Blockcipher-Based Double-Length Hash Functions for Pseudorandom Oracles

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Abstract. PRO (Pseudorandom Oracle) is an important security of hash functions because it ensures that the PRO hash function inherits all properties of a random oracle in single stage games up to the PRO bound (e.g., collision resistant security, preimage resistant security and so on). In this paper, we propose new blockcipher-based double-length hash functions, which are PROs up to $\mathcal{O}(2^n)$ query complexity in the ideal cipher model. Our hash functions use a single blockcipher, which encrypts an n-bit string using a $2n$-bit key, and maps an input of arbitrary length to an n-bit output. Since many blockciphers supports a $2n$-bit key (e.g. AES supports a 256-bit key), the assumption to use the $2n$-bit key length blockcipher is acceptable. To our knowledge, this is the first time double-length hash functions based on a single (practical size) blockcipher with birthday PRO security.

1 Introduction

The blockcipher-based design (e.g. [19,26]) is the most popular method for constructing a cryptographic hash function. A hash function is designed by the following two steps: (1) designing a blockcipher and (2) designing a mode of operation. MD-family [28,29], SHA-family [23] and SHA-3 candidates follow the design method. Another design method is to utilize a practical blockcipher such as AES. Such hash functions are useful in size restricted devices such as RFID tags and smart cards: when implementing both a hash function and a blockcipher, one has only to implement a blockcipher. However, the output length of practical blockciphers is far too short for a collision resistant hash function, e.g., 128 bits for AES. Thus designing a collision resistant double length hash function (CR-DLHF) is an interesting topic. The core of the design of the CR-DLHF is to design a collision resistant double-length compression function (CR-DLCF) which maps an input of fixed length (more than $2n$-bits) to an output of $2n$-bit length when using an n-bit output length blockcipher. Then the hash function combined a domain extension (e.g. strengthened Merkle-Damgård (SMD) [5,21]), which preserves CR security, with the CR-DLCF yields a CR-DLHF. Many DLCFs, e,g,. [2,22,11,14,24,16,18], have been designed and the security is proven in the ideal cipher (IC) model [8,11,17,9,24,15,30].

The indifferentiability framework was introduced by Maurer *et al.* [20], which considers the reducibility of one system to another system. Roughly speaking,

A. Miri and S. Vaudenay (Eds.): SAC 2011, LNCS 7118, pp. 338–355, 2012.

if a system F is indifferentiable from another system G up to q query complexity, in single-stage games (e.g., IND-CCA, EUF-CMA, Collision game, Second Preimage game, Preimage game and many others), any cryptosystem is at least as secure under F as under G up to q query complexity. Recent proposed hash functions, e.g. SHA-3 candidates, considered the security of the indifferentiability from a random oracle (RO) (or Pseudorandom Oracle (PRO)). It ensures that in single stage games the hash function has no structural design flows in composition and has security against any generic attacks up to the PRO query complexity. So it is important to consider PRO security when a DLHF is designed.

Hereafter a blockcipher which encrypts an n-bit string using a k-bit key is denoted by (k,n)-BC. Gong *et al.* [10] proved that the prefix-free Merkle-Damgård using the PBGV compression function [25] is PRO up to $\mathcal{O}(2^{n/2})$ query complexity as long as the ($2n$,n)-BC is IC. The PRO security is not enough because the query complexity is 2^{64} when $n = 128$. Chang *et al.* [3] and Hirose *et al.* [12] proposed $2n$-bit output length DLHFs using a compression function $h : \{0,1\}^d \rightarrow \{0,1\}^n$ where $d > 2n$. Their proposals are PROs up to $\mathcal{O}(2^n)$ query complexity as long as h is a fixed input length RO (FILRO). Since IC where the plain text element is fixed by a constant is FILRO, these hash functions can be modified to blockcipher-based schemes which use a (d,n)-BC. However, practical blockciphers (such as AES) don't support d-bit key where $d > 2n$. Many other practical size[1] blockcipher-based DLHFs were proposed, e.g., [2,22,11,14,24,16,18], while none of them achieves PRO security.[2] There is no hash function with birthday PRO security, and thus, we rise the following question:

Can We Construct a DLHF from a "single practical size blockcipher" with "birthday PRO security"?

In this paper, we propose DLHFs using a single ($2n$,n)-BC, which are PROs up to $\mathcal{O}(2^n)$ query complexity in the IC model. Since many blockciphers support $2n$-bit key length, e.g., AES supports 256-bit key length, and the existing DLCFs (e.g., Hirose's compression function [11], Tandem-DM [14], Abreast-DM [14], and generalized DLCF [24]) use a ($2n$,n)-BC, the assumption to use a ($2n$,n)-BC is acceptable. To our knowledge, our hash functions are the first time DLHFs based on a practical size blockcipher with birthday PRO security.[3] When $n = 128$ which is supported by AES, our hash functions have 2^{128} security. Since our hash functions use only a *single* blockcipher, it is useful on size restricted devices when implementing both a hash function and a blockcipher. (the hybrid encryption

[1] "Practical size" is the size supported by practical blockciphers such as AES.

[2] Since PRO security is stronger security than CR security, CR security does not guarantee PRO security.

[3] Our hash functions don't satisfy the stronger notion called reset indifferentiability from RO, which ensure security in the multi-stage games [27]. Note that there is no hash function satisfying the notion. Thus to propose the hash function is an open problem.

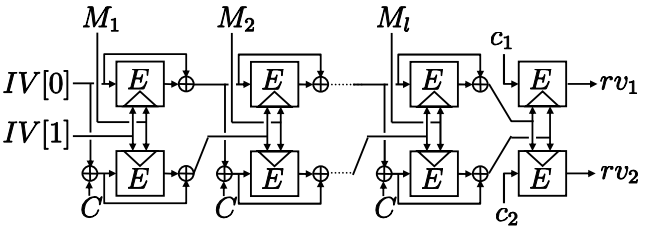

Fig. 1. Our DLHF using Hirose's compression function

schemes use both a blockcipher and a hash function used in a key derivation function, for example.)

Our DLHF. Our DLHFs, which use each of Hirose's compression function, Tandem-DM and Abreast-DM, iterate the compression function and use a new post-processing function f at the last iteration which calls a $(2n, n)$-BC twice. Our DLHFs are slightly lesser for speed than existing CR-DLHFs but have higher security (birthday PRO security).

Let $\mathsf{BC}_{2n,n} = (E, D)$ be a $(2n,n)$-BC where E is an encryption function and D is a decryption function. Let $\mathrm{DLCF}^{\mathsf{BC}_{2n,n}}$ be a DLCF: Hirose's compression function, Tandem-DM, or Abreast-DM. Let $\mathrm{SMD}^{\mathrm{DLCF}^{\mathsf{BC}_{2n,n}}} : \{0,1\}^* \to \{0,1\}^{2n}$ be the SMD hash function using the compression function $\mathrm{DLCF}^{\mathsf{BC}_{2n,n}}$. Our DLHF is defined as follows:

$$F^{\mathsf{BC}_{2n,n}}(M) = f^{\mathsf{BC}_{2n,n}}(\mathrm{SMD}^{\mathrm{DLCF}^{\mathsf{BC}_{2n,n}}}(M))$$

where $f^{\mathsf{BC}_{2n,n}}(x) = E(x, c_1)||E(x, c_2)$ and c_1 and c_2 are n-bit constant values. Note that the first element of the encryption function is the key element and the second element is the plain text element. The DLHF using Hirose's compression function is illustrated in Fig. 1 where each line is n bits and $IV[0], IV[1], C, c_1$ and c_2 are constant values. Note that in this figure we omit the suffix free padding function sfpad. So the hash function takes as its input a message M, $\mathsf{sfpad}(M) = M_1||M_2||\cdots||M_l$ with each block of n bits, and outputs the final value $rv_1||rv_2$. We use the DLHF $\mathrm{SMD}^{\mathrm{DLCF}^{\mathsf{BC}_{2n,n}}}$ to compress an arbitrary length input into an fixed input length value. Since SMD hash functions cannot be used as ROs [4], the post-processing function $f^{\mathsf{BC}_{2n,n}}$ is used to guarantee PRO security.

