Fast Finsler Active Contours and Shape Prior Descriptor

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Abstract. In this paper we proposed a new segmentation method based Fast Finsler Active Contours (FFAC). The FFAC is formulated in the Total Variation (TV) framework incorporating both region and shape descriptors. In the Finsler metrics, the anisotropic boundary descriptor favorites strong edge locations and suitable directions aligned with dark to bright image gradients. Strong edges are not required everywhere along. We prove the existence of a solution to the new binary Finsler active contours model and we propose a fast and easy algorithm in characteristic function framework. Finally, we show results on some MR challenging images to illustrate accurate.

Keywords: Finsler Active contours, Wulff Shape, characteristic function, Shape prior, Primal dual.

1 Introduction

The Finsler Active Contours (FAC) has been proposed as natural way for adding directionality to the Active Contours [1, 2, 10]. The utilization of general Finsler metrics instead of Riemannian metrics allows the boundary descriptor to favor appropriate locations and suitable directions [10, 12]. The boundary descriptor is weighted by some position and direction-dependent local image information. The local image information can be obtained from a direction-a learned dependent pattern detector. In order to obtain fast and optimal segmentation we proposed both to use local information such anisotropic boundary descriptor [11] and global information such as statistic and geometric shape prior knowledge in the formulation of FAC. The formulation of FAC in Total Variation framework [4, 7], allows the convert non-convex segmentation problem into a convex problem segmentation. The goal is to use an anisotropic boundary descriptor that forces attracting AC at distinct specific positions with particular orientations. Integration position and orientation leads to TV primal dual formulation of segmentation problem. The formulation of the segmentation problem in the characteristic function leads to a fast globally segmentation procedure. We demonstrate the powerful of the proposed segmentation method on some challenging MR images.

2 Review of Finsler Active Contours

In this work, we are interested in a fast segmentation based Finsler Active Contours (FAC) model. We proposed to reformulate traditional FAC in characteristic function framework ($\chi \in [0, 1]$) as combination of finsler, region and shape descriptors in Total variation framework:

$$\left\{ \begin{array}{l} E(\chi) = \underbrace{\int_{\Omega} k_f(\mathbf{x}, \mathbf{p}_{\perp}) |\nabla \chi(\mathbf{x})| d\mathbf{x}}_{E_b(\partial\Omega)} \\ + \underbrace{\lambda_1 \int_{\Omega} k_r(\mathbf{x}, \Omega) \chi(\mathbf{x}) d\mathbf{x}}_{E_{data}(\mathbf{x}, \Omega)} + \underbrace{\lambda_2 \int_{\Omega} k_s(\mathbf{x}, \Omega_{ref}) \chi(\mathbf{x}) d\mathbf{x}}_{E_{shape}(\mathbf{x}, \Omega_{ref})} \\ \chi = \mathbf{1}_{\{\mathbf{x}: \lambda_1 k_r(\mathbf{x}, \Omega) + \lambda_2 k_s(\mathbf{x}, \Omega_{ref}) < 0; \lambda_1, \lambda_2 \in \mathbb{R}^+\}} \end{array} \right. \quad (1)$$

where λ_1, λ_2 are the calibration factors, k_b is the boundary descriptor, k_r is the regions descriptor defined in the same manner as in [9] and k_s is the shape prior descriptor defined as in [13] and Ω_{ref} the reference shape. In Finsler metrics [10], the traditional isotropic descriptor $k_b(\cdot)$ is replaced by an anisotropic Finsler descriptor $k_f(\mathbf{x}, \mathbf{p})$ by adding the directionality to the traditional boundary descriptor:

$$\left\{ \begin{array}{l} k_f(\mathbf{x}, \mathbf{p}) = k_b(\mathbf{x}, \mathbf{p}_{\perp}) \max\left(\left|\mathbf{p}_{\parallel}^1 - \mathbf{p}_{\parallel}^2\right|_2, \left|\mathbf{p}_{\parallel}^1 - \mathbf{p}_{\parallel}^2\right|_2\right)^{m=1 or 2} \\ \mathbf{p} = [\mathbf{p}_{\perp}, \mathbf{p}_{\parallel}] \end{array} \right. \quad (2)$$

where $\mathbf{p}_{\parallel} = [\mathbf{p}_{\parallel}^1, \mathbf{p}_{\parallel}^2]^T$ denote the tangential direction to the curve $\partial\Omega$ and $\mathbf{p}_{\perp} = [\mathbf{p}_{\perp}^1, \mathbf{p}_{\perp}^2]^T$ denote the normal direction to the curve $\partial\Omega$. Since the desired result is a curve in higher dimensions, a dynamic programming approach is used to determine the implicit convex minimizer of FAC energy [10]. In the next section we are interested in a definition of new convex Finsler descriptor in the TV framework ensuring a globally and optimal segmentation. When other convex or non-convex descriptors such as regions or shape prior descriptors are incorporated within Finsler descriptor, the solution is also implicitly convex.

