotid LIB and AB; Kerwin *et al.* [4] proposed to utilize B-spline snakes to extract both boundaries; Adame *et al.* [5] applied fuzzy clustering for carotid LIB segmentation and ellipse fitting with dynamic programming for carotid AB segmentation. However, such methods provide only locally optimal results which are often inefficient and sensitive to the initialization; in addition, they need to explicitly handle changes in topology at the carotid bifurcations and require a great amount of user interaction.

Contributions. In this paper, we describe a novel approach to segment the AB and LIB of common carotid artery (CCA), internal carotid artery (ICA), and external carotid artery (ECA) from T1-weighted (T1w) black blood MR images efficiently and robustly. The segmentation of the carotid AB and LIB is achieved by simultaneously evolving two surfaces with the enforcement of their anatomical order. We show the simultaneous evolution of the coupled surfaces can be solved globally and exactly at each discrete time step, by means of convex relaxation. We propose a continuous max-flow model for the coupled surface evolution, which introduces a dual model to the convex relaxed formulation and derives a fast and fully parallelized algorithm. The results of the experiments demonstrate that our method provides high accuracy and repeatability with significantly less user interaction. The method is mainly intended for monitoring atherosclerosis in patients during medical treatment.

2 Method

The segmentation task of the carotid AB and LIB partitions an MR image into 3 regions: the lumen $\mathcal{R}_l$, the outer-wall $\mathcal{R}_w$, and the background $\mathcal{R}_b$ [see Fig. 1 (a) & (b)]. Since the lumen region is always enclosed within the outer wall region, we model such spatial consistency by a geometrical constraint such that

$$\mathcal{R}_l \subset \mathcal{R}_w, \quad (1)$$

where the background region $\mathcal{R}_b = \Omega \setminus \mathcal{R}_w$: i.e. only two regions $\mathcal{R}_l$ and $\mathcal{R}_w$ need to be segmented. Here, we propose a novel global optimization-based approach

for the simultaneous evolution of two surfaces ($\mathcal{R}_l$ and $\mathcal{R}_w$ in this case) subject to histogram-matching and the overlap constraint (1).

2.1 Optimization Model

PDF Matching. The probability density functions (PDFs) of intensities are regional descriptors of the objects of interest, thus matching PDFs provides a robust partitioning of regions with similar PDFs [6, 7]. In practice, model PDFs (i.e. histogram) can be obtained either from a training set or from sampled voxels. In this work, we generated the model PDFs using sampled voxels.

Let $I(x) \in \mathcal{Z}$ be a given 3D carotid MR image, where $\mathcal{Z}$ is the set of image intensity values. $u_i(x)$, $i = l, w, b$, be the indicator function of the estimated region $\mathcal{R}_i$ such that

$$u_i(x) := \begin{cases} 1, & \text{where } x \text{ is inside } \mathcal{R}_i \\ 0, & \text{otherwise} \end{cases}, \quad i = l, w, b. \quad (2)$$

Given that $\mathcal{R}_b = \Omega \setminus \mathcal{R}_w$, we have $u_b = 1 - u_w$, i.e. only two indicator functions $u_l(x)$ and $u_w(x)$ are used in our optimization problem. The PDF $p_i(u, z)$, where $z \in \mathcal{Z}$ and $i = l, w, b$, of the estimated region $\mathcal{R}_i$ is computed using the Parzen method such that

$$p_i(u, z) = \frac{\int_{\Omega} K(z - I(x)) u \, dx}{\int u \, dx}, \quad i = l, w, b,$$

where $K(\cdot)$ is the Gaussian kernel function $K(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-x^2/2\sigma^2)$.

Let $q_i(z)$, $i = l, w, b$, be the intensity PDF of region $\mathcal{R}_i$, where $z \in \mathcal{Z}$. We use the statistical divergence metric, e.g. Bhattacharyya or Kullback-Leibler distance etc., to measure the distance between the estimated PDFs $p_i(u, z)$, $i = l, w, b$, of the three regions and their respective model PDFs $q_i(z)$. In this work, the Bhattacharyya distance [7] is used for PDF matching:

$$E_{\text{match}}(u) = - \sum_{i=l,w,b} \sum_{z \in \mathcal{Z}} \sqrt{p_i(u, z) q_i(z)}. \quad (3)$$

Optimization Formulation. Using the binary indicator functions $u_{l,w}(x)$, the geometrical overlap prior (1) for regions $\mathcal{R}_l$ and $\mathcal{R}_w$ reduces to the linear inequality constraint

$$u_w(x) \geq u_l(x), \quad \forall x \in \Omega, \quad (4)$$

which enforces the label order of $u_l(x)$ and $u_w(x)$.