The use of the constant values c_1 and c_2 in the post-processing function is inspired by the design technique of EMD proposed by Bellare and Ristenpart [1]. This realizes the fact that we can treat our hash function as a NMAC-like hash function. Note that the security of EMD is proven when the compression function is FILRO, while the security of our hash functions is proven when the compression function is the DLCF in the IC model. So additional analyses are needed due to the invertible property of IC and the structures of DLCFs.We thus prove the PRO security of $F^{\mathsf{BC}_{2n,n}}$ by using three techniques: the PrA (Preimage Aware) design framework of Dodis *et al.* [6], PRO for a small function [4], and

indifferentiability from a hash function. The first two techniques are existing techniques and the last technique is a new application of the indifferentiability framework [20].

First, we prove that the DLCFs are PrA up to $\mathcal{O}(2^n)$ query complexity. The PrA design framework offers the hash functions which are PROs up to $\mathcal{O}(2^n)$ query complexity where FILRO is used as the post-processing function. Second, we convert FILRO into the blockcipher-based post-processing function. We prove that the post-processing function is PRO up to $\mathcal{O}(2^n)$ query complexity in the IC model (PRO for a small function). Then, we prove that the PRO security of the post-processing function and the first PRO result ensure that the converted hash functions are PROs up to $\mathcal{O}(2^n)$ query complexity. We note that the hash functions use two blockciphers.[4] Finally, we consider the single-blockcipher-based hash functions $F^{\mathsf{BC}_{2n,n}}$. We prove that the single blockcipher-based hash functions are indifferentiable from the two-blockciphers-based hash functions in the IC model up to $\mathcal{O}(2^n)$ query complexity (indifferentiability from a hash function). Then we show that the indifferentiable security result and the second PRO result ensure that our hash functions are PROs up to $\mathcal{O}(2^n)$ query complexity in the IC model.

2 Preliminaries

Notation. For two values x, y, $x||y$ is the concatenated value of x and y. For some value y, $x \leftarrow y$ means assigning y to x. $\oplus$ is bitwise exclusive or. $|x|$ is the bit length of x. For a set (list) $\mathcal{T}$ and an element W, $\mathcal{T} \leftarrow W$ means to insert W into $\mathcal{T}$ and $\mathcal{T} \xleftarrow{\cup} W$ means $\mathcal{T} \leftarrow \mathcal{T} \cup \{W\}$. For some $2n$-bit value x, $x[0]$ is the first n bit value and $x[1]$ is the last n-bit value. $\mathsf{BC}_{d,n} = (E, D)$ be a blockcipher where $E : \{0,1\}^d \times \{0,1\}^n \rightarrow \{0,1\}^n$ is an encryption function, $D : \{0,1\}^d \times \{0,1\}^n \rightarrow \{0,1\}^n$ is a decryption function, the key size is d bits and the cipher text size is n bits. $\mathcal{C}_{d,n} = (E_I, D_I)$ be a ideal cipher (IC) where $E_I : \{0,1\}^d \times \{0,1\}^n \rightarrow \{0,1\}^n$ is an encryption oracle, $D_I : \{0,1\}^d \times \{0,1\}^n \rightarrow \{0,1\}^n$ is a decryption oracle, the key size is d bits and the cipher text size is n bits. $\mathcal{F}_{a,b} : \{0,1\}^a \rightarrow \{0,1\}^b$ is a random oracle (RO). An arbitrary input length random oracle is denoted by $\mathcal{F}_b : \{0,1\}^* \rightarrow \{0,1\}^b$. For any algorithm A, we write $\mathsf{Time}(A)$ to mean the sum of its description length and the worst-case number of steps.

Merkle-Damgård [5,21]. Let $h : \{0,1\}^{2n} \times \{0,1\}^d \rightarrow \{0,1\}^{2n}$ be a compression function using a primitive P (more strictly h^P) and $\mathsf{pad} : \{0,1\}^* \rightarrow (\{0,1\}^d)^*$ be a padding function. The Merkle-Damgård hash function MD^h is described as follows where IV is a $2n$-bit initial value.

[4] Two independent ideal cipher can be obtained from a single ideal cipher by victimizing one bit of the key space. So using a blockcipher with the $2n + 1$-bit key space and the n-bit key space, the hash functions which uses a single blockcipher can be realized. But the size of the blockcipher is not a practical size.

$\underline{\mathrm{MD}^h(M)}$
$z \leftarrow IV$;
Break $\mathsf{pad}(M)$ into d-bit blocks, $\mathsf{pad}(N) = M_1 || \cdots || M_l$;
for $i = 1, \ldots, l$ do $z \leftarrow h(z, M_i)$;
Ret z;

We denote MD^h, when padding pad is a suffix-free padding sfpad, by SMD^h, called strengthened Merkle-Damgård. We assume that it is easy to strip padding, namely that there exists an efficiently computable function $\mathsf{unpad} : (\{0,1\}^d)^* \rightarrow \{0,1\}^* \cup \{\perp\}$ such that $x = \mathsf{unpad}(\mathsf{pad}(x))$ for all $x \in \{0,1\}^*$. Inputs to unpad that are not valid outputs of pad are mapped to $\perp$ by unpad.

Pseudorandom Oracle [20]. Let $H^P : \{0,1\}^* \rightarrow \{0,1\}^n$ be a hash function that utilizes an ideal primitive P. We say that H^P is PRO if there exists an efficient simulator S that simulates P such that for any distinguisher A outputting a bit it is the case that

$$\mathsf{Adv}^{\mathsf{pro}}_{H^P,S}(A) = |\Pr[A^{H^P,P} \Rightarrow 1] - \Pr[A^{\mathcal{F}_n,S^{\mathcal{F}_n}} \Rightarrow 1]|$$

is small where the probabilities are taken over the coins used the experiments. S can make queries to $\mathcal{F}_n$. The S's task is to simulate P such that relations among responses of (H^P, P) hold in responses of $(\mathcal{F}_n, S)$ as well.

Preimage Awareness [6,7]. The notion of preimage awareness is useful for PRO security proofs of NMAC hash functions. We only explain the definition of preimage awareness. Please see Section 3 of [7] for the spirit of the notion. Let F^P be a hash function using an ideal primitive P. The preimage awareness of F^P is estimated by the following experiment.

$\underline{\mathsf{Exp}^{\mathsf{pra}}_{F^P,P,\mathcal{E},A}}$
$x \xleftarrow{\$} A^{\mathsf{P},\mathsf{Ex}}$;
$z \leftarrow F^P(x)$;
Ret $(x \neq \mathsf{V}[z] \wedge \mathsf{Q}[z] = 1)$;

$\underline{\textbf{oracle } \mathsf{P}(m)}$
$c \leftarrow P(m)$;
$\alpha \xleftarrow{\cup} (m, c)$;
Ret c;

$\underline{\textbf{oracle } \mathsf{Ex}(z)}$
$\mathsf{Q}[z] \leftarrow 1$;
$\mathsf{V}[z] \leftarrow \mathcal{E}(z, \alpha)$;
Ret $\mathsf{V}[z]$;

Here an adversary A is provided two oracles P and Ex. The oracle P provides access to the ideal primitive P and records a query history α. The extraction oracle Ex provides an interface to an extractor $\mathcal{E}$, which is a deterministic algorithm that uses z and the query history α of P, and returns either $\perp$ or an element x' such that $F^P(x') = z$. If x' can be constructed from α, it returns x' and otherwise returns $\perp$. In this experiment, the (initially everywhere $\perp$) array Q and the (initially empty) array V are used. When z is queried to Ex, $\mathsf{Q}[z] \leftarrow 1$ and then the output of $\mathcal{E}(z, \alpha)$ is assigned to $\mathsf{V}[z]$. For the hash function F^P, the adversary A, and the extractor $\mathcal{E}$, we define the advantage relation

$$\mathsf{Adv}^{\mathsf{pra}}_{F^P,P,\mathcal{E}} = \Pr[\mathsf{Exp}^{\mathsf{pra}}_{F^P,P,\mathcal{E},A} \Rightarrow \mathsf{true}]$$

where the probabilities are over the coins used in running the experiments. When there exists an efficient extractor $\mathcal{E}$ such that for any adversary A the above advantage is small, we say that F^P is preimage aware (PrA).

The pra-advantage can be evaluated from the cr-advantage (collision resistance advantage) and the 1-wpra (1-weak PrA) advantage [7]. The 1-WPrA experiment is described as follows.

