Multi Body Kalman Filtering with Articulation Constraints for Humanoid Robot Pose and Motion Estimation

Daniel Hauschildt, Sören Kerner, Stefan Tasse, and Oliver Urbann

Robotics Research Institute
Section Information Technology
TU Dortmund University
44221 Dortmund, Germany

Abstract. In this paper, a concept for articulated rigid body state estimation is proposed. The articulated body, for instance a humanoid robot, is modeled in a maximal coordinate formulation and the articulations between the rigid bodies as nonlinear position and linear motion constraints. At first, the individual state of each particular rigid body is estimated with a Kalman filter, which leads to an unconstrained state estimate. Subsequently, the correct state estimate for the articulated rigid body is derived by projecting the unconstrained estimate onto the constraint surface.

1 Introduction

Nowadays robots are primarily used in assembly line production. The deployed robots are mostly fixed-based manipulators. However, researchers all over the world are trying to extend the robots task spectrum into areas requiring more mobility, such as search and rescue services, military operations or carrying out assistant tasks in everyday life. In order to fulfill those tasks, robots must be capable of navigating in various environments of which some are inhabited and designed for human beings. Especially the latter environments are particularly challenging for conventional wheeled robots as stairs or even small objects can become insurmountable obstacles. Thus legged and especially humanoid robots are becoming more and more focus of research. Since humanoid robots try to imitate human appearance in respect of their body design, they have a good foundation to navigate in the proposed environments and therefore are believed to have high potential for future applications. Despite this, currently available humanoid robots have a lack of mobility due to the fact that the generation of stable biped motion is still a major problem, which is not yet solved completely. To control the motion of a robot it is required to determine its current state, including among other things its ego-pose and motion. Exact orientation and velocity information is especially useful for balancing walking motions while measurements of translational movement can also complement the odometry information for improved global localization. Humans determine their state by

T. Röfer et al. (Eds.): RoboCup 2011, LNCS 7416, pp. 415–426, 2012.

proprioception - the unconscious perception of movement and spatial orientation arising from stimuli within the body itself - which a robot simply does not have, complicating the process of ensuring motion stability. Proprioception is therefore imitated by estimating the body state with the help of proprioceptive sensors, among them accelerometers, gyroscopes or a compass. Since high-grade sensors are, if available, expensive, low-grade sensors are commonly used in commercially availably humanoid robot designs. Since their sensor reading are mostly not very accurate they in turn lead to erroneous state estimation results further influencing the motion stability of the robot. To overcome this flaw a novel approach to multi body state estimation is proposed in this paper, reducing the estimation error with the help of rigid body simulation. Furthermore this algorithm is capable to combine different sensor readings, allowing to further improve the state estimation by using a multitude of low-cost sensors at different parts of the robot.

After presenting a short overview of current research on this topic in section 2, the mathematical background is formulated in section 4. The derivation of the proposed algorithm is explained in sections 5 and 6 and finally evaluated in section 7.

2 Related Work

Various solutions have been proposed to solve the problem of ego-state estimation of a legged robot. Most of them use some sort of Bayesian filtering technique such as Kalman-filters or one of its variations. The vast majority of those approaches use a greatly simplified robot model in combination with stochastic information about the reliability of the sensor measurement to estimate the state with the highest probability. But only little work towards using a more sophisticated model of the robot has been done (see [1]) so far.

Latter approaches have in common that they try to model the robot as a chain of rigid bodies, the limbs, which are connected via various types of joints. In contrast to the approach, proposed in this paper, so-called reduced coordinate formulations are employed. Indeed this has the advantage that the number of such coordinates only equals the number of the degrees of freedom of the system, which are removed by the joints, and therefore yield in a more compact and intuitive robot model. Nevertheless, this also generally leads to a highly nonlinear model, which makes the utilization of well-known state estimation approaches, such as Kalman-filters difficult or not applicable at all.