3 Finsler Active Contours in the Total Variation Framework

In this section, we replace the isotropic boundary descriptor in (1) by anisotropic descriptor in TV framework [4, 6, 8, 12]. More formally, let $(\psi_{\mathbf{x}})_{\mathbf{x} \in \Omega}$ be a family of weighted anisotropic, positively 1-homogeneous functions, the segmentation problem can be formulated as:

$$E_{FAC}(\chi) = \underbrace{\int_{\Omega} \psi_{\mathbf{x}}(\nabla \chi) d\mathbf{x}}_{E_{finsler}(\cdot)} + \underbrace{\lambda_1 \int_{\Omega} k_r(\mathbf{x}, \Omega) \chi(\mathbf{x}) d\mathbf{x}}_{E_{data}(\mathbf{x}, \Omega)} + \underbrace{\lambda_2 \int_{\Omega} k_s(\mathbf{x}, \Omega_{ref}) \chi(\mathbf{x}) d\mathbf{x}}_{E_{shape}(\mathbf{x}, \Omega_{ref})} \quad (3)$$

Where ψ is induced by potential shapes noted as Wulff shapes W_{ψ} [6, 8, 11].

Definition

Let $\psi: \mathbb{R}^2 \rightarrow \mathbb{R}$ be a convex, positively 1-homogeneous function $\psi(\lambda \nabla \chi) = \lambda \psi(\nabla \chi)$, for $\lambda > 0$.

The Wulff shape W_{ψ} convex, bounded and closed set ($0 \in W_{\psi}$) is defined as:

$$\begin{aligned} W_{\psi} &= \{ \forall \mathbf{x} \in \Omega \subset \mathbb{R}^2, \nabla \chi \in \mathbb{R}^2 : \langle \mathbf{p}, \nabla \chi \rangle \leq \psi(\nabla \chi) \} \\ \text{such } W_{\psi} &= \{ -\mathbf{p} : \mathbf{p} \in W_{\psi} \} \\ \text{and } \psi(\nabla \chi) &= \max_{\mathbf{p} \in -W_{\psi}} (\langle \mathbf{p}, \nabla \chi \rangle) = \max_{\mathbf{p} \in W_{\psi}} (\langle -\mathbf{p}, \nabla \chi \rangle) \end{aligned} \quad (4)$$

where $\langle \cdot, \cdot \rangle$ denote the inner product in $\mathbb{R}^2$.

The nature of solution of (3) was already shown for the TV [4, 6, 11] by rewriting the anisotropic TV in terms of characteristic function and extended to general families Wulff shape.

Lemma

Let ψ be a positively 1-homogeneous function, and the descriptors k_r and k_s . Then any global minimizer of (3) can be considered as a global minimize.

Proof: Assume $\chi^*: \Omega \rightarrow [0, 1]$ is a global minimizer of (3). The corresponding characteristic function $\hat{\chi}$ is then defined as:

$$\hat{\chi}(\mathbf{x}) = \begin{cases} 1 & \text{if } \chi^*(\mathbf{x}) \geq 0 \\ 0 & \text{if } \chi^*(\mathbf{x}) < 0 \end{cases} \quad (5)$$

If $\nabla \hat{\chi} \neq 0$ has the same direction as $\nabla \chi^*$, then we can write $\nabla \hat{\chi} = c \nabla \chi^*$, $c \in \mathbb{R}^{**}$. The dual energy of (3) is given by:

$$E_{FAC}(\mathbf{p}) = \int_{\Omega} \min(0, \text{div}(\mathbf{p}) + \lambda_1 k_r(\mathbf{x}, \Omega) + \lambda_2 k_s(\mathbf{x}, \Omega_{ref})) d\mathbf{x} \quad (6)$$

which is maximized with respect to a vector field $\mathbf{p}$ subject to $-\mathbf{p} \in W_{\psi_{\mathbf{x}}}$. By inserting the respective constraints on χ and $\mathbf{p}$ using the dirac function δ , the primal (3) and dual (6) energies can be stated as:

$$E^*(\chi) = E_{FAC}(\chi) + \int_{\Omega} \delta_{[0,1]}(\chi(\mathbf{x})) d\mathbf{x} \quad (7)$$

and we can expressed the dual energy as:

$$E_{FAC}^*(\mathbf{p}) = E_{FAC}(\mathbf{p}) - \int_{\Omega} \delta_{W_{\psi}}(-\mathbf{p}) d\mathbf{x} \quad (8)$$

We use the Karush-Kuhn-Tucker (KKT) conditions to prove the optimality of $\hat{\chi}$ [3].