$\underline{\mathsf{Exp}^{\mathsf{1wpra}}_{F^P,P,\mathcal{E}^+,A}}$
$x \xleftarrow{\$} A^{\mathsf{P},\mathsf{Ex}^+}$;
$z \leftarrow F^P(x)$;
Ret $(x \notin L \wedge \mathsf{Q}[z] = 1)$;

$\underline{\textbf{oracle } \mathsf{P}(m)}$
$c \leftarrow P(m)$;
$\alpha \xleftarrow{\cup} (m, c)$;
Ret c;

$\underline{\textbf{oracle } \mathsf{Ex}^+(z)}$
$\mathsf{Q}[z] \leftarrow 1$;
$L \leftarrow \mathcal{E}^+(z, \alpha)$;
Ret L;

The difference between the 1-WPrA experiment and the PrA experiment is the extraction oracle. In the 1-WPrA experiment, a multi-point extractor oracle Ex^+ is used. Ex^+ provides an interface to a multi-point extractor $\mathcal{E}^+$, which is a deterministic algorithm that uses z and α, and returns either $\perp$ or a set of an element in the domain of F^P. The output (set) of $\mathcal{E}^+$ is stored in list L. Thus, if $L \neq \{\perp\}$, for any $x' \in L$ $F^P(x') = z$. In this experiment, an adversary A can make only a single query to Ex^+. For a hash function F^P, an adversary A, and a multi-point extractor $\mathcal{E}^+$, we define the advantage relation

$$\mathsf{Adv}^{\mathsf{1wpra}}_{F^P,P,\mathcal{E}} = \Pr[\mathsf{Exp}^{\mathsf{1wpra}}_{F^P,P,\mathcal{E}^+,A} \Rightarrow \mathsf{true}]$$

where the probabilities are over the coins used in running the experiments. When there exists an efficient multi-point extractor $\mathcal{E}^+$ such that the above advantage is small for any adversary A, we say that F^P is 1-WPrA.

The definition of the cr-advantage as follows. Let A be an adversary that outputs a pair of values x and x'. To hash function F^P using primitive P and adversary A we associate the advantage relation

$$\mathsf{Adv}^{\mathsf{cr}}_{F^P,P}(A) = \Pr[(x, x') \xleftarrow{\$} A^P : F^P(x) = F^P(x') \wedge x \neq x']$$

where the probability is over the coins used by A and primitive P.

Then the pra-advantage can be evaluated as follows.

Lemma 1 (Lemmas 3.3 and 3.4 of [7]). *Let $\mathcal{E}^+$ be an arbitrary multi-point extractor. There exists an extractor $\mathcal{E}$ such that for any pra-adversary A^{pra} making q_e extraction queries and q_P primitive queries there exists 1-wpra adversary A^{1wpra} and cr-adversary A^{cr} such that*

$$\mathsf{Adv}^{\mathsf{pra}}_{F^P,P,\mathcal{E}}(A^{\mathsf{pra}}) \leq q_e \cdot \mathsf{Adv}^{\mathsf{1wpra}}_{F^P,P,\mathcal{E}^+}(A^{\mathsf{1wpra}}) + \mathsf{Adv}^{\mathsf{cr}}_{F^P,P}(A^{\mathsf{cr}}).$$

A^{1wpra} runs in time at most $\mathcal{O}(q_e\mathsf{Time}(\mathcal{E}^+))$ and makes the same number of P queries as A^{pra}. A^{cr} asks q_P queries and run in time $\mathcal{O}(q_e \cdot \mathsf{Time}(\mathcal{E}^+))$. $\mathcal{E}$ runs in the same time as $\mathcal{E}^+$. ♦

NMAC Hash Function. Let $g : \{0,1\}^n \to \{0,1\}^n$ be a function and $H^P : \{0,1\}^* \to \{0,1\}^n$ be a hash function using primitive P such that g is not used in H^P. Dodis *et al.* [7] proved that the PRO security of the NMAC hash function $g \circ H^P$ can be reduced into the PrA security of H^P.

Lemma 2 (Theorem 4.1 of [7]). *Let P be an ideal primitive, g be a random oracle and $\mathcal{E}$ be any extractor for H^P. Then there exists a simulator $S = (S_P, S_g)$ such that for any PRO adversary A making at most q_F, q_P, q_g queries to its three oracles $(\mathcal{O}_F, \mathcal{O}_P, \mathcal{O}_g)$ where $(\mathcal{O}_F, \mathcal{O}_P, \mathcal{O}_g) = (g \circ H^P, P, g)$ or $(\mathcal{O}_F, \mathcal{O}_P, \mathcal{O}_g) = (\mathcal{F}_n, S_P, S_g)$, there exists a PrA adversary B such that*

$$\mathsf{Adv}^{\mathsf{pro}}_{g \circ H^P, S}(A) \leq \mathsf{Adv}^{\mathsf{pra}}_{H^P, P, \mathcal{E}}(B).$$

S runs in time $\mathcal{O}(q_P + q_g \cdot \mathsf{Time}(\mathcal{E}))$. Let l be the length of the longest query made by A to $\mathcal{O}_H$. B runs in time $\mathcal{O}(\mathsf{Time}(A) + q_F t_H + q_P + q_g)$, makes $q_P + q_H q_F$ queries, q_g extraction queries, and outputs a preimage of length at most l where for any input M to H^P the output of $H^P(M)$ can be calculated within at most t_H times and q_H queries to P. ♦

Dodis *et al.* proved that the SMD construction preserves the PrA security as follows. Therefore, the PRO security of the NMAC hash function using the SMD hash function can be reduced into the PrA security of the compression function.

Lemma 3 (Theorem 4.2 of [7]). *Let h^P be a compression function using an ideal primitive P. Let $\mathcal{E}_h$ be an arbitrary extractor for h^P. There exists an extractor $\mathcal{E}_H$ for SMD^{h^P} such that for any adversary A_H making at most q_P primitive queries and q_e extraction queries and outputting a message at most l blocks there exists an adversary A_h such that*

$$\mathsf{Adv}^{\mathsf{pra}}_{\mathrm{SMD}^{h^P}, P, \mathcal{E}_H}(A_H) \leq \mathsf{Adv}^{\mathsf{pra}}_{h^P, P, \mathcal{E}_h}(A_h)$$

$\mathcal{E}_H$ runs in time at most $l(\mathsf{Time}(\mathcal{E}_h) + \mathsf{Time}(\mathsf{unpad}))$. A_h runs in time at most $\mathcal{O}(\mathsf{Time}(A_H) + q_e l)$, makes at most $q_H + q_P$ ideal primitive queries, and makes at most $q_e l$ extraction queries where q_H is the maximum number of P queries to calculate SMD^{h^P}. ♦

3 Blockcipher-Based Double-Length Hash Functions for PROs

Let $\mathsf{BC}_{2n,n} = (E, D)$, $\mathsf{BC}^1_{2n,n} = (E1, D1)$, $\mathsf{BC}^2_{2n,n} = (E2, D2)$, and $\mathsf{BC}^3_{2n,n} = (E3, D3)$ be blockciphers. Let $g : \{0,1\}^{2n} \to \{0,1\}^{2n}$ be a function. In this section, we propose the following DLHFs using a single blockcipher and prove that our hash functions are PROs up to $\mathcal{O}(2^n)$ query complexity in the IC model.

$$F^{\mathsf{BC}_{2n,n}}(M) = f^{\mathsf{BC}_{2n,n}}(\mathrm{SMD}^{\mathrm{DLCF}^{\mathsf{BC}_{2n,n}}}(M))$$

where $f^{\mathsf{BC}_{2n,n}}(x) = E(x,c_1)||E(x,c_2)$ such that c_1 and c_2 are n-bit different constant values and are different from values which are defined by the compression function (see subsection 3.3). The hash functions use Hirose's compression function, Tandem-DM, and Abreast-DM as the underlying DLCF, respectively. We prove the PRO security by the three steps. Each step uses the PrA design framework, PRO for a small function and indifferentiability from a hash function, respectively.