In view of the histogram matching energy function (3) and the geometrical overlap constraint (4), we propose to segment 3D carotid MR images by minimizing the energy functional

$$\min_{u_{l,w}(x) \in \{0,1\}} E_{\text{match}}(u) + \sum_{i=l,w} \int_{\Omega} g_i(x) |\nabla u_i(x)| \, dx, \quad \text{s.t. } u_w(x) \geq u_l(x), \quad (5)$$

where $g_i(x) = \lambda_1 + \lambda_2 \times \exp(-\lambda_3 \times |\nabla I(x)|)$ and λ_1, λ_2 , and λ_3 are positive constants and $|\nabla I(x)|$ is the normalized image gradient. The anisotropic total-variation term encodes the segmentation with the minimal geodesic length or area. In this work, we propose to minimize the energy functional (5) by means of contour evolution.

2.2 Evolution of Coupled Contours

Distinct from the conventional local optimization-based contour evolution methods, e.g. [7–9] etc, we simultaneously propagate two coupled contours by means of global optimization. The global optimization approaches to evolve a single contour was previously studied by using graph-cuts [10] and by a total variation-based method [11]. In this work, we extend the single contour evolution [10] to evolve two coupled surfaces simultaneously in a spatially continuous setting.

For each region $\mathcal{R}^t$ at time t , we consider its changes w.r.t. its new position $\mathcal{R}$ at the next time frame $t + 1$, in terms of two distinct regions $\mathcal{R}^+$ and $\mathcal{R}^-$ with their respective cost functions $e^+(x)$ and $e^-(x)$ (see Fig. 1)

1. $\mathcal{R}^+$ denotes the increased area w.r.t. $\mathcal{R}$, i.e. for $\forall x \in \mathcal{R}^+$, it is initially outside $\mathcal{R}^t$, but ‘jumps’ to be inside $\mathcal{R}$; for such ‘jump’, it pays the cost $e^+(x)$.
2. $\mathcal{R}^-$ denotes the reduced area w.r.t. $\mathcal{R}^t$: for $\forall x \in \mathcal{R}^-$, it is initially inside $\mathcal{R}^t$, but ‘jumps’ to be outside $\mathcal{R}$ at $t + 1$; for such ‘jump’, it pays the cost $e^-(x)$.

$\mathcal{R}^t$ is propagated to $\mathcal{R}$ by optimizing the following energy globally and exactly:

$$\min_{\mathcal{R}} \int_{\mathcal{R}^+} e^+(x) dx + \int_{\mathcal{R}^-} e^-(x) dx + \int_{\partial \mathcal{R}} g(s) ds. \quad (6)$$

In view of (6), we propose to minimize (5) by evolving the current two regions $\mathcal{R}_l^t$ and $\mathcal{R}_w^t$ to their new positions $\mathcal{R}_l$ and $\mathcal{R}_w$ such that

$$\min_{\mathcal{R}_l, \mathcal{R}_w} \sum_{i=l, w} \left\{ \int_{\mathcal{R}_i^+} e_i^+(x) dx + \int_{\mathcal{R}_i^-} e_i^-(x) dx \right\} + \sum_{i=l, w} \int_{\partial \mathcal{R}_i} g_i(s) ds \quad (7)$$

subject to the geometrical overlap constraint $\mathcal{R}_l \subset \mathcal{R}_w$.

By using the indicator functions (2), we can equally reformulate (7) as follows

$$\min_{u_l, u_w \in \{0, 1\}} \langle u_l, C_l \rangle + \langle u_w - u_l, C_w \rangle + \langle 1 - u_w, C_b \rangle + \sum_{i=l, w} \int_{\Omega} g_i(x) |\nabla u_i| dx \quad (8)$$

subject to the labeling order constraint (4), i.e. $u_w(x) \geq u_l(x)$ for $\forall x \in \Omega$. The above cost functions $C_i(x)$, $i = l, w, b$, are defined as the first-order variation of the histogram matching term (3) w.r.t. u_l and u_w (see [7] for details).