Let $\mathbf{p}^*$ be the corresponding dual solution for χ^* . The KKT conditions stated as:

$$\nabla \chi^* \in \partial \int \delta_{W_{\psi}}(-\mathbf{p}^*) - \text{div}(\mathbf{p}^*) \in \lambda_1 \partial \int k_r(\mathbf{x}, \Omega) \chi^* d\mathbf{x} + \lambda_2 \partial \int k_s(\mathbf{x}, \Omega_{ref}) \chi^* d\mathbf{x} + \delta_{[0,1]}(\chi^*) \quad (9)$$

Hence the KKT conditions can be applied point-wise. Therefore, $(\chi^*, \mathbf{p}^*)$ are minimizers of the primal energy (7) and its corresponding dual energy (8) if and only if:

$$\nabla \chi^* \in \delta_{W_{\psi}}(-\mathbf{p}^*) - \text{div}(\mathbf{p}^*) \in \lambda_1 k_r(\mathbf{x}) \chi^* + \lambda_2 k_s(\mathbf{x}, \Omega_{ref}) \chi^* + \delta_{[0,1]}(\chi^*) \quad (10)$$

Then, we established that $\nabla \hat{\chi}$ is sub gradient of $\delta_{W_{\psi}}(-\mathbf{p}^*)$:

$$\delta_{W_{\psi}}(-\mathbf{p}^*) + (\nabla \hat{\chi})^T (\mathbf{p} - \mathbf{p}^*) \leq \delta_{W_{\psi}}(-\mathbf{p}) \quad (11)$$

Since $-\mathbf{p}^* \in W_{\psi}$ and $-\mathbf{p} \in W_{\psi} \Rightarrow \delta_{W_{\psi}}(-\mathbf{p}^*) = 0$ and $\delta_{W_{\psi}}(-\mathbf{p}) = 0$ and we can write:

$$(\nabla \hat{\chi})^T (\mathbf{p} - \mathbf{p}^*) = c (\nabla \chi^*)^T (\mathbf{p} - \mathbf{p}^*) \leq 0, c \in \mathbb{R}^+ \quad (12)$$

We can establish that $\nabla \hat{\chi} \in \delta_{W_{\psi}}(-\mathbf{p}^*)$.

To establish that $-\text{div}(\mathbf{p}^*)$ is sub gradient [3] in (12), we consider two cases. In the first case χ^* is already either 0 or 1, then $\hat{\chi} = \chi^*$. The second case χ^* is in the open interval $]0, 1[$, then $\partial \delta_{[0,1]}(\chi^*) = 0$, since $\delta_{[0,1]}(\cdot)$ is constant in $[0, 1]$:

$$\delta_{[0,1]}(\hat{\chi}) \leq \delta_{[0,1]}(\chi) + \text{div}(\mathbf{p}^*)(\chi - \hat{\chi}) \quad (13)$$

Then $-\text{div}(\mathbf{p}^*) \in \lambda_1 k_r(\mathbf{x}, \Omega) \hat{\chi} + \lambda_2 k_s(\mathbf{x}, \Omega_{ref}) \hat{\chi} + \delta_{[0,1]}(\hat{\chi})$, thus $(\hat{\chi}, \mathbf{p}^*)$ also satisfies the KKT conditions and $\hat{\chi}$ is therefore a global binary minimizer.

4 Finsler Active Contours in the TV Framework

In this section, we consider that $\psi(\nabla \chi) = \max_{\mathbf{p} \in W_{\psi}} (-\mathbf{p}^T \nabla \chi)$, the energy in (7) and (8) can be formulated in a primal-dual setting as:

$$E(\chi, \mathbf{p}) = \int_{\Omega} \underbrace{\max_{\mathbf{p} \in W_{\psi}} \langle \mathbf{p}, \nabla \chi(\mathbf{x}) \rangle}_{\psi_{\mathbf{x}}(\chi)} d\mathbf{x} + \int_{\Omega} \left\{ \begin{array}{l} \lambda_1 k_r(\mathbf{x}, \Omega) \\ + \lambda_2 k_s(\mathbf{x}, \Omega_{ref}) \end{array} \right\} \chi(\mathbf{x}) d\mathbf{x} \quad (14)$$

and the respective gradient descent (for χ) and ascent (for $\mathbf{p}$) equations are:

$$\left\{ \begin{array}{l} \frac{\partial \chi(\mathbf{x}, \tau)}{\partial \tau} = \text{div}(\mathbf{p}) - \lambda_1 k_r(\mathbf{x}, \Omega) - \lambda_2 k_s(\mathbf{x}, \Omega_{ref}), \quad \chi(\cdot, \tau=0) = \mathbf{1}_{\min(\text{dist}(\mathbf{x}, \partial\Omega_0))} \\ \frac{\partial \mathbf{p}}{\partial \tau} = -\nabla \chi(\mathbf{x}, \tau), \quad \mathbf{p}_0 = \text{design as in sect 5.2} \end{array} \right. \quad (15)$$

where τ is an artificial time parameter, $\partial\Omega_0$ is the initial contour curve. Enforcing the constraints on χ and $\mathbf{p}$ is both done by clamping $[0,1]$, and reprojecting $\mathbf{p}$ onto the feasible set $W_{\psi_{\mathbf{x}}}$.