- **Step 1.** We prove that Hirose's compression function, Tandem-DM, and Abreast-DM are PrA up to $\mathcal{O}(2^n)$ query complexity in the IC model. Lemma 2 and Lemma 3 then ensure that the following NMAC hash function is PRO up to $\mathcal{O}(2^n)$ query complexity as long as the blockcipher is IC and g is FILRO.

$$F_1^{g,\mathsf{BC}^1_{2n,n}}(M) = g(\mathrm{SMD}^{\mathrm{DLCF}^{\mathsf{BC}^1_{2n,n}}}(M))$$

- **Step 2.** We prove that $f^{\mathsf{BC}^3_{2n,n}}$ is PRO up to $\mathcal{O}(2^n)$ query complexity in the IC model where c_1 and c_2 are n-bit different values. Then, we prove that the PRO security of F_1 and the PRO security of f ensure that the following hash function is PRO up to $\mathcal{O}(2^n)$ query complexity in the IC model.

$$F_2^{\mathsf{BC}^2_{2n,n},\mathsf{BC}^3_{2n,n}}(M) = f^{\mathsf{BC}^3_{2n,n}}(\mathrm{SMD}^{\mathrm{DLCF}^{\mathsf{BC}^2_{2n,n}}}(M))$$

- **Step 3.** This is the final step. We use the indifferentiability from a hash function: we prove that $F^{\mathsf{BC}_{2n,n}}$ is indifferentiable from $F_2^{\mathsf{BC}^2_{2n,n},\mathsf{BC}^3_{2n,n}}$ up to $\mathcal{O}(2^n)$ query complexity in the IC model. Then, we prove that the indifferentiable result and the PRO security of F_2 ensure that $F^{\mathsf{BC}_{2n,n}}$ is PRO up to $\mathcal{O}(2^n)$ query complexity in the IC model.

3.1 Step 1

We prove that Hirose's compression function [11] is PrA up to $\mathcal{O}(2^n)$ query complexity as long as the blockcipher is an ideal cipher. Similarly, we can prove that Abreast-DM and Tandem-DM [14] are PrA. We prove the PrA security in the full version.

Definition 1 (Hirose's Compression Function). *Let* $\mathsf{BC}^1_{2n,n} = (E1, D1)$ *be a blockcipher. Let* $\mathsf{CF}^{\mathsf{Hirose}}[\mathsf{BC}^1_{2n,n}] : \{0,1\}^{2n} \times \{0,1\}^n \to \{0,1\}^{2n}$ *be a compression function such that* $(G_i, H_i) = \mathsf{CF}^{\mathsf{Hirose}}[\mathsf{BC}^1_{2n,n}](G_{i-1}||H_{i-1}, M_i)$ *where* $G_i, H_i, G_{i-1}, H_{i-1} \in \{0,1\}^n$ *and* $M_i \in \{0,1\}^n$. (G_i, H_i) *is calculated as follows:*

$$G_i = G_{i-1} \oplus E1(H_{i-1}||M_i, G_{i-1}) \quad (1)$$

$$H_i = C \oplus G_{i-1} \oplus E1(H_{i-1}||M_i, G_{i-1} \oplus C,) \quad (2)$$

We call the procedure 1 "first block" and the procedure 2 "second block". ♦

Lemma 4 (Hirose's Compression Function is PrA). *Let* $\mathcal{C}^1_{2n,n} = (E1_I, D1_I)$ *be an ideal cipher. There exists an extractor* $\mathcal{E}$ *such that for any adversary* A *making at most* q_P *queries to* $\mathcal{C}_{2n,n}$ *and* q_e *extraction queries we have*

$$\mathsf{Adv}^{\mathsf{pra}}_{\mathsf{CF}^{\mathsf{Hirose}}[\mathcal{C}^1_{2n,n}],\mathcal{C}^1_{2n,n},\mathcal{E}}(A) \leq \frac{2q_P^2}{(2^n - 2q_P)^2} + \frac{2q_P}{2^n - 2q_P} + \frac{2q_P q_e}{(2^n - q_P)^2}$$

where $\mathcal{E}$ *runs in time at most* $\mathcal{O}(q_e q_P)$. ♦

Proof. We prove that Hirose's compression function is 1-WPrA, and then Lemma 1 gives the final bound. We note that Theorem 3 of [9] upperbounds the cr-advantage of A by $2q_P^2/(2^n - 2q_P)^2 + 2q_P/(2^n - 2q_P)$, yielding the first two terms.

Intuitively, the 1-WPrA game for the compression function is that A declares a value z then an extractor outputs preimages, stored in L, of z which can be constructed from input-output values of A's queries to $\mathcal{C}^1_{2n,n}$. Then A outputs a new preimage of z which is not stored in L. Note that A can adaptively query to $\mathcal{C}^1_{2n,n}$. We define the multi-point extractor to utilize the preimage resistant bound, proven in [9], of Hirose's compression function as follows.

algorithm $\mathcal{E}^+(z, \alpha)$
Let L be an empty list;
Parse $(k_1, x_1, y_1), \ldots, (k_i, x_i, y_i) \leftarrow \alpha$; $//E1(k_j, x_j) = y_j$
For $j = 1$ to i do
  If $z[0] = x_j \oplus y_j$ then
    $y \leftarrow E1_I(k_j, x_j \oplus C)$;
    If $z[1] = C \oplus x_j \oplus y$ then $L \xleftarrow{\cup} (x_j || k[0], k[1])$;
  If $z[1] = x_j \oplus y_j$ then
    $y \leftarrow E1_I(k_j, x_j \oplus C)$;
    If $z[0] = C \oplus x_j \oplus y$ then $L \xleftarrow{\cup} ((x_j \oplus C) || k[0], k[1])$;
If L is not an empty list then return L otherwise return $\perp$;

If an input-output triple of the first block is defined, automatically the input of the second block is defined, and vice versa, from the definition of the compression function. For a query (z, α) to $\mathcal{E}^+$, when there is an input-output triple (k, x, y) such that $x \oplus y = z[0]$, $\mathcal{E}^+$ checks whether the output of the second block is equal to $z[1]$ or not and if this holds the multi-point extractor stores it in the return list L, and vice versa. Therefore, A must find a new preimage of z to win the 1-WPrA experiment. Thus one can straightforwardly adapt the preimage resistant advantage of the compression function (described in Theorem 5 of [9])[5] because the proof of Theorem 5 of [9] can be applied to the case that an adversary selects an image z of the compression function and then finds the preimage of z. The advantage is at most $2q_P/(2^n - q_P)^2$. □

[5] Note that while the 1-WPrA bound is equal to the preimage bound, this is not trivial because one needs to construct the extractor that converts the preimage bound into the 1-WPrA bound.

Lemma 4 ensures the following theorem via Lemma 2 and Lemma 3 where F_1 is PRO up to $\mathcal{O}(2^n)$ query complexity.

Theorem 1. *There exists a simulator $S_1 = (S1_g, S1_{\mathcal{C}})$ where $S1_{\mathcal{C}} = (S1_E, S1_D)$ such that for any distinguisher A_1 making at most (q_H, q_g, q_E, q_D) queries to four oracles which are $(F_1, g, E1, D1)$ or $(\mathcal{F}_{2n}, S1_g, S1_E, S1_D)$, we have*

$$\mathsf{Adv}^{\mathsf{pro}}_{F_1^{g,\mathcal{C}^1_{2n,n}},S1}(A_1) \leq \frac{2Q_1^2}{(2^n - 2Q_1)^2} + \frac{2Q_1}{2^n - 2Q_1} + \frac{2lq_gQ_1}{(2^n - Q_1)^2}$$

where S_1 works in time $\mathcal{O}(q_E + q_D + lq_gQ_1) + lq_g \times \mathsf{Time}(\mathsf{unpad})$ and $S1_g$ makes q_g queries to $\mathcal{F}_{2n}$ where l is the maximum number of n-bit blocks of a query to $F_1/\mathcal{F}_{2n}$, $Q_1 = 2l(q_H + 1) + q_E + q_D$. $S1_g$ simulates g, which makes one query to $\mathcal{F}_{2n}$ for one $S1_g$ query, and $S1_{\mathcal{C}}$, which makes no query, simulates the ideal cipher. ♦

3.2 Step 2

Lemma 5 ($f^{\mathcal{C}^3_{2n,n}}$ is PRO). *Let $\mathcal{C}^3_{2n,n} = (E3_I, D3_I)$ be an ideal cipher. Let $g = \mathcal{F}_{2n,2n}$. There exists a simulator $\mathsf{S} = (\mathsf{S}_E, \mathsf{S}_D)$ such that for any distinguisher A_2 making at most q_f, q_E and q_D queries to oracles $(\mathcal{O}_f, \mathcal{O}_E, \mathcal{O}_D)$ where $(\mathcal{O}_f, \mathcal{O}_E, \mathcal{O}_D) = (f^{\mathcal{C}^3_{2n,n}}, E3_I, D3_I)$ or $(\mathcal{O}_f, \mathcal{O}_E, \mathcal{O}_D) = (g, \mathsf{S}_E, \mathsf{S}_D)$, we have*

$$\mathsf{Adv}^{\mathsf{pro}}_{f^{\mathcal{C}^3_{2n,n}},\mathsf{S}}(A_2) \leq \frac{q_f + q_E + q_D}{2^n}$$

where S works in time $\mathcal{O}(q_E + q_D)$ and makes at most queries $q_E + q_D$. S simulates the ideal cipher. ♦

We explain the intuition of the result of Lemma 5. The proof is given the full version. An ideal cipher where the plain text is fixed by a constant value is RO. So the first half value y_1 of an output of f is randomly chosen from $\{0,1\}^n$ and the last half value is chosen from $\{0,1\}^n \backslash \{y_1\}$, while an output of RO is randomly chosen from $\{0,1\}^{2n}$. The statistical distance appears in the PRO bound.

