Urgent Epidemic Control Mechanism for Aviation Networks

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Abstract. In the current century, the highly developed transportation system can not only boost the economy, but also greatly accelerate the spreading of epidemics. While some epidemic diseases may infect quite a number of people ahead of our awareness, the health care resources such as vaccines and the medical staff are usually locally or even globally insufficient. In this research, with the network of major aviation routes as an example, we present a method to determine the optimal locations to allocate the medical service in order to minimize the impact of the infectious disease with limited resources. Specifically, we demonstrate that when the medical resources are insufficient, we should concentrate our efforts on the travelers with the objective of effectively controlling the spreading rate of the epidemic diseases.

Keywords: Epidemic Control, Aviation Networks, SIS Model.

1 Introduction

Although the human beings have made so many achievements, infectious diseases continue to pose a major global public health challenge. They have afflicted humankind ever since the emergence of Homo sapiens despite advances in science and the social environment around the world. Epidemic diseases are a major cause of death ever since cold war years. For example, in 2002 they caused 14.9 million deaths, accounting for 26% of total global mortality and almost 30% of the total disability-adjusted life years (DALYs) lost worldwide.

Today, with the highly development of transportation system, infectious diseases can be spread by travelers within a very short time via the aviation network. As the transportation network has an enormous influence on people's economical and social activities, it is unwise and impossible to confine the disease spreading by simply blocking all the traffics.

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We start our research from a simple susceptible-infected-susceptible (SIS) model [1], in which people only have two types: infected and susceptible. This model assumes each type of individuals can change their roles by infection or recovery.

We combine the model to an aviation network, and find out an optimal solution for the epidemic controlling: we should immune the travelers first before we apply it on the other citizens.

2 Model for Epidemic Spreading

2.1 Model Description

In our model, we divide the epidemic spreading procedure into two parts.

The first part is inner city transmitting. To simplify the problem, we may classify the whole population in a city into S susceptible individuals and Iinfectious individuals, to adopt the SIS model [1, 2, 3]. Based on this model, we assume the two types of people inside the city are well mixed and well connected. Then the SIS model can be described by two differential equations in Eq. (1) [1], where β is the infectious rate and α is the recover rate.

$$\begin{aligned}\frac{dS}{dt} &= -\beta SI + \alpha I \\ \frac{dI}{dt} &= \beta SI - \alpha I\end{aligned} \tag{1}$$

The second part is the inter city connection. Transportation between cities is also crucial for the disease spreading nowadays. The highly developed transportation system is fundamental for social and economical activities, but this system also contributes to the faster and easier spreading of epidemic diseases. Take SARS as an example, theoretical and simulation results verifies that the aviation network played an important role in SARS spreading [4], while that network is also a critical infrastructure with an enormous impact on local, national, and international economies [5]. Therefore, it is very expensive and difficult for us to isolate cities brutally to control epidemics [4]. These facts make it very practical and crucial to find an optimal solution for immunization or isolation, to reduce the negative influences on society, economy, etc.

To find optima for this problem, we first define several notations. Let $\mathbf{I}$ be a column vector, in which the ith element corresponds to the number of infected people in city i. Let $\mathbf{S}$ be a similar vector, but representing the number of susceptible individuals. We also define a vector $\mathbf{P}$ with ith element as the population of city i. $\mathbf{A}$ is similar as an adjacency matrix, with $\mathbf{A}(i,j)\,(i \neq j)$ to represent the daily traffic from city j to i.

We also assume people in each city have the same infection rate β and recover rate α. Compared to the whole population, the passengers transferred everyday by plane is negligibly small. Therefore, among these four categories of travels: import/ export infected/susceptible people, only the import infected travelers are important, especially when the target city has no infected citizens. In short, every city can be described as a single SIS model, with only one additional factor: imported infectious travelers. The SIS model for city i is:

$$\begin{aligned}\frac{d\mathbf{S}_i}{dt} &= -\beta \mathbf{S}_i \mathbf{I}_i + \alpha \mathbf{I}_i \\ \frac{d\mathbf{I}_i}{dt} &= \beta \mathbf{S}_i \mathbf{I}_i - \alpha \mathbf{I}_i + \mathbf{A}(i,:)\mathbf{I}\end{aligned} \tag{2}$$

where $\mathbf{A}(i,:)$ is the ith row of $\mathbf{A}$.

