Fast 3D Spine Reconstruction of Postoperative Patients Using a Multilevel Statistical Model

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Abstract. Severe cases of spinal deformities such as scoliosis are usually treated by a surgery where instrumentation (hooks, screws and rods) is installed to the spine to correct deformities. Even if the purpose is to obtain a normal spine curve, the result is often straighter than normal. In this paper, we propose a fast statistical reconstruction algorithm based on a general model which can deal with such instrumented spines. To this end, we present the concept of multilevel statistical model where the data are decomposed into a within-group and a between-group component. The reconstruction procedure is formulated as a second-order cone program which can be solved very fast (few tenths of a second). Reconstruction errors were evaluated on real patient data and results showed that multilevel modeling allows better 3D reconstruction than classical models.

Keywords: 3D reconstruction, spine, statistical shape model, multilevel modeling.

1 Introduction

Three-dimensional reconstruction of the spine is a valuable process to study spinal deformities such as scoliosis. It allows to determine clinical indices helping diagnosis and treatment. It normally needs to be performed based on radiographs to allow a natural standing position for the patient and to reduce as much as possible the exposition of young patients to ionizing radiations.

Usual approaches to reconstruct the spine from two radiographs consists in manually identifying corresponding landmarks on the views and matching them in a three-dimensional space. These methods required to locate at least six points per vertebra [1]. Other authors proposed to increase the number of points to be located by considering landmarks that are only visible in one radiograph [8]. These methods are very time-consuming and are hardly transposable to a medical practice.

In order to reduce the amount of manual intervention, statistical reconstructions methods later appeared. In those methods, a reduced set of input is provided by the user and the rest of the model is inferred with the help of a statistical shape model. For example, Humbert *et al.* [5] proposed to evaluate a parametric model based on the spinal centerline, Moura *et al.* [9] inferred an articulated model of the spine based on splines, and Boisvert *et al.* [2] formulated the estimation of the spine shape as a second-order cone program.

Patients with spinal deformities can be treated in various ways depending on the severity of the deformation. For severe cases, surgery can be recommended. All the methods we have just described are valuable, but are always focused on patients who have not undergone any surgery. Surgical treatment of scoliosis consists in applying instrumentation to the spine in order to redress the spine and maintain the correction. Even if the purpose is to obtain a normal spine curve, the result is often neither a normal spinal curve nor a scoliotic one (see Fig. 1 for instance). Therefore, it is difficult to capture these specific deformations with a classical statistical model. In this context, we propose to use a more general model adapted to hierarchical structures like the spine: a multilevel statistical shape model. The advantage of such a model is to represent the dependency between one vertebra and the others. As a result, several sub-models are built and can be treated separately, each level characterizing one sub-model. In the literature, the inter-vertebra dependence between pairs of vertebræ has already been modeled in [3]. Our multilevel framework is however more generic since various group structures can be selected. We can, for instance, represent the dependence between individual vertebra, between duos of vertebræ, between triplets, *etc.*

To the best of our knowledge, this paper is the first report of an interactive and fast 3D reconstruction method of the spine when surgical instrumentation is present. Furthermore, our approach introduces the use of the multilevel statistical shape modeling to the problem of 3D shape reconstruction. We will show that our method provides better results than classical statistical models.

2 Method

2.1 Multilevel Statistical Shape Model

While principal component analysis (PCA) is usually required to build a statistical shape model, multilevel component analysis (MCA) is the basis to design a multilevel statistical shape model. The concept of MCA was introduced in [10] as an extension of PCA for hierarchical structures. If we consider a model with 2 levels, the idea is to decompose the data into a within-individual and a between-individual component. Let us assume a sample with N items, divided into K groups of size K_k . An item i belonging to the group k according to the variable j is denoted by: x_{ijk} , with $i \in [1, \dots, K_k]$, $k \in [1, \dots, K]$, and $j \in [1, \dots, J]$. Based on the Cronbach and Webb's model [4], x_{ijk} can be decomposed into a within-group and a between-group term, such as:

Fig. 1. Radiographs of a postoperative patient and the associated reconstruction. Left: Lateral view. Center: Postero-anterior view. Right: 3D reconstruction based on our approach.

$$x_{ijk} = x_{\bullet j \bullet} + (x_{\bullet jk} - x_{\bullet j \bullet}) + (x_{ijk} - x_{\bullet jk}), \quad (1)$$

where $x_{\bullet j \bullet} = \frac{1}{N} \sum_{k=1}^K \sum_{i=1}^{K_k} x_{ijk}$, and $x_{\bullet jk} = \frac{1}{K_k} \sum_{i=1}^{K_k} x_{ijk}$. In the relation (1), $(x_{\bullet jk} - x_{\bullet j \bullet})$ is the between-group term, while $(x_{ijk} - x_{\bullet jk})$ is the within-group one.