Theorem 1 and Lemma 5 ensure the following theorem where F_2 using Hirose's compression function is PRO up to $\mathcal{O}(2^n)$ query complexity in the IC model. We prove the theorem in the full version. Similarly, we can prove the PRO security of the hash functions using Tandem-DM and Abreast-DM, respectively.

Theorem 2 (F_2 is PRO). *There exists a simulator $S_2 = (S2, S3)$ where $S2 = (S2_E, S2_D)$ and $S3 = (S3_E, S3_D)$ such that for any distinguisher A_3 making at most $(q_H, q_{E2}, q_{D2}, q_{E3}, q_{D3})$ queries to five oracles which are $(F_2, E2, D2, E3, D3)$ or $(\mathcal{F}_{2n}, S2_E, S2_D, S3_E, S3_D)$, we have*

$$\mathsf{Adv}^{\mathsf{pro}}_{F_2^{\mathcal{C}^2_{2n,n},\mathcal{C}^3_{2n,n}},S_2}(A_3) \leq \frac{2Q_2^2}{(2^n - 2Q_2)^2} + \frac{2Q_2}{2^n - 2Q_2} + \frac{2lq_3Q_2}{(2^n - Q_2)^2} + \frac{q_H + q_3}{2^n}$$

where S_2 works in time $\mathcal{O}(q_2 + lq_3Q_2) + lq_3 \times \mathsf{Time}(\mathsf{unpad})$ and $S3$ makes q_3 queries to $\mathcal{F}_{2n}$ where l is the maximum number of n-bit blocks of a query to $F_2/\mathcal{F}_{2n}$, $Q_2 = 2l(q_H + 1) + q_{E2} + q_{D2}$, $q_2 = q_{E2} + q_{D2}$ and $q_3 = q_{E3} + q_{D3}$. ♦

3.3 Step 3

In this section, we consider the hash function using Hirose's compression function. The same discussion can be applied to the hash functions using Tandem-DM and Abreast-DM, respectively. The discussions are given in the full version.

When using Hirose's compression function, we use the constant values c_1 and c_2 of the post-processing function f such that c_1 and c_2 are not equal to $C \oplus IV[0]$ and $IV[0]$ where IV is the initial value of $\mathrm{SMD}^{\mathrm{DLCF}^{\mathrm{BC}_{2n,n}}}$ and C is the constant value used in Hirose's compression function. If c_1 and c_2 which are equal to $C \oplus IV[0]$ or $IV[0]$ are used, we cannot prove the security of the hash function. In this case, we fail to construct a simulator.

First, we define the indifferentiability from a hash function as follows.

Definition 2. *Let $H_1^{P_1} : \{0,1\}^* \to \{0,1\}^{2n}$ and $H_2^{P_2} : \{0,1\}^* \to \{0,1\}^{2n}$ be hash functions using ideal primitives P_1 and P_2, respectively. $H_1^{P_1}$ is indifferentiable from $H_2^{P_2}$ if there exists a simulator $\mathcal{S}$ such that for any distinguisher A_4 outputting a bit it is the case that*

$$\mathsf{Adv}^{\mathsf{indif}}_{H_1^{P_1},H_2^{P_2},\mathcal{S}}(A_4) \leq |\Pr[A_4^{H_1^{P_1},P_1} \Rightarrow 1] - \Pr[A_4^{H_2^{P_2},\mathcal{S}^{P_2}} \Rightarrow 1]|$$

is small where the probabilities are taken over the coins used the experiments. ♦

The following lemma is that F is indifferentiable from F_2 up to $\mathcal{O}(2^n)$ query complexity in the IC model.

Lemma 6. *Let $\mathcal{C}_{2n,n} = (E_I, D_I)$ be an ideal cipher. Let $\mathcal{C}^2_{2n,n} = (E2_I, D2_I)$ and $\mathcal{C}^3_{2n,n} = (E3_I, D3_I)$ be different ideal ciphers. There exists a simulator $\mathcal{S} = (\mathcal{S}_E, \mathcal{S}_D)$ such that for any distinguisher A_4 making at most q_F, q_E and q_D queries to its oracles $(\mathcal{O}_F, \mathcal{O}_E, \mathcal{O}_D)$ which are $(F^{\mathcal{C}_{2n,n}}, E_I, D_I)$ or $(F_2^{\mathcal{C}^2_{2n,n},\mathcal{C}^3_{2n,n}}, \mathcal{S}_E, \mathcal{S}_D)$, we have*

$$\mathsf{Adv}^{\mathsf{indif}}_{F^{\mathcal{C}_{2n,n}},F_2^{\mathcal{C}^2_{2n,n},\mathcal{C}^3_{2n,n}},\mathcal{S}}(A_4) \leq \frac{14 \times (2(lq_F + 1) + q_E + q_D)}{2^n - (2(lq_F + 1) + q_E + q_D)}$$

where $\mathcal{S}$ works in time $\mathcal{O}(3(q_E + q_D))$ and makes at most ideal cipher queries $q_E + q_D$. l is the maximum number of n-bit blocks of a query to $\mathcal{O}_F$. ♦

Proof. Without loss of generality, we omit the padding function of our hash function which is more general case than including the padding function. In Fig. 2, we define a simulator $\mathcal{S} = (\mathcal{S}_E, \mathcal{S}_D)$ such that it simulates the ideal cipher $\mathcal{C}_{2n,n} = (E_I, D_I)$ and the relation among responses of $(F^{\mathcal{C}_{2n,n}}, E_I, D_I)$ holds in responses of $(F_2^{\mathcal{C}^2_{2n,n},\mathcal{C}^3_{2n,n}}, \mathcal{S}_E, \mathcal{S}_D)$ as well, namely, $F^{\mathcal{S}}(M) = F_2^{\mathcal{C}^2_{2n,n},\mathcal{C}^3_{2n,n}}(M)$.

simulator $\mathcal{S}_E(k, x)$
E01 If $\mathsf{E}[k, x] \neq \perp$ then ret $\mathsf{E}[k, x]$;
E02 If $\mathsf{E}[k, c_1] = \perp$,
E03 $y \leftarrow E3_I(k, c_1)$;
E04 $\mathsf{E}[k, c_1] \leftarrow y$; $\mathsf{D}[k, y] \leftarrow c_1$;
E05 $y \leftarrow E3_I(k, c_2)$;
E06 $\mathsf{E}[k, c_2] \leftarrow y$; $\mathsf{D}[k, y] \leftarrow c_2$;
E07 If $x \neq c_1$ and $x \neq c_2$,
E08 $y \leftarrow E2_I(k, x)$;
E09 $\mathsf{E}[k, x] \leftarrow y$; $\mathsf{D}[k, y] \leftarrow x$;
E10 Ret $\mathsf{E}[k, x]$;

simulator $\mathcal{S}_D(k, y)$
D01 If $\mathsf{D}[k, y] \neq \perp$ then ret $\mathsf{D}[k, y]$;
D02 If $\mathsf{E}[k, c_1] = \perp$,
D03 $y \leftarrow E3_I(k, c_1)$;
D04 $\mathsf{E}[k, c_1] \leftarrow y$; $\mathsf{D}[k, y] \leftarrow c_1$;
D05 $y \leftarrow E3_I(k, c_2)$;
D06 $\mathsf{E}[k, c_2] \leftarrow y$; $\mathsf{D}[k, y] \leftarrow c_2$;
D07 If $\mathsf{D}[k, y] = \perp$,
D08 $x \leftarrow D2_I(k, y)$;
D09 $\mathsf{E}[k, x] \leftarrow y$; $\mathsf{D}[k, y] \leftarrow x$;
D10 Ret x;

Fig. 2. Simulator

Since $E2_I$ is used in inner calculations and $E3_I$ is used in the post-processing calculations, if for a query (k, x) to $\mathcal{S}_E$ (k, x) is used in the post-processing calculations, it returns the output of $E3_I(k, x)$, and otherwise it returns the output of $E2_I(k, x)$. Since in post-processing calculation the second value x of a E query is c_1 or c_2, we define $\mathcal{S}$ such that $\mathcal{S}_E(k, x)$ is defined by $E3_I(k, x)$, if $x = c_1$ or $x = c_2$, and is defined by $E2_I(k, x)$ otherwise.[6] E and D are (initially everywhere $\perp$) arrays.