2.3 Convex Relaxation and Continuous Max-Flow Approach

It has been proven that the non-convex optimization problem (8) can be solved globally and exactly by its convex relaxation [12]:

$$\min_{u_l, u_w \in [0, 1]} \langle u_l, C_l \rangle + \langle u_w - u_l, C_w \rangle + \langle 1 - u_w, C_b \rangle + \sum_{i=l, w} \int_{\Omega} g_i(x) |\nabla u_i| dx \quad (9)$$

Fig. 2. Example expert initializations and 2D segmentation results. The only user interaction used in the pipeline is the choice of sample seeds on a single transverse slice. Green, red, and blue seeds correspond to lumen, wall and background regions respectively. (a) & (b): T1w 3T MR image. (c) & (d) T1w 1.5T MR image.

subject to the label ordering constraint $u_w(x) \geq u_l(x)$ for $\forall x \in \Omega$. In other words, the two regions $\mathcal{R}_l^t$ and $\mathcal{R}_w^t$ can be evolved to their globally optimal positions $\mathcal{R}_l$ and $\mathcal{R}_w$ at each time frame from t to $t+1$, subject to the geometrical overlap constraint (1). In this work, we follow the continuous min-cut/max-flow theory proposed by Yuan *et al.* [13] and Bae *et al.* [12] to solve the proposed optimization problem (9), globally and exactly. To this end, we adopt their flow configuration [12] and propose the continuous max-flow as follows:

$$\max_{p_b, p_l, p_w} \int_{\Omega} p_l(x) dx \quad (10)$$

subject to the flow capacities

$$|q_i(x)| \leq g_i(x), \quad i = l, w; \quad p_i(x) \leq C_i(x), \quad i = b, l, w; \quad (11)$$

and the flow conservation conditions

$$(\operatorname{div} q_l - p_w + p_l)(x) = 0, \quad (\operatorname{div} q_w - p_b + p_w)(x) = 0, \quad \forall x \in \Omega. \quad (12)$$

The continuous max-flow model is dual/equivalent to the convex relaxation problem (9) [12]. By (10), we derive an efficient continuous max-flow based algorithm which is different from the ones proposed by Bae *et al.* [12]. Our model explores the optimization over all the dual flow functions in parallel, instead of sequentially or group-wise sequentially. In practice, the new parallelized scheme achieves a faster convergence.

3 Experiments and Results

Segmentation Pipeline. Initially, the anisotropic voxels are interpolated into isotropic voxels. In our approach, the carotid wall, lumen and the background regions were initialized using an interactive user interface by the expert *only on a single transverse slice* as shown in Fig. 2 (a) and (c). The sampled voxels are used to generate model PDFs for histogram matching [see (3)] and are considered

Fig. 3. Example segmentations of a T1w 3T MR image [Fig. 3(b)] and a 1.5T MR image [Fig. 3(d)] by the proposed coupled surface evolution approach

as region-based hard constraints. The carotid AB and LIB are segmented using 2D multi-surface evolution by minimizing the objective function (5). The 2D segmentation result is then used to generate a more representative prior PDFs for each region for the 3D coupled segmentation. We perform a region growing segmentation of the LIB to obtain a crude initial guess [see Fig. 3 (a) & (c)]. We used region growing method for its simplicity; however, the operator could also provide additional sampled seeds on the long-axis direction of the artery. We obtain an initial estimate of the AB surface by dilating the LIB initial surface. Finally, we use the coupled surface evolution in 3D for the segmentation of carotid AB and LIB by minimizing the objective function (5). To incorporate an anatomically-motivated separation of the carotid AB and LIB, we assigned $e^+(x)$ infinite cost for the voxels that are within a minimum separation distance (0.6 mm is used for experiments) outside to the LIB.

Data and Validation. The data comprise of 16 left and right carotid artery T1w black blood MR images from eight subjects: eight 3T (voxel size $\approx 0.2 \times 0.2 \times 2 \text{ mm}^3$) and eight 1.5T (voxel size $\approx 0.5 \times 0.5 \times 2 \text{ mm}^3$) MR images. Subjects were scanned using a GE Excite HD MRI (Milwaukee, WI, USA) with a custom-built six-element carotid-bifurcation-optimized receive-only phased-array coil. The imaging parameters are as follows: TR is 1RR and TEs are 11.4 ms and 12 ms for 3T and 1.5T images respectively with FSE and fat saturation.

The performance of the algorithm was evaluated with respect to manual segmentations in terms of accuracy and reproducibility. Manual segmentations were performed on a slice-by-slice basis on transverse view using a multi-planar reformatting software with 1 mm inter-slice distance up to 4 cm along carotid including CCA, ICA, and ECA. We used Dice coefficient (DSC) as the region-based metric, root mean square error (RMSE), and Hausdorff distance (MAXD) as distance-based metrics.