Based on the decomposition of the equation (1), a multilevel model is defined as several sub-models that can be treated separately. Let us consider the spine as a hierarchical structure such as in Fig. 2.

Let us assume a sample of I patients characterized by K vertebrae. In [7], the authors proposed a multilevel modelization of the vertebrae. Here, we develop a deformable model that can represent all the spine. As a consequence, in the

Fig. 2. Multilevel representation of the spine for a sample of patients

proposed modelization, each patient of the hierarchical structure can be viewed as a group at the first level. As a consequence, the within-group part of the model represents the variability between vertebrae while the between-group part concerns the inter-patient variability. If we assume that a patient (a spine) is represented according to J variables, a deformable model of a given vertebra x_i with $i \in [1, \dots, K]$ is defined by:

$$x_i = \bar{x} + \phi_{W,i}d_W + \phi_B d_B, \quad (2)$$

where $\bar{x}$ is a column-vector of size J standing for the mean computed over all the objects in the sample, $\phi_{W,i}$ denotes the lines of ϕ_W , a $J \times R_W$ matrix containing the within-group principal components, d_W is a $R_W \times 1$ vector representing the weights controlling the deformation of the within-group term, ϕ_B stands for the $J \times R_B$ matrix representing the between-group principal components, d_B is the $R_B \times 1$ vector of weights controlling the deformation of the between-group term.

The interest of MCA is that the parameters of the equation (2) can be determined separately (the demonstration can be found in [10]).

Let X , be a $N \times J$ matrix including all the vertebrae of the sample ($N = KI$). First, the within-group parameters are obtained after a particular decomposition of the matrix X . This decomposition consists in mean-centering all the sub-matrices X_i of size $K \times J$, where X_i is the partition of the matrix X belonging to the group i . Let $X_{c,i}$ be the resulting matrix. $X_{c,i}$ actually represents the within-patient variability. Therefore, each line of the matrix defines a vertebra. To obtain a global representation of the spine, matrices $X_{c,i}$ are transformed in a column-vector by concatenating the lines of $X_{c,i}$. Let X_c be the matrix built from the vertical concatenation of the resulting column-vectors. The matrix ϕ_W is composed of the eigenvectors of the covariance matrix related to X_c . Furthermore, the variance of the weight d_W , which limits the deformation of the within-group sub-model, is determined by the eigenvalues of the covariance matrix related to X_c .

In order to determine the between-group parameters of the model, let us subtract the overall mean of the matrix X . Let $\tilde{X}$ be this matrix. Let us note $\tilde{X}_i$, the partition of the matrix $\tilde{X}$ belonging to the group i . Moreover, let us consider the I vectors $\tilde{m}_i$, each of them representing the mean of the associated matrix $\tilde{X}_i$. In fact, these vectors characterize the between-patient differences. Therefore, we note M , the matrix resulting from the vertical concatenation of the vectors $\tilde{m}_i$. As a consequence, the matrix ϕ_B of the between-group principal components is built by the eigenvectors of the covariance matrix related to M . In addition, the deformation limits of the between-group sub-model are given by the eigenvalues of the covariance matrix related to M .

These concepts can naturally be extended to a greater number of hierarchical levels. Since we want to represent the variability between the vertebrae (*i.e. within-patient*), the extra levels are within-group terms. We can generalize the relation (2) in accordance with:

$$x_i = \bar{x} + \sum_{l=1}^{L-1} \phi_{W_l, i} d_{W_l} + \phi_B d_B, \quad (3)$$

where L is the number of levels and i_l is the index of the group to which the object belongs at the level l .

2.2 Reconstruction Algorithm

The principle of the reconstruction algorithm is to deform a 3D shape of the spine so that it matches the multiple views of the object to be reconstructed. In our case, two views are available: a posteroanterior (PA) and a lateral (LAT) radiograph. However, the final solution needs to be in conform to the model defined in section 2.1. In other words, the model constitutes a statistical *a priori* that the reconstruction algorithm has to take advantage of. A common metric to determine the degree of similarity between a shape and a shape model is the Mahalanobis distance.

The minimization of the Mahalanobis distance during the reconstruction process allows that the final shape fits the statistical distribution of the model. Another constraint needs to be met in the optimization problem. In order to match the 3D shape with the PA and LAT views, the Euclidean distance between the projection of a 3D point x_i and its theoretical location on the radiograph has to be minimized. Authors showed [2] that this distance can be computed given:

$$\|u_i^j - \tilde{u}_i^j\|_2 = \frac{1}{P_3^j (x_i \ 1)^T} \left\| \begin{pmatrix} P_1^j - P_3^j u_{i,x}^j \\ P_2^j - P_3^j u_{i,y}^j \end{pmatrix} \begin{pmatrix} x_i \\ 1 \end{pmatrix} \right\|_2. \quad (4)$$

where P_i^j is the i^{th} line of the matrix P^j and u_i^j is the projection of the point x_i . These authors also proposed to limit this error to a given constant e_{max} while optimizing for the point position. They formulated the problem as a second-order cone optimization program, just as it is demonstrated in [6]. Since second-order cone programming expects to operate with the norm of expressions, the Mahalanobis distance requires to be formulated with L , a Cholesky decomposition of Σ^{-1} , the inverse covariance matrix of the sample [2].