When t is small, Eq. (2) is enough, but when t is large, we should balance the population of the city, so that the equation becomes:

$$\begin{aligned}\frac{d\mathbf{S}_i}{dt} &\approx -\beta \mathbf{S}_i \mathbf{I}_i + \alpha \mathbf{I}_i - \mathbf{A}(i,:)\mathbf{I} \\ \frac{d\mathbf{I}_i}{dt} &= \beta \mathbf{S}_i \mathbf{I}_i - \alpha \mathbf{I}_i + \mathbf{A}(i,:)\mathbf{I}\end{aligned} \tag{3}$$

If we use the fact that $\mathbf{P} = \mathbf{S} + \mathbf{I}$, and put $\mathbf{P}$, $\mathbf{S}$ and $\mathbf{I}$ to the diagonals of matrix $\mathbf{P_D}$, $\mathbf{S_D}$ and $\mathbf{I_D}$, leaving other entries to be 0, we have the following vector form equation:

$$\begin{aligned}\frac{d(\mathbf{I})}{dt} &= \beta \mathbf{S_D} \mathbf{I} - \alpha \mathbf{I} + \mathbf{A}\mathbf{I} \\ &= (\beta \mathbf{S_D} - \alpha \mathbf{E} + \mathbf{A})\mathbf{I} \\ &= (\beta(\mathbf{P_D} - \mathbf{I_D}) - \alpha \mathbf{E} + \mathbf{A})\mathbf{I} \\ &= (\beta \mathbf{P_D} - \alpha \mathbf{E} + \mathbf{A})\mathbf{I} - \beta \mathbf{I_D}\mathbf{I}\end{aligned} \tag{4}$$

where $\mathbf{E}$ represents the identity matrix.

By discretizing the procedure and use the superscript t to represent the time, the infection number is

$$\mathbf{I}^{t+1} = \mathbf{I}^t + (\beta \mathbf{P_D} - \alpha \mathbf{E} + \mathbf{A})\mathbf{I}^t - \beta {\mathbf{I_D}}^t \mathbf{I}^t \tag{5}$$

Consider the situation that in early stage of the outbreak, we may only able to immune a limited number p_{imn} of population due to the limitation of vaccines, medical staff and facilities. With a definition of matrix $\mathbf{X_{in}}$ and $\mathbf{X_{be}}$ for the number of inner and between city immunizations respectively, we can design a linear programming for the epidemic controlling. Here we assume that we only have the knowledge of infection number inferred from the reported infectors, but cannot identify whether it is infected or not from person to person, so that have to distribute vaccine randomly.

$$\begin{aligned}\min\ f(\mathbf{X_{in}}, \mathbf{X_{be}}) = \mathbf{1}^{\mathrm{T}}[&\beta(\mathbf{P_D} - \mathbf{X_{in}}) - (1-\alpha)\mathbf{E} \\ &+ (\mathbf{A} - \mathbf{X_{be}})]\mathbf{I} - \beta \mathbf{1}^{\mathrm{T}} {\mathbf{I_D}}^t \mathbf{I}^t \\ \text{s.t.} \begin{cases} \mathbf{P_D} \geq \mathbf{X_{in}} \\ \mathbf{A} \geq \mathbf{X_{be}} \\ \mathbf{X_{in}} \geq \mathbf{0} \\ \mathbf{X_{be}} \geq \mathbf{0} \\ p_{imn} \geq \mathbf{1}^{\mathrm{T}}(\mathbf{X_{in}} + \mathbf{X_{be}})\mathbf{1} \end{cases}&\end{aligned} \tag{6}$$

where $\mathbf{0}$ is a column vector of 0s, $\mathbf{1}$ is a column vector of 1s, and the superscript T to represents the matrix transpose. The inequality operators in Eq. (6) here indicate an element-wise operation.

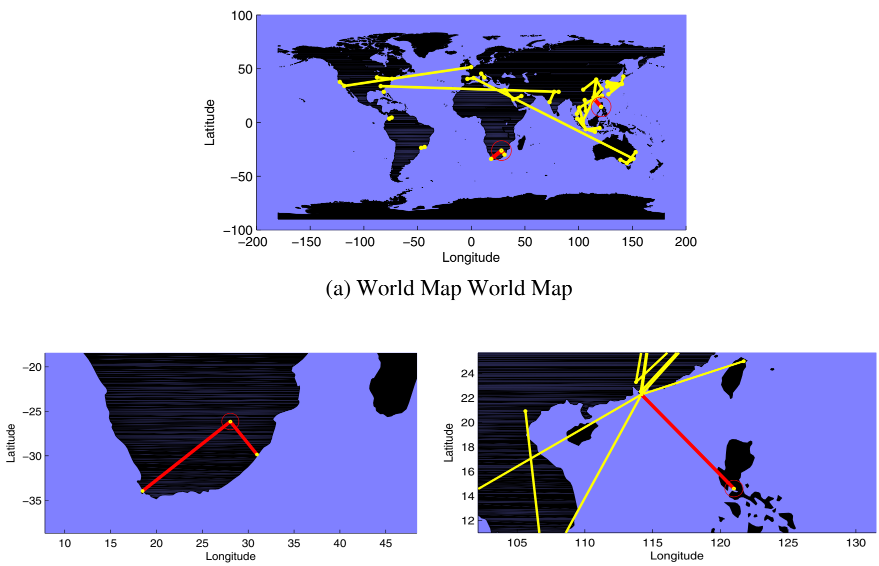
Eq. (6) can be rewritten as follows without change the solution.