3 Proposed Approach

Commonly the estimation of a humanoid robot's position change and orientation in space is estimated using standard Kalman approaches modeling the robot as a single rigid body and using inertia measurements as sensor information. For biped robots however the inertia measurements originating from the periodic walking motions dominate those correlated to actually changing speed or heading. Common approaches simply handle those former signal components as noise, thus having to cope with a significant noise to signal ratio. To be able to use those walking motion signal components as well as the joint sensor information it is necessary to model the complete kinematic structure of the robot to be used for estimation.

To do this and to improve the state estimation of a rigid multi body model a novel approach is presented in this paper, using a maximal coordinate formulation scheme. In this approach the robot is modeled using a maximal coordinate formulation scheme, so that sophisticated prediction methods, such as Kalman-filters, can be applied to improve the pose and motion estimation. In contrast to reduced coordinate formulations, no reduction of the number of required parameters is performed. Instead the connections between the rigid bodies - the joints - are modeled as constraints, which actually introduce additional parameters. While this at first appears to have a counter-productive effect, the modeling allows subdividing the articulated rigid body state estimation problem into a set of smaller single rigid body state estimation problems and therefore can be handled more efficiently.

Furthermore, this approach not only allows the use of a more sophisticated model and therefore a better estimate for the robot's state, but also yields additional benefits. By treating every limb of the robot separately, additional sensor informations can be integrated in the filter. For instance, inertia sensors need not be attached to the torso of the robot only, but could also be mounted to any limb, to improve the overall grade of the estimated state. Furthermore using the robots structural information it is possible to include the knowledge about the current environment the robot is located in. Hence, for example, ground contact or collision information can be modeled as a special type of joints, resulting in contact and collision constraints, which in turn can be included in the state estimation with ease.

4 Definitions

A humanoid robot can be modeled as a collection of n rigid bodies connected by joints. These joints impose a number of $n_{c,p}$ position constraints and $n_{c,m}$ motion constraints that restrict the possible motion of each rigid body. The state x_i of a rigid body can be described as

$$x_i = (p_i, q_i, v_i, \omega_i)^T \tag{1}$$

whereby p_i and q_i are the position vector and orientation unit quaternion of a rigid body's center of mass. Furthermore, v_i and w_i are the linear and angular velocities. When not mentioned differently all coordinates are relative to a common world coordinate system (WCS). In addition, the state of a rigid body is integrated over time Δt by the following transfer function

$$x_i^{k+1} = \begin{pmatrix} p_i^k + \Delta \mathrm{t}\ v_i^k + \frac{1}{2}\Delta t^2 M_i^{-1} f_i^k \\ q_i\left(\Delta t \omega_i^k + \frac{1}{2}\Delta t^2\ I_i^{-1}\ \tau_i^k\ \right) * q_i^k \\ v_i^k + \Delta t\ M_i^{-1}\ f_i^k i \\ \omega_i^k + \Delta t\ I_i^{-1}\ \tau_i^k \end{pmatrix} = f\left(x_i^k, u_i^k\right) \tag{2}$$

with $u_i = (f_i,\, \tau_i)^T$ being the vector of external forces f_i and torques τ_i. Thereby, M_i and I_i are the mass matrix and inertia matrix which describe the first and second order mass distribution of the rigid body in the WCS. While M_i is independ of the current state, I_i depends on the current orientation $R_i = f(q_i)$. Consequently, the angular mass matrix is derived by $I_i = R_i D_i R_i^T$ with D_i being the angular mass matrix in the body local coordinate system (BCS).