Results. The convex max-flow algorithm was implemented using parallel computing architecture (CUDA, NVIDIA Corp., Santa Clara, CA) and the user interface in Matlab (Natick, MA). The experiments were conducted on a Quad core Windows workstation with 2.8 GHz and a GPU of NVIDIA GTX580. Our algorithm required ≈ 25 s of time for expert initialization. The computational

Table 1. Results for eight 3T MR images and eight 1.5T MR images

Metric	CCA		ICA		ECA	
	AB	LIB	AB	LIB	AB	LIB
3T DSC (%)	94 ± 2	94 ± 4	87 ± 6	90 ± 5	80 ± 6	83 ± 6
3T RMSE (mm)	0.3 ± 0.1	0.4 ± 0.3	0.3 ± 0.1	0.3 ± 0.1	0.3 ± 0.1	0.3 ± 0.2
3T MAXD (mm)	1.2 ± 0.2	0.5 ± 0.6	2 ± 1.6	0.7 ± 0.2	1.4 ± 1.5	0.8 ± 0.3
1.5T DSC (%)	90 ± 5	93 ± 3	75 ± 9	92 ± 4	69 ± 12	88 ± 4
1.5T RMSE (mm)	0.5 ± 0.2	0.8 ± 1.2	0.6 ± 0.3	0.4 ± 0.1	0.6 ± 0.3	0.4 ± 0.1
1.5T MAXD (mm)	1.6 ± 1.0	1.5 ± 2.0	3.1 ± 5.3	1.1 ± 1.1	6 ± 4.0	0.9 ± 0.3

Table 2. Results for observer variability using DSC(%) for 8 1.5T MR images

Repetition #	CCA		ICA		ECA	
	AB	LIB	AB	LIB	AB	LIB
1	89.8 ± 5.1	93.3 ± 3.2	75.5 ± 9.0	91.8 ± 3.9	70.4 ± 12.0	87.9 ± 3.8
2	89.6 ± 5.0	93.3 ± 3.3	75 ± 8.9	92 ± 4.0	68.2 ± 11.5	88.4 ± 4.5
3	89.7 ± 5.0	93.6 ± 3.4	75.2 ± 9.1	92.2 ± 4.3	69.3 ± 10.7	88.3 ± 4.1

time for convergence of the algorithm was ≈ 40 s (8s for max-flow in a GPU and 32s for cost computation using non-optimized Matlab code) which was achieved within 10-15 iterations for a single 3D MR image with 110 slices. Figure 3 (b) & (d) show the carotid AB and LIB surfaces generated using our algorithm for some example 3T and 1.5T MR images. The performance results of the algorithm are shown in Table 1. The algorithm yielded high DSC and low RMSE for CCA, ICA, and ECA for both data sets except for the ECA AB where a DSC of 70% was obtained for 1.5T MR images. We also assessed the intra-observer variability by repeatedly segmenting the same image set three times with different initializations. The results of the intra-observer variability analysis of the algorithm is shown in Table 2. The algorithm yielded approximately similar DSCs in all three repetitions, which suggests a high reproducibility of our approach.

4 Discussion and Conclusion

We developed a novel global optimization approach for coupled surface evolution for carotid AB and LIB segmentations. The coupling permits the integration of image information derived from both surfaces to drive their optimization. The algorithm provided robust and efficient segmentation results for the AB and LIB in terms of accuracy and intra-observer reproducibility. The 2D method proposed by Adame *et al.* [5] is currently used in clinical trials, which requires 40s to segment a single 2D slice. Our method provides substantial improvement in speed for segmenting 3D images over previous methods. Li *et al.* [14] also proposed a single-shot graph-cut approach to segment coupled surfaces, but their method need to unwrap the image domain, in which handling carotid bifurcations or changes in topology are challenging.

Most previous papers segment only the carotid CCA and report accuracy in terms of area measurements but not DSC or RMSE [5, 4]. Our algorithm reports a higher accuracy in terms of DSC for the CCA and ICA than ECA. The reduced accuracy for the ECA is mostly due to the weak image information of the ECA AB, in which the algorithm attempts to maintain a minimal length surface without splitting at the carotid bifurcations. Clinically, CCA and ICA segmentations are more important than ECA as the plaque tend to be present mostly in CCA and ICA [2]. The expert may initialize on more than one 2D slice to achieve further accuracy of the segmentations, especially proximal to the carotid bifurcations. The performance results of our algorithm suggest that it may be suitable for use in clinical trials involving the monitoring of carotid atherosclerosis using 3D MR imaging-based biomarkers.

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