The second-order cone program is expressed by minimizing the Mahalanobis distance while constraining the solution to result in a projection error smaller than e_{max} using:

$$\begin{cases} \min & t \\ s.t. & \left\| L^T (x - \bar{x}) \right\|_2 \leq t \\ & \left\| \begin{pmatrix} P_1^j - P_3^j u_{i,x}^j \\ P_2^j - P_3^j u_{i,y}^j \end{pmatrix} \begin{pmatrix} x_i \\ 1 \end{pmatrix} \right\|_2 \leq e_{max} P_3^j \begin{pmatrix} x_i \\ 1 \end{pmatrix} \end{cases} \quad (5)$$

To reduce the number of variables, it is possible to optimize MCA weights instead of point coordinates. To this end, let us define some specific notations. Let x_{ik} be a point k belonging to the vertebra x_i . Equation (3) allows to express:

$$x_{ik} = \bar{x}_k + \sum_{l=1}^{L-1} \phi_{W_l,ik} d_{W_l} + \phi_{B,k} d_B, \quad (6)$$

where $\bar{x}_k$, $\phi_{W_l,ik}$ and $\phi_{B,k}$ are respectively the lines of $\bar{x}$, $\phi_{W_l,i}$, and ϕ_B , associated to the point x_{ik} . Let us also consider $\sigma_{W_l}^2$ and σ_B^2 , the variances associated to, respectively, the within-group and the between-group sub-models. Moreover, for simplicity of writing, let us define:

$$\psi(d_{W_l}, d_B)_k = \bar{x}_k + \sum_{l=1}^{L-1} \phi_{W_l,ik} d_{W_l} + \phi_{B,k} d_B. \quad (7)$$

Finally, the second-order cone programming optimization problem to match a multilevel statistical model with radiographic views is formulated as:

$$\begin{cases} \min & t \\ \text{s.t.} & \left(\sum_{l=1}^{L-1} \left\| \text{diag}\left(\frac{1}{\sigma_{W_l}}\right) d_{W_l} \right\|_2^2 + \left\| \text{diag}\left(\frac{1}{\sigma_B}\right) d_B \right\|_2^2 \right)^{\frac{1}{2}} \leq t \\ & \left\| \begin{pmatrix} P_1^j - P_3^j u_{k,x}^j \\ P_2^j - P_3^j u_{k,y}^j \end{pmatrix} \begin{pmatrix} \psi(d_{W_l}, d_B)_k \\ 1 \end{pmatrix} \right\|_2 \leq e_{max} P_3^j \begin{pmatrix} \psi(d_{W_l}, d_B)_k \\ 1 \end{pmatrix} \end{cases} \quad (8)$$

3 Results

In order to process a reconstruction, two radiographs are presented to the user. He then has to point out some anatomical landmarks on the images to initiate the optimization of the problem (8). To validate our approach, we used a sample of 307 scoliotic patients for building the multilevel model. We considered 17 vertebrae of the spine: T1 to L5. Each vertebra is represented by 6 points of reference, *i.e.* the center of inferior and superior endplates, and the inferior and superior extremities of pedicles. 3D reconstructions based on a 2-level and a 3-level model have been performed for 25 post-operative patients whose spine was previously reconstructed following a reference method [1]. Let us note that tests were performed on an Intel Core 2 Duo 2.53 GHz.

We first present at Table 1 the decomposition of the total variability for a 2-level and a 3-level model. These values are computed in the same way as for the variance decomposition in ANOVA. Results of Table 1 show that the magnitude of the within-group and the between-group variability is sufficient to use a 2-level and a 3-level model in the reconstruction algorithm.