Starting with equations 1 and 2 the state of n independent rigid bodies can be comprised as

$$\begin{aligned} x^k &= \left(x_0^k, \ldots, x_{n-1}^k\right)^T \\ u^k &= \left(u_0^k, \ldots, u_{n-1}^k\right)^T \\ f\left(x^k, u^k\right) &= \left(f\left(x_0^k, u_0^k\right), \ldots, f\left(x_{n-1}^k, u_{n-1}^k\right)\right)^T. \end{aligned} \tag{3}$$

For now, the joints between the rigid bodies are not considered. However, joints can easily be formulated as constraints in a maximal coordinate formulation. Thereby, it has to be distinguished between pure position constraints $C_p(p, q)$ and pure motion constraints $C_m(v, w)$. Other types of constraints are not considered, however, most common joint types can be modeled in this way. As a result, the motion of n articulated rigid bodies can be formulated as

$$x^{k+1} = f\left(x^k, u^k\right) \tag{4}$$

subject to

$$C_p(p, q) - d_p = 0 \tag{5}$$

$$C_m(v, \omega) - d_m = 0 \tag{6}$$

where d_p and d_m are constant offsets, for example the fixed distance between two joints.

5 Unconstrained State Estimation

Consider the following nonlinear time invariant system given by equation 3 and sensor measurements

$$z^k = \left(z_0^k, \ldots,\, z_{n-1}^k\right)^T \tag{7}$$

whereby

$$z_i^k = g(x_i^k) \tag{8}$$

measures a sub state of the i-th rigid body at time $k\ \Delta t$. Measurements regarding multiple rigid bodies are not considered and thereby are neglected. However, they can be modeled as constraints if the measurement noise is small and can be neglected, as for instance is true for accurate joint state measurements provided by most servo motors.

Furthermore, it is assumed that the control input u_i and the measurements z_i are either unknown or disturbed by noise and therefore modeled as Gaussian random variables. Consequently, the states x_i of the rigid bodies become random variables with unknown probability distributions. Considering this, the first order central moments are

$$E\left(a\right) = \left(E\left(a_0\right), \ldots, E\left(a_{n+1}\right)\right)^T = \widehat{a} \tag{9}$$

and the second order moments are

$$E\left(\left(a - \hat{a}\right)^2\right) = \begin{pmatrix} \Sigma_{a_0,a_0} & \cdots & \Sigma_{a_0,a_{n-1}} \\ \vdots & \ddots & \vdots \\ \Sigma_{a_{n-1},a_0} & \cdots & \Sigma_{a_{n-1},a_{n-1}} \end{pmatrix} = \Sigma_a \tag{10}$$

with $E(\cdot)$ being the expectation operator, $a \in \{x, u, z\}$ and $a^2 = a\ a^T$.

In this manner, it must be noted that the covariances

$$\Sigma_{x_i,x_j} = \Sigma_{u_i,u_j} = \Sigma_{z_i,z_j} = 0\ for\ i \neq j \tag{11}$$

are assumed zero and therefore will be disregarded. This assumption allows us to tackle the multi body state estimation problem more efficiently as the problem can be decomposed into n single body state estimation problems. Besides, the influence among the rigid bodies is already handled by the constrained projection (see section 6). Thus, well known rigid body state estimation algorithms such as the Extended Kalman Filter [5] and the Unscented Kalman Filter [6] can be applied. Since these types of state estimators have already been discussed in detail in the literature, they are not explained here and further treated as black box algorithms. The completely unconstrained estimation process can be depicted as shown in figure 1.

6 Constrained Projection

In the previous step, an unconstrained state estimate $\hat{x}^{k+1}$ has been derived. The constrained state estimate $\hat{x}_c^{k+1}$ can be found by projecting the unconstrained estimate onto the constrained surface. Therefore, a constrained weighed least squares optimization problem can be formulated by