We give the proof via a game-playing argument on the game sequences Game 0, Game 1, and Game 2. Game 0 is the F scenario and Game 2 is the F_2 scenario. In each game, A_4 can make queries to three oracles $(\mathcal{O}_F, \mathcal{O}_E, \mathcal{O}_D)$. Let Gj be the event that in Game j the distinguisher A_4 outputs 1. Therefore, $\Pr[A_4^{F^{\mathcal{C}_{2n,n}}, E_I, D_I} \Rightarrow 1] = \Pr[G0]$ and $\Pr[A_4^{F_2^{\mathcal{C}^2_{2n,n}, \mathcal{C}^3_{2n,n}}, \mathcal{S}_E, \mathcal{S}_D} \Rightarrow 1] = \Pr[G2]$. Thus

$$\mathsf{Adv}^{\mathsf{indif}}_{F^{\mathcal{C}_{2n,n}}, F_2^{\mathcal{C}^2_{2n,n}, \mathcal{C}^3_{2n,n}}, \mathcal{S}}(A_4) \leq |\Pr[G1] - \Pr[G0]| + |\Pr[G2] - \Pr[G1]|$$

Game 0: Game 0 is the F scenario. So $(\mathcal{O}_F, \mathcal{O}_E, \mathcal{O}_D) = (F^{\mathcal{C}_{2n,n}}, E_I, D_I)$.

Game 1: We modify the underlying functions $(\mathcal{O}_E, \mathcal{O}_D)$ from (E_I, D_I) to $(\mathcal{S}_E, \mathcal{S}_D)$. So $(\mathcal{O}_F, \mathcal{O}_E, \mathcal{O}_D) = (F^{\mathcal{S}}, \mathcal{S}_E, \mathcal{S}_D)$ where only $\mathcal{S}$ has oracle access to $(\mathcal{C}^2_{2n,n}, \mathcal{C}^3_{2n,n})$.

We must show that the A_4's view has statistically close distribution in Game 0 and Game 1. Since the difference between the games is the underlying function, we show that the output of the functions is statistically close; this in turn shows that the A_4's view has statistically close distribution in Game 0 and Game 1. First we rewrite $\mathcal{S}$ in Fig. 3. $\mathcal{C}^3_{2n,n}$ is hard-coded in the steps e03-e05, e06-e08, d03-05 and d06-08 where $\mathsf{E2}$ and $\mathsf{D2}$ are (initially everywhere $\perp$) arrays to store the output of the ideal cipher and $\mathcal{T}_{E2}$ and $\mathcal{T}_{D2}$ are (initially everywhere empty) tables. Similarly, $\mathcal{C}^3_{2n,n}$ is hard-coded in Steps e10-e11 and d10-d11 where $\mathsf{E3}$ and $\mathsf{D3}$ are (initially everywhere $\perp$) arrays to store the output of the ideal cipher. For any k, if $\mathsf{E2}[k, x] \neq \perp$, $\mathsf{E2}[k, x] \in \mathcal{T}_{E2}[k]$, and if $\mathsf{D2}[k, y] \neq \perp$, $\mathsf{D2}[k, y] \in \mathcal{T}_{D2}[k]$.

[6] If c_1 and c_2 which are equal to $C \oplus IV[0]$ or $IV[0]$ are used, $\mathcal{S}$ cannot decide whether using $E2_I$ or $E3_I$.

simulator $\mathcal{S}_E(k,x)$

e01 If $\mathsf{E}[k,x] \neq \perp$ then ret $\mathsf{E}[k,x]$;
e02 If $\mathsf{E}[k,c_1] = \perp$,
e03 $\quad y_1 \xleftarrow{\$} \{0,1\}^n$;
e04 $\quad \mathsf{E3}[k,c_1] \leftarrow y_1$; $\mathsf{D3}[k,y_1] \leftarrow c_1$;
e05 $\quad \mathsf{E}[k,c_1] \leftarrow \mathsf{E3}[k,c_1]$; $\mathsf{D}[k,y_1] \leftarrow c_1$;
e06 $\quad y_2 \xleftarrow{\$} \{0,1\}^n \backslash \{y_1\}$;
e07 $\quad \mathsf{E3}[k,c_2] \xleftarrow{\$} y_2$; $\mathsf{D3}[k,y_2] \leftarrow c_2$;
e08 $\quad \mathsf{E}[k,c_1] \leftarrow \mathsf{E3}[k,c_1]$; $\mathsf{D}[k,y_2] \leftarrow c_1$;
e09 If $x \neq c_1$ and $x \neq c_2$,
e10 $\quad y \xleftarrow{\$} \{0,1\}^n \backslash \mathcal{T}_{E2}[k]$;
e11 $\quad \mathsf{E2}[k,x] \leftarrow y$; $\mathsf{D2}[k,y] \leftarrow x$;
e12 $\quad \mathsf{E}[k,x] \leftarrow y$; $\mathsf{D}[k,y] \leftarrow x$;
e13 Ret $\mathsf{E}[k,x]$;

simulator $\mathcal{S}_D(k,y)$

d01 If $\mathsf{D}[k,y] \neq \perp$ then ret $\mathsf{D}[k,y]$;
d02 If $\mathsf{E}[k,c_1] = \perp$,
d03 $\quad y_1 \xleftarrow{\$} \{0,1\}^n$;
d04 $\quad \mathsf{E3}[k,c_1] \leftarrow y_1$; $\mathsf{D3}[k,y_1] \leftarrow c_1$;
d05 $\quad \mathsf{E}[k,c_1] \leftarrow \mathsf{E3}[k,c_1]$; $\mathsf{D}[k,y_1] \leftarrow c_1$;
d06 $\quad y_2 \xleftarrow{\$} \{0,1\}^n \backslash \{y_1\}$;
d07 $\quad \mathsf{E3}[k,c_2] \xleftarrow{\$} y_2$; $\mathsf{D3}[k,y_2] \leftarrow c_2$;
d08 $\quad \mathsf{E}[k,c_1] \leftarrow \mathsf{E3}[k,c_1]$; $\mathsf{D}[k,y_2] \leftarrow c_1$;
d09 If $\mathsf{D}[k,y] = \perp$,
d10 $\quad x \xleftarrow{\$} \{0,1\}^n \backslash \mathcal{T}_{D2}[k]$;
d11 $\quad \mathsf{E2}[k,x] \leftarrow y$; $\mathsf{D2}[k,y] \leftarrow x$;
d12 $\quad \mathsf{E}[k,x] \leftarrow y$; $\mathsf{D}[k,y] \leftarrow x$;
d13 Ret x;

Fig. 3. Revised Simulator

Encryption Oracle $E_I(k,x)$

01 If $\mathsf{E}[k,x] \neq \perp$, ret $\mathsf{E}[k,x]$;
02 If $\mathsf{E}[k,c_1] = \perp$,
03 $\quad \mathsf{E}[k,c_1] \xleftarrow{\$} \{0,1\}^n$;
04 $\quad \mathsf{D}[k,\mathsf{E}[c_1,x]] \leftarrow c_1$;
05 $\quad \mathsf{E}[k,c_2] \xleftarrow{\$} \{0,1\}^n \backslash \{\mathsf{E}[k,c_1]\}$;
06 $\quad \mathsf{D}[k,\mathsf{E}[c_2,x]] \leftarrow c_2$;
07 If $x \neq c_1$ and $x \neq c_2$,
08 $\quad \mathsf{E}[k,x] \xleftarrow{\$} \{0,1\}^n \backslash \mathcal{T}_E[k]$;
09 $\quad \mathsf{D}[k,\mathsf{E}[k,x]] \leftarrow x$;
10 Ret $\mathsf{E}[k,x]$;