Mean RMS reconstruction error has been evaluated as a function of the number of points per radiograph (see Fig. 3). The parameter e_{max} was set to 8 pixels. The reconstruction of pedicles and plates are distinguished. One can remark that

Table 1. Decomposition of the total variability

Levels	Var. With. (%)	Var. Between (%)
2	88.83	11.17
3	23.12	65.71

plates reconstruction is better than pedicles reconstruction. This observation is actually similar in the case of reconstruction of the spine with no instrumentation [9]. Moreover, the reconstruction error decreases as the number of points per radiograph increases. However, this effect is reduced after approximatively 6 to 7 points per radiograph. Finally, Fig. 3 indicates that, both for plates and pedicles, the 3-level model converges more slowly than the 2-level model or the 1-level model. Adding extra levels in the model requires more constraints in the optimization problem. As a consequence, more control points per radiograph are needed. We have also compared the mean reconstruction error based on the 2-level model with a classical statistical model (with only one level) for 17 control points. A mean error of $2.12mm$ for endplates and $4.02mm$ for pedicles was obtained. For the 2-level model, these values are, respectively, $2.05mm$ and $3.70mm$. A paired *t-test* shows that these differences are significant (at level $\alpha = 0.05$). Actually, when a few control points are considered, the difference between the mean error of the 2-level model and the 1-level model is low. This difference increases with the number of control points. Furthermore, if we only consider instrumented vertebrae, the difference between the 2-level model and the classical model is increased. In this context, the 2-level model shows a mean error of $2.09mm$ for endplates and of $3.64mm$ for pedicles. The classical model is characterized by a mean error of $2.19mm$ for endplates and of $4.37mm$ for pedicles. This demonstrates that our approach based on a multilevel model is to be preferred in the case of 3D reconstruction of the post-operative patient spine.

Finally, we propose at Fig. 3 the execution times for a reconstruction based on a PCA model, a 2-level and a 3-level model. Since more constraints are considered

Fig. 3. Left: Evolution of the mean RMS 3D reconstruction error as a function of the number of points per radiograph. Right: Evolution of the elapsed time as a function of the number of points per radiograph

for the 3-level model, computing times are logically higher. Nevertheless, the results are an order of magnitude fast than most current methods, for example Humbert *et al.* [5] take about 4000ms to generate a reconstruction and Moura *et al.* [9] about 3000ms. These results tend to show that our approach could be used interactively in the clinic.

4 Conclusion

In this manuscript, we proposed an algorithm to perform 3D reconstructions of the spine from bi-planar radiographs when surgical instrumentation is present. Our approach is based on a multilevel statistical model. Results showed that this model allows better reconstruction than classical models. The separation into several levels allows to deal with discontinuities characterizing the spine of post-operative patients. Since 3D spine reconstructions are obtained in real-time, preliminary results tend to show that our approach could be transposable to medical practice.

References

1. Aubin, C.E., Dansereau, J., Parent, F., Labelle, H., de Guise, J.A.: Morphometric evaluations of personalised 3D reconstructions and geometric models of the human spine. *Medical & Biological Engineering & Computing* 35(6), 611–618 (1997)
2. Boisvert, J., Moura, D.C.: Interactive 3D Reconstruction of the Spine from Radiographs Using a Statistical Shape Model and Second-Order Cone Programming. In: 33rd Annual International Conference of the IEEE EMBS, pp. 5726–5729 (2011)
3. de Bruijne, M., Lund, M.T., Tankó, L.B., Pettersen, P.C., Nielsen, M.: Quantitative vertebral morphometry using neighbor-conditional shape models. *Medical Image Analysis* 11(5), 503–512 (2007)
4. Cronbach, L.J., Webb, N.: Between-class and within-class effects in a reported aptitude treatment interaction: Reanalysis of a study by G. L. Anderson. *Journal of Educational Psychology* 67(6), 717–724 (1975)
5. Humbert, L., De Guise, J.A., Aubert, B., Godbout, B., Skalli, W.: 3D reconstruction of the spine from biplanar X-rays using parametric models based on transversal and longitudinal inferences. *Medical Engineering & Physics* 31(6), 681–687 (2009)
6. Kahl, F., Hartley, R.: Multiple-View Geometry Under the L_∞ -Norm. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 30(9), 1603–1617 (2009)
7. Lecron, F., Boisvert, J., Benjelloun, M., Labelle, H., Mahmoudi, S.: Multilevel statistical shape models: a new framework for modeling hierarchical structures. In: *IEEE International Symposium on Biomedical Imaging*, pp. 1284–1287 (2012)
8. Mitulescu, A., Semaan, I., De Guise, J.A., Leborgne, P., Adamsbaum, C., Skalli, W.: Validation of the non-stereo corresponding points stereoradiographic 3D reconstruction technique. *Med. Biol. Eng. Comput.* 39(2), 152–158 (2001)
9. Moura, D.C., Boisvert, J., Barbosa, J.G., Labelle, H., Tavares, J.M.R.S.: Fast 3D reconstruction of the spine from biplanar radiographs using a deformable articulated model. *Medical Engineering & Physics* 33, 924–933 (2011)
10. Timmerman, M.E.: Multilevel component analysis. *British Journal of Mathematical and Statistical Psychology* 59(2), 301–320 (2006)