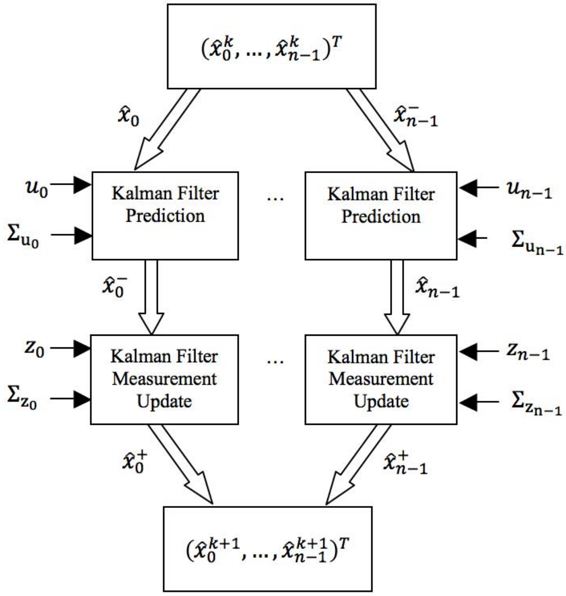


Fig. 1. Unconstrained estimation process

$$\hat{x}_c = \arg\min_x (x - \hat{x}) W (x - \hat{x}) \tag{12}$$

subject to

$$C_p(p, q) - d_p = 0 \tag{13}$$
$$C_m(v, \omega) - d_m = 0. \tag{14}$$

whereby W is a positive definite weight matrix. However, the position and motion constraints are often contradictory and thus both constraints may not be fulfilled concurrently. As a result equation 12 is solved consecutively for each constraint type.

Nonlinear Position Constraint Projection

For the sake of generality, the position constraints are assumed nonlinear. A solution to equation 12 is found by applying the method of Lagrange-multipliers λ which results in

$$\hat{x}_c = arg \min_x (x - \hat{x}) \ W (x - \hat{x}) - \lambda^T (C_p(p, q) - d_p) \tag{15}$$

subject to

$$C_{p(p,q)} - d_p = 0 \tag{16}$$

Consequently, the optimal $\hat{x}_c$ can be determined by solving

$$W\left(x-\hat{x}\right) - J_p^T\left(\hat{x}\right)\lambda = 0 \tag{17}$$

$$C_p\left(x\right) - d_p = 0 \tag{18}$$

with

$$J_p(x) = \frac{\partial C_p\left(p,q\right)}{\partial x} \tag{19}$$

being the Jacobian of $C_p(p,q)$.

The solution to the nonlinear problem is derived by linearizing the nonlinear position constraint around the current operating point and iterating until convergence is achieved. The linearization yields

$$\begin{aligned} C_p\left(x+\Delta x\right) &\approx C_p\left(\hat{x}\right) + J_p\left(\hat{x}\right)\Delta x - d_p \\ \Delta x &= \left(x-\hat{x}\right). \end{aligned} \tag{20}$$

After replacing the nonlinear constraint with its linear counterpart and rearranging equation 17 to Δx, equation

$$\Delta x = W^{-1} J_p^T\left(\hat{x}\right)\ \lambda \tag{21}$$

is derived. Furthermore, replacing Δx in equation 20 leads to

$$J_p\left(\hat{x}\right) W^{-1} J_p^T\left(\hat{x}\right)\lambda = \mathrm{d_p} - \mathrm{C_p}\left(\widehat{\mathrm{x}}\right). \tag{22}$$

Hence, the complexity for solving equation 22 does not depend on the dimension of x but instead it depends on the dimension of λ. As a result, the effort for solving the minimization problem depends solely on the number of constraints and not on the number of rigid bodies involved.

Regarding to [8] the choice of the weight matrix W determines the meaning of the result. On the one hand, choosing

$$W = diag\left(\begin{pmatrix} M_0 & 0 \\ 0 & I_0 \end{pmatrix}, \ldots, \begin{pmatrix} M_{n-1} & 0 \\ 0 & I_{n-1} \end{pmatrix}\right) \tag{23}$$

results in an almost physically correct position change (see [4]). On the other hand,

$$W = \Sigma_x^{-1} \tag{24}$$

favors a maximum likelihood estimate and is therefore chosen. Exploiting the fact that the partial derivatives are