Decryption Oracle $D_I(k,y)$

11 If $\mathsf{D}[k,y] \neq \perp$, ret $\mathsf{D}[k,y]$;
12 If $\mathsf{E}[k,c_1] = \perp$,
13 $\quad \mathsf{E}[k,c_1] \xleftarrow{\$} \{0,1\}^n$;
14 $\quad \mathsf{D}[k,\mathsf{E}[c_1,x]] \leftarrow c_1$;
15 $\quad \mathsf{E}[k,c_2] \xleftarrow{\$} \{0,1\}^n \backslash \{\mathsf{E}[k,c_1]\}$;
16 $\quad \mathsf{D}[k,\mathsf{E}[c_2,x]] \leftarrow c_2$;
17 If $\mathsf{D}[k,y] \neq \perp$,
18 $\quad \mathsf{D}[k,y] \xleftarrow{\$} \{0,1\}^n \backslash \{\mathcal{T}_D[k]\}$;
19 $\quad \mathsf{E}[k,\mathsf{D}[k,y]] \leftarrow y$;
20 Ret $\mathsf{D}[k,y]$;

Fig. 4. Lazily-Sample Ideal Cipher

In the following, we use the lazily-sample ideal cipher in Fig. 4. E and D are (initially everywhere $\perp$) arrays and $\mathcal{T}_E$ and $\mathcal{T}_D$ (initially empty) tables. For any (k,x) such that $\mathsf{E}[k,x] \neq \perp$, $\mathsf{E}[k,x]$ is stored in $\mathcal{T}_E[k]$, and for any (k,y) such that $\mathsf{D}[k,y] \neq \perp$, $\mathsf{D}[k,y]$ is stored in $\mathcal{T}_D[k]$. On a query which the key element is k, first the output of $E_I(k,c_1)$ is determined (steps 03-04 or steps 13-14) and second the output of $E_I(k,c_2)$ is determined (Steps 05-06 or Steps 15-16). Then the outputs of $E_I(k,x)$ such that $x \neq c_1$ and $x \neq c_2$ are determined. Since no adversary (distinguisher) learns $E_I(k,c_1)$ and $E_I(k,c_2)$ until querying the corresponding value, the procedures of the steps 03-06 and 13-16 do not affect the lazily-sample ideal cipher simulation.

We compare the simulator with the lazily-sample ideal cipher. In the simulator and the ideal cipher, $\mathsf{E}[k,c_1]$ and $\mathsf{E}[k,c_2]$ (and also $\mathsf{D}[k,\mathsf{E}[k,c_1]]$ and $\mathsf{D}[k,\mathsf{E}[k,c_2]]$) are chosen from the same distribution, while $\mathsf{E}[k,x]$ (and $\mathsf{D}[k,\mathsf{E}[k,x]]$) where $x \neq c_1$ and $x \neq c_2$ is chosen different distribution. If in the step e10 y is randomly chosen from $\mathcal{T}_{E2}[k] \cup \{\mathsf{E}[k,c_1], \mathsf{E}[k,c_2]\}$ and in the step d10 x is randomly chosen

from $\mathcal{T}_{D2}[k] \cup \{c_1, c_2\}$, then the output distribution of the simulator and the ideal cipher is the same. That is, if any value y randomly chosen from $\{0,1\}^n \backslash \mathcal{T}_{E2}[k]$ does not collide $\mathsf{E}[k, c_1]$ and $\mathsf{E}[k, c_2]$ and any value x randomly chosen from $\{0,1\}^n \backslash \mathcal{T}_{D2}[k]$ does not collide c_1 and c_2, then the output distribution between them is the same. Since for any k, the number of values in $\mathcal{T}_{E2}[k]$ and $\mathcal{T}_{D2}[k]$ is at most $2lq_F + q_E + q_D$, the statistical distance of $\mathsf{E}[k, x]$ (and $\mathsf{D}[k, \mathsf{E}[k, x]]$) where $x \neq c_1$ and $x \neq c_2$ is at most $2/(2^n - (2lq_F + q_E + q_D))$. So the statistical distance of the simulator and the ideal cipher is at most $(2lq_F + q_E + q_D) \times 2/(2^n - (2lq_F + q_E + q_D))$. We thus have that

$$|\Pr[G1] - \Pr[G0]| \leq \frac{2 \times (2lq_F + q_E + q_D)}{2^n - (2lq_F + q_E + q_D)}.$$

Game 2: We modify $\mathcal{O}_F$ from $F^{\mathcal{S}}$ to $F_2^{\mathcal{C}^2_{2n,n}, \mathcal{C}^3_{2n,n}}$. So $(\mathcal{O}_F, \mathcal{O}_E, \mathcal{O}_D) = (F_2^{\mathcal{C}^2_{2n,n}, \mathcal{C}^3_{2n,n}}, \mathcal{S}_E, \mathcal{S}_D)$ and this is the F_2 scenario.

We show that unless the following bad events occur, the A_4's view of Game 1 and Game 2 is the same.

- Event B1: On some query (k, x) to $\mathcal{S}_E$, the output y is such that $y \oplus x$ is equal to c_1 or c_2.
- Event B2: On some query (k, x) to $\mathcal{S}_E$, the output y is such that $y \oplus x \oplus C$ is equal to c_1 or c_2.
- Event B3: On some query (k, y) to $\mathcal{S}_D$, the output x is equal to c_1 or c_2 such that x is defined in the step D08.

To prove this, we use the proof method in [4,13]. Specifically, we prove the following two points.

1. In Game 1, unless the bad events occur, for any query M the output of $\mathcal{O}_F(M)$ is equal to that of $F_2^{\mathcal{C}^2_{2n,n}, \mathcal{C}^3_{2n,n}}(M)$. If this holds, the output distribution of $\mathcal{O}_F$ in Game 1 and Game 2 is equivalent.
2. In Game 2, unless the bad events occur, $\mathcal{O}_E$ and $\mathcal{O}_D$ are consistent with $\mathcal{O}_F$ as in Game 1. $\mathcal{O}_F$ uses $\mathcal{O}_E$ in Game 1 while does not in Game 2 (note that in both games $(\mathcal{O}_E, \mathcal{O}_D) = (\mathcal{S}_E, \mathcal{S}_D)$). So if this holds, the difference does not affect the output distribution of $\mathcal{O}_E$ and $\mathcal{O}_D$, namely, the output distribution of $\mathcal{O}_E$ and $\mathcal{O}_D$ in Game 1 and Game 2 is the same.

In the following, for input-output triple (k, x, y) of $\mathcal{S}$ we denote $x \oplus y$ by w, namely, $w = x \oplus y$. Before proving the above two points, we define chain triples and give a useful lemma.

Definition 3. $(k_1, x_1, y_1), \ldots, (k_i, x_i, y_i), (k'_1, x'_1, y'_1), \ldots, (k'_i, x'_i, y'_i), (k, x, y), (k', x', y')$ *stored in the simulator's tables* E, D *are chain triples if for some* M *the output of* $F^{\mathcal{S}}(M)$ *can be obtained from the triples. That is,* $x_1 = IV[0], k_1[0] = IV[1]$, $k_j = k'_j$ $(j = 1, \ldots, i)$, $w_j = x_{j+1}$ $(j = 1, \ldots, i-1)$, $w_j \oplus C = x'_{j+1}$ $(j = 1, \ldots, i-1)$, $w'_j = k_{j+1}[0]$ $(j = 1, \ldots, i-1)$, $x = c_1$, $x' = c_2$, $k = k'$, $k[0] = w_i$, $k[1] = w'_i$, $M = k_1[1] || \cdots || k_i[1]$, *and* $y || y' = F^{\mathcal{S}}(M)$.