$$\min f(\mathbf{X_{in}}, \mathbf{X_{be}}) = \mathbf{1}^{\mathrm{T}}(-\beta \mathbf{X_{in}} - \mathbf{X_{be}})\mathbf{I}$$
$$\text{s.t.} \begin{cases} \mathbf{P_D} \geq \mathbf{X_{in}} \\ \mathbf{A} \geq \mathbf{X_{be}} \\ \mathbf{X_{in}} \geq \mathbf{0} \\ \mathbf{X_{be}} \geq \mathbf{0} \\ p_{imn} \geq \mathbf{1}^{\mathrm{T}}(\mathbf{X_{in}} + \mathbf{X_{be}})\mathbf{1} \end{cases} \tag{7}$$

It is an integer programming, but can be relaxed to a linear one and solve approximately and efficiently. Intuitively, we can see that when there are only a few number of vaccines, immune on travelers of the infected city is more useful than immune the randomly selected citizens.

2.2 Examples Solved by the Model

In this section, we present an example about how our model works. We use the aviation network constructed from the world's top 50 busiest routes [6].We consider the problem on daily basis, so the capacity data from Wikipedia is divided by 30.



(a) World Map World Map

(b) Enlarged Map 1

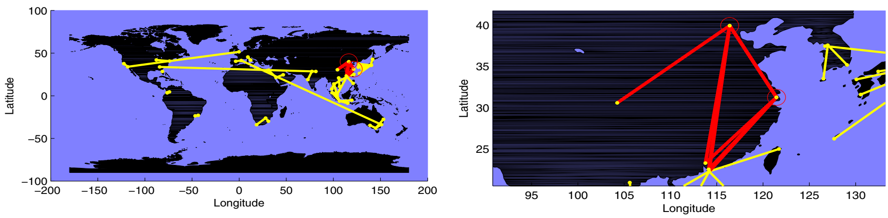
(c) Enlarged Map 2

Fig. 1. Example 1: Two Distant Cities Find Infectors

To make the problem more interesting, for each entry of $\mathbf{P}$, rather than choose a static census result, we randomly pick a value from $(0, 2 \times 10^7)$ and assign it to the entry. Theoretically, the population size would not matters a lot if it is large enough. We also use $\beta = 0.1/(2 \times 10^7)$, although it is not quite sensitive. p_{imn} is set to be 100, indicating that not all the population can get the vaccines.

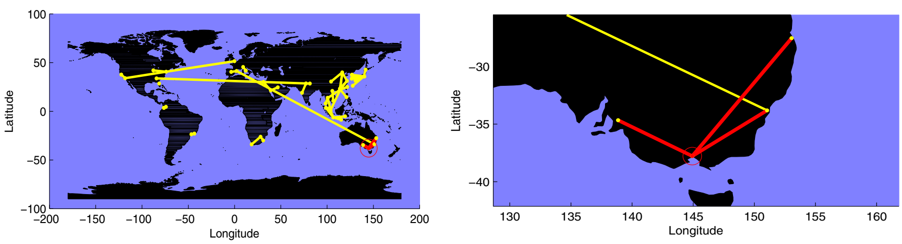
The cities we picked that initially find infectors are circled by red, and the cities or airlines we plan to immune according to our proposed mechanism are red color dots or lines respectively. From Fig. 1 and 2, we can see that when the vaccine is very limited while the city population is very large, it is wise to provide more health care service to the travelers, rather than to the citizens.

When the available vaccines become quite sufficient, for example, $p_{imn} = 1 \times 10^6$, to immunize the citizens becomes a good choice, as in Fig. 3.



(a) World Map (b) Enlarged Map

Fig. 2. Example 2: Two Neighboring Cities Find Infectors



(a) World Map (b) Enlarged Map

Fig. 3. Example 3: When the Medical Service is Sufficient

3 Future Work

Assume there are several demand cities, and several pharmaceutical manufacturing plants (suppliers). They are located in a widespread geometric graph. We can assume Each demand location has a fixed demand related to the epidemic situation and the population. We may also try to establish a transportation schedule between the supplier and demander, so that vaccines and other medical facilities, even medical staffs can reach the most critical site in time.

4 Conclusion

In this research, we propose an optimal mechanism to control the spreading of the epidemic diseases when they suddenly outbreaks. We use the network of the aviation routes to demonstrate that the best way for controlling the infection rate is to put the travelers in highest priority. We also successfully illustrate that this mechanism works well especially in our current highly advanced traffic system. Also in our future work, we present an efficiently potential approach for optimally transporting and allocating the limited resources. Controlling the epidemic diseases has always been, and will still be a major challenge in health care science and people's everyday life. It will become more critical as new types of diseases continue emerging in the future. Finally, we hope our contributions can help protect people's health and life.

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