$$\frac{\partial C_p(x)}{\partial (p,q)^T} = J_p(p,q) \neq 0 \tag{25}$$

$$\frac{\partial C_p(x)}{\partial (v,\omega)^T} = J_p(v,\omega) = 0 \tag{26}$$

and that the covariance matrix Σ_x can be subdivided into

$$\Sigma_x = \begin{pmatrix} \Sigma_{(p,q)} & \Sigma_{(p,q),(v,w)} \\ \Sigma_{(v,w),(p,q)} & \Sigma_{(v,w)} \end{pmatrix} \tag{27}$$

with

$$\Sigma_{(a,b),(c,d)} = E\left(\left(\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}\right)\left(\begin{pmatrix} c \\ d \end{pmatrix} - \begin{pmatrix} \hat{c} \\ \hat{d} \end{pmatrix}\right)\right) \tag{28}$$

being the covariance of random vectors $(a,b)^T$ and $(c,d)^T$, equation 22 can be further reduced to

$$J_p(p,q)\ \Sigma_{p,q}\ J_p^T(p,q)\ \lambda = d_p - \mathrm{C_p}(\widehat{\mathrm{x}}). \tag{29}$$

Altogether, the estimate $\hat{x}_c^j$ at the j-th iteration becomes

$$\hat{x}_c^0 = \hat{x} \tag{30}$$

$$\hat{x}_c^j = \hat{x}_c^{j-1} + \begin{pmatrix} \Sigma_{(p,q)} \\ \Sigma_{(p,q),(v,w)} \end{pmatrix} J_p^T(p,q)\ \lambda. \tag{31}$$

Linear Motion Constraint Projection

After enforcing the position constraints, the motion constraints $C_m(v,w)$ can be applied. Fortunately, motion constraints are mostly linear constraints, such that $C_m(v,w)$ becomes

$$\mathrm{J_m}\ (v,w)^T - d_m = 0 \tag{32}$$

which is linear in $(v,w)^T$. Besides, in the case that the motion constraints are also nonlinear one needs to proceed as described in section 6. However, for linear constraints equation 12 simplifies to

$$\Sigma_{(\mathrm{v,w})}^{-1}\begin{pmatrix} v - \hat{v} \\ w - \hat{w} \end{pmatrix} - \mathrm{J_m^T}\ \lambda_{\mathrm{v,w}} = 0 \tag{33}$$

$$J_m \begin{pmatrix} v \\ w \end{pmatrix} - d_m = 0 \tag{34}$$

whereby $\Sigma_{v,w}$ is the motion covariance matrix. Further rearranging equation 33 to $(v, w)^T$ and inserting it into equation 34 yields

$$J_m \Sigma_{v,w} \; J_m^T \; \lambda = d_m - J_m^T \begin{pmatrix} \hat{v} \\ \hat{w} \end{pmatrix} \tag{35}$$

which is similar to equation 22 and can therefore be solved likewise.

7 Evaluation

To proof the concept of the described state estimation of a multi body system the algorithm is exemplarily applied to predict the ego-state of the humanoid robot NAO manufactured by Aldebaran Robotics. This robot model has 21 degrees of freedom and an inertia measurement unit in its chest consisting of a 3-axis accelerometer and a 2-axis gyroscope which can not measure the robot's rotational velocity around the z-axis.

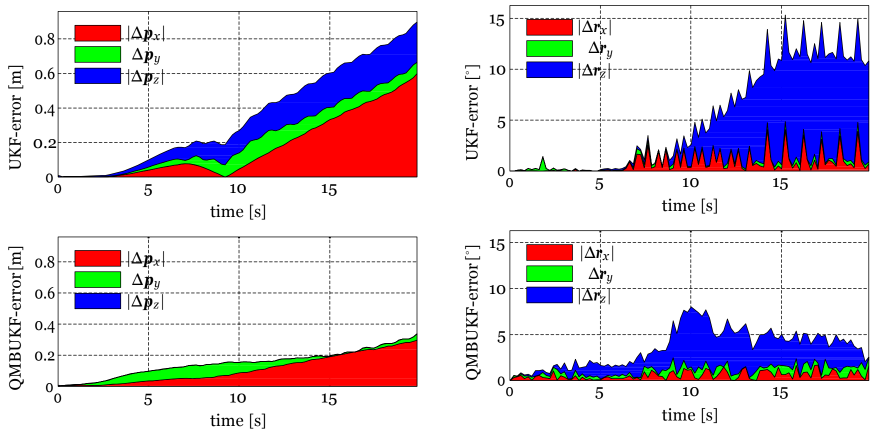
To demonstrate the benefit of the multi body model, the trajectory of the robot's center of mass is predicted using the presented approach and compared to a classic *Unscented-Kalman-Filter* (UKF). Furthermore, the proposed approach will be referred to as *Quaternion MultiBody Unscented-Kalman-Filter* (QMB-UKF) for the remainder of this paper. While the UKF can only take the acceleration sensor and gyroscope informations into account, the QMB-UKF additionally uses the robots structural information and joint angle informations provided by the servo motors as constraints.