Lemma 7. *For any chain triple* $(k_1, x_1, y_1), \ldots, (k_i, x_i, y_i), (k'_1, x'_1, y'_1), \ldots, (k'_i, x'_i, y'_i), (k, x, y), (k', x', y')$*, unless the bad events occur,* $F^{\mathcal{S}}(M) = F_2^{\mathcal{C}^2_{2n,n}, \mathcal{C}^3_{2n,n}}(M)$ *where* $M = k_1[1]||\cdots||k_i[1]$.

Proof. To contrary, assume that there exist chain triples $(k_1, x_1, y_1), \ldots, (k_i, x_i, y_i), (k'_1, x'_1, y'_1), \ldots, (k'_i, x'_i, y'_i), (k, x, y), (k', x', y')$ such that $F^{\mathcal{S}}(M) \neq F_2^{\mathcal{C}^2_{2n,n}, \mathcal{C}^3_{2n,n}}(M)$ where $M = k_1[1]||\cdots||k_i[1]$. Then, since the output of $\mathcal{S}$ is defined by $E2_I$ or $E3_I$, one of the following events occur.

- Event 1: In the inner calculation of $F^{\mathcal{S}}(M)$, some triple is defined by $E3_I$. That is, some of $(k_1, x_1, y_1), \ldots, (k_i, x_i, y_i), (k'_1, x'_1, y'_1), \ldots, (k'_i, x'_i, y'_i)$, is defined by $E3_I$.
- Event 2: In the post-processing function calculation of $F^{\mathcal{S}}(M)$, some triple is defined by $E2_I$. That is, (k, x, y) or (k', x', y') is defined by $E2_I$.

Consider Event 1. First consider the case that (k_j, x_j, y_j) is defined by $E3_I$. Since $x_1 = IV[0]$, $j \neq 1$. When the output of $\mathcal{S}_E(k_j, x_j)$ is defined by $E3_I$, $x_j = c_1$ or $x_j = c_2$. Which means that $w_{j-1} = c_1$ or $w_{j-1} = c_2$. So the bad event 1 occurs. Second consider the case that (k'_j, x'_j, y'_j) is defined by $E3_I$. Similarly, since $x_1 = IV[0] \oplus C$, $j \neq 1$. When the output of $\mathcal{S}_E(k_j, x_j)$ is defined by $E3_I$, $x_j = c_1$ or $x_j = c_2$. Which means that $w'_{j-1} \oplus C = c_1$ or $w'_{j-1} \oplus C = c_2$. So the bad event 2 occurs.

Next consider Event 2. First consider the case that (k, x, y) is defined by $E2_I$. Then the triple is defined in $\mathcal{S}_D$ because $x = c_1$ (if the triple is defined in $\mathcal{S}_E$, it is defined by $E2_I$ due to the condition of the step E07). So the triple is defined in the step D08. The bad event 3 occurs. Finally, consider the case that (k', x', y') is defined by $E2_I$. Then the triple is defined in $\mathcal{S}_D$ because $x = c_2$. So the triple is defined in the step D08. The bad event 3 occurs. □

Proof of Point 1. From the above lemma, unless the bad event occurs, the output of $\mathcal{O}_F(M) = F^{\mathcal{S}}(M) = F_2^{\mathcal{C}^2_{2n,n}, \mathcal{C}^3_{2n,n}}(M)$.

Proof of Point 2. Since in Game 1 for any M the output of $\mathcal{O}_F(M)$ is calculated by $F^{\mathcal{S}}(M)$, we must show that in Game 2 the relation also holds, that is, unless the bad events occur, for any chain triples $(k_1, x_1, y_1), \ldots, (k_i, x_i, y_i), (k'_1, x'_1, y'_1), \ldots, (k'_i, x'_i, y'_i), (k, x, y), (k', x', y')$ the output of $F^{\mathcal{S}}(M)$ is equal to $\mathcal{O}_F(M)$ $(= F_2^{\mathcal{C}^2_{2n,n}, \mathcal{C}^3_{2n,n}}(M))$ where $M = k_1[1]||\cdots||k_i[1]$. From the above lemma, unless the bad event occurs, this holds.

The Bound of $|\Pr[G2] - \Pr[G1]|$**.** The above two points imply that unless the bad events occur, the A_4's view of Game 1 and Game 2 is the same, and so we have that

$$|\Pr[G2] - \Pr[G1]| \leq 2 \times \max\{\Pr[B1_1] + \Pr[B2_1] + \Pr[B3_1], \Pr[B1_2] + \Pr[B2_2] + \Pr[B3_2]\}$$

where Bi_j is the event Bi in Game j. Since the number of queries to $\mathcal{S}$ in Game 1 is more than that in Game 2,

$$|\Pr[G2] - \Pr[G1]| \leq 2 \times (\Pr[B1_1] + \Pr[B2_1] + \Pr[B3_1]).$$

First, we evaluate the probability $\Pr[B1_1]$. In Game 1, the number of queries to $\mathcal{S}$ is at most $2(lq_F + 1) + q_E + q_D$. So the output is randomly chosen from at least $2^n - (2(lq_F + 1) + q_E + q_D)$ values. We thus have that

$$\Pr[B1_1] \leq \frac{2 \times (2(lq_F + 1) + q_E + q_D)}{2^n - (2(lq_F + 1) + q_E + q_D)}.$$

Second, we evaluate the probability $\Pr[B2_1]$. From the same discussion as $\Pr[B1_1]$,

$$\Pr[B2_1] \leq \frac{2 \times (2(lq_F + 1) + q_E + q_D)}{2^n - (2(lq_F + 1) + q_E + q_D)}.$$

Finally, we evaluate the probability $\Pr[B3_1]$. A value in the step D08 is defined by $D2_I$. That is, in this case, the output of $D2_I$ is equal to c_1 or c_2. Since the number of queries to $\mathcal{C}^2_{2n,n}$ is at most $2(lq_F+1)+q_E+q_D$, So the output of $D2_I$ is randomly chosen from at least $2^n - (2(lq_F + 1) + q_E + q_D)$ values. We thus have that

$$\Pr[B1_1] \leq (2(lq_F + 1) + q_E + q_D) \times \frac{2}{2^n - (2(lq_F + 1) + q_E + q_D)}.$$

We thus have that

$$|\Pr[G2] - \Pr[G1]| \leq 2 \times \frac{6 \times (2(lq_F + 1) + q_E + q_D)}{2^n - (2(lq_F + 1) + q_E + q_D)}$$

Consequently, we can obtain the following bound.

$$\mathsf{Adv}^{\mathsf{indif}}_{F^{\mathcal{C}_{2n,n}}, F_2^{\mathcal{C}^2_{2n,n}, \mathcal{C}^3_{2n,n}}, \mathcal{S}}(A_4) \leq \frac{14 \times (2(lq_F + 1) + q_E + q_D)}{2^n - (2(lq_F + 1) + q_E + q_D)}.$$

□

Theorem 2 and Lemma 6 ensure the following theorem where F is PRO up to $\mathcal{O}(2^n)$ query complexity in the IC model. We prove the theorem in the full version.

Theorem 3 (F is PRO). *There exists a simulator $S = (S_E, S_D)$ such that for any distinguisher A making at most (q_H, q_E, q_D) queries to three oracles which are (F, E_I, D_I) or $(\mathcal{F}_{2n}, S_E, S_D)$, we have*

$$\mathsf{Adv}^{\mathsf{pro}}_{F^{\mathcal{C}_{2n,n}}, S}(A) \leq \frac{2Q^2}{(2^n - 2Q)^2} + \frac{2Q}{2^n - 2Q} + \frac{2l(2q)Q}{(2^n - Q)^2} + \frac{q_H + 2q}{2^n} + \frac{14Q}{2^n - Q}$$

where S works in time $\mathcal{O}(q + 2lqQ) + 2lq \times \mathsf{Time}(\mathsf{unpad})$ and makes $2q$ queries to $\mathcal{F}_{2n}$ where $Q = 2l(q_H + 1) + q_E + q_D$ and $q = q_E + q_D$. ♦

4 Conclusion

We proposed new DLHFs constructed from a single practical size blockcipher, where the key size is twice of the plaintext size. The security was proven by that (1) the PRO security of F_1 is proven by the PrA framework, (2) the PRO security of F_2 is proven by PRO for a small function and the result (1), and (3) the PRO security of F is proven by indifferentiability from a hash function and the result (2). Our schemes are the first time DLHFs achieving birthday PRO security. This paper only considers PRO security, while the performance evaluation is not considered. So the evaluation is a future work.

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