Since the purpose of this paper is to provide a proof of concept of the QMB-UKF, only a MATLAB implementation is available so far. Due to the fact that this implementation is non optimal in term of runtime, it can not be run on the Nao's AMD Geode processor in real-time at the moment. But given the fact, that algorithm runs near real-time using common desktop hardware, an application to the real hardware using an optimized implementation in addition to mire efficient constraint solvers.

In turn only simulative studies were carried out and thus reference data for the experiments is generated by the simulator *SimRobot* [7]. While it also uses a time discrete rigid body simulation, the ODE [9], the method applied to ensure position stability, the Baumgarte method [2], varies, resulting in a difference and thereby prediction error between SimRobot and the prediction base of the algorithm. This difference is further increased by the fact that the ODE simulation time step of $h_{SimRobot} = 0.005s$ is chosen to yield results closer to reality. The sensors are modeled to mimic the characteristics of the ones used in the real Nao as close as possible in simulation, emphasizing the noise value to yield non-optimal measurements. A detailed overview on sensor modeling can not be given in this paper.

Experiment 1 - Walking in x-Direction

During the first experiment the robot walks straight ahead in x-direction with a constant walking speed of $v_x = 0.07m/s$ for the duration of 20 seconds. The trajectory of the robots center of mass is predicted by the UKF and with help of the proposed algorithm using the QMB-UKF. The results regarding the position and rotation error of the center of mass can be found in figure 2. The errors are displayed additively to demonstrate their relative influence.



(a) Position error. (b) Rotation error.

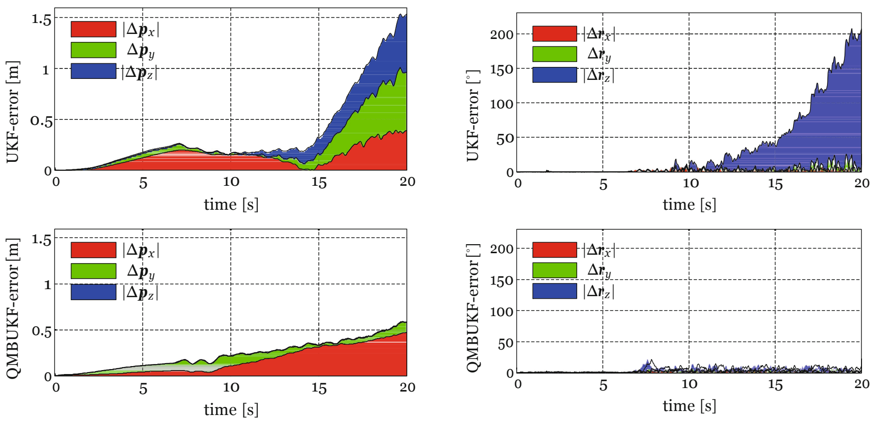
Fig. 2. Experiment 1 - Walking straight in x-direction

The evaluation of figure 2(a) clearly demonstrates the benefit of taking the multi body prediction into account. While the error over time, using the QMB-UKF, still sums up to roundabout 35 cm, this result is obviously superior to the error of over 90 cm using the UKF. The overall rotation error, see figure 2(b), is higher in both cases, with the rotational error around the z-axis having the most influence. Taking into account that the sensors cannot measure the speed or rotation around z-axis of the robot this result is reasonable. Nevertheless evaluating figure 2(b) again demonstrates the benefit of the QMB-UKF.

Experiment 2 - Unforeseen Collision

Since walking with a constant speed represents a favorable use case for a Kalman-filter the second experiment is chosen to evaluate the influence of an unforeseen disturbance. Again the robot walks straight ahead in x-direction with a speed of $v_x = 0.07m/s$, but collides with an unforeseen object which results in sudden change of both horizontal and angular speed. The results of this experiment are demonstrated in figure 3. Again they are displayed additively to show their relative influence.

As expected both filters have problems tracking the state of the robot since they cannot predict the sudden change of acceleration. But again it is clearly visible that the QMB-UKF highly benefits from taking the multi body model into account and thereby reducing the error in comparison to the UKF.



(a) Position error. (b) Rotation error.

(c) Speed error. (d) Angular speed error.

Fig. 3. Experiment 2 - Collision with an unforeseen object

8 Conclusion and Future Works

In this paper, an approach towards articulated rigid body pose and motion estimation has been proposed. As a prove of concept, application of the algorithm to a simulated humanoid robot model, has clearly demonstrated an accuracy benefit over conventional body state prediction methods. Preliminary tests have shown that the accuracy of the state estimation only improves if the constraint violations are small, since the position state estimation performs poorly, if the constraint violations are large. However this can be neglected if the iteration steps are small. Unfortunately, this leads to a high count of calculation iterations, which in turn results in an computational demand, which cannot be easily handled by current mobile devices. However, using more efficient constraint solvers it should be possible to be usable in real-time applications such as online humanoid pose and motion estimation.

To further reduce the computational demand, future work has to focus on the nonlinear constraint projection. Thus, new and more reliable ways have to be explored to solve the constraint equation. Current research is trying to adapt the ideas of impulse based position stabilization [3] often used in physics simulation for the position constraint projection.

Testing the algorithm on a real robot is yet to be done and should include the use of additional sensors evaluate the usefulness of the proposed sensor fusion.

References

1. Aoustin, Y., Plestan, F., Lebastard, V.: Experimental comparison of several posture estimation solutions for biped robot rabbit. In: IEEE International Conference on Robotics and Automation, ICRA 2008, pp. 1270–1275 (May 2008)
2. Baumgarte, J.: Stabilization of constraints and integrals of motion in dynamical systems. In: Computer Methods in Applied Mechanics and Engineering (1972)
3. Bender, J.: Impulse-based dynamic simulation in linear time. Computer Animation and Virtual Worlds 18(4-5), 225–233 (2007)
4. Cline, M.B., Pai, D.K.: Post-stabilization for rigid body simulation with contact and constraints. In: IEEE Intl. Conf. on Robotics and Automation (2003)
5. Halvorsen, K., Söderström, T., Stokes, V., Lanshammar, H.: Using an extended Kalman filter for rigid body pose estimation. Journal of Biomechanical Engineering 127(3), 475–483 (2005), `http://link.aip.org/link/?JBY/127/475/1`
6. Kraft, E.: A quaternion-based unscented Kalman-filter for orientation tracking. Tech. rep. (2003)
7. Laue, T., Spiess, K., Röfer, T.: SimRobot – A General Physical Robot Simulator and Its Application in RoboCup. In: Bredenfeld, A., Jacoff, A., Noda, I., Takahashi, Y. (eds.) RoboCup 2005. LNCS (LNAI), vol. 4020, pp. 173–183. Springer, Heidelberg (2006)
8. Simon, D., Chia, T.L.: Kalman filtering with state equality constraints. IEEE Transactions on Aerospace and Electronic Systems 38(1), 128–136 (2002)
9. Smith, R.: Opensource dynaminc engine (2004), `http://www.ode.org`