The Energy based region term is usually defined as a domain integral of the region descriptor k_r :

$$E_{data}(I, \Omega) = \int_{\Omega} \sqrt{p(I|\Omega) p(I|\Omega \setminus \Omega_I)} d\mathbf{x} \quad (16)$$

Where the region descriptor based on Bhattacharyya distance [9] is calculated using the gradient shape tool [5]:

$$k_r(\mathbf{x}, \Omega) = \frac{1}{2} \left\{ \begin{array}{l} \sqrt{p_{in}(I(\mathbf{x}), \Omega) p_{out}(I, \Omega)} \left(\frac{1}{|\Omega_{in}|} - \frac{1}{|\Omega_{out}|} \right) + \\ \int_{R^+} \frac{1}{|\Omega_{out}|} \sqrt{\frac{p_{in}(I, \Omega)}{p_{ref}(I, \Omega)}} \cdot (G_{\sigma_{ker}}(I - I(\Omega)) - \sqrt{p_{out}(I, \Omega)}) dI \\ - \int_{R^+} \frac{1}{|\Omega_{in}|} \sqrt{\frac{p_{out}(I, \Omega)}{p_{in}(I, \Omega)}} \cdot (G_{\sigma_{ker}}(I - I(\Omega)) - \sqrt{p_{in}(I, \Omega)}) dI \end{array} \right\} \quad (17)$$

We therefore estimate density by Parzen kernel, which can better describe the regions. This method estimates the probability density function based on the histograms, using a smoothed Gaussian kernel:

$$p(I, \Omega_I) = \frac{1}{|\Omega_I|} \int_{\Omega} G_{\sigma}(I - I(\Omega_I)) d\mathbf{x} \quad (18)$$

where G_{σ} denote the Gaussian kernel and σ^2 the variance.

The shape prior descriptor is defined as the Euclidean distance between the evolving Legendre moment region $\eta(\Omega)$ and the reference shapes $\{\chi_{ref}^i, i = 1, \dots, N\}$:

$$E_{shape}(\mathbf{x}, \Omega_{ref}) = \int_{\Omega} k_{shape}(\mathbf{x}, \chi_{ref}^i) d\mathbf{x} = \sum_{p,q}^{p+q \leq N} \left| \eta_{pq}(\chi) - \eta_{pq}(\chi_{ref}^i) \right|^2 \quad (19)$$

where the η_{pq} are defined as follows, using the geometric moments M_{pq} and the coefficients a_{pq} of the Legendre polynomials [13]:

$$\eta_{pq} = \frac{(2p+1)(2q+1)}{4} \sum_{u=0}^p \sum_{v=0}^q a_{pu} a_{qv} M_{uv} \quad (20)$$

Where $M_{pq}(\chi) = \frac{1}{\chi^{\frac{p+q+2}{2}}} \int_{\Omega} (x - \bar{x})^p (y - \bar{y})^q dx dy$ and $(\bar{x}, \bar{y})$ are the shape barycenter coordinates. In the next section we established our based fast FAC segmentation method.

5 Results

5.1 Data and Protocol

In this section, we provide prostate segmentation results for two data sets obtained from Saint Philibert Hospital Lille France. The MR images are pre-processed through the following pipeline: 1) spatial registration, 2) noise removal and 3) intensity standardization. We use the T1 weighted and T2 weighted MR sequences. The image sizes are 256x256 pixels, each slice thickness is 3.5mm with spacing between slices of 3.9 mm.

5.2 Finsler Shape Design

There is a wide spread of the possibilities to design ψ_x . In Fig. 1(a), the Wulff shape is composed by a half-circle with radius 1 and a circular segment combining gradient direction with gradient magnitude. Designing ψ_x in such manner allows straightforward reprojection for the vector field after the gradient ascent update (15).

Fig. 1. Construction of Wulff shape for Finsler Active Contours

5.3 Learned Shapes

The prostate learned shapes are designed by an expert using manual segmentation. In the figure 2, the object shape is segmented using 20 learned shapes and the segmentation is done using statistical and geometric shape prior.

$$\chi_{ref}^1, \chi_{ref}^2, \dots, \chi_{ref}^{20}$$

Fig. 2. Construction of Wulff shape for Finsler Active Contours

In the next section we introduced our interactive user term used to suitability segmentation method.

5.4 Segmentation Results

To quantify the accuracy of the segmentation, we measured the Dice Similarity Coefficient (DSC) between the manually segmented prostate and our segmentation method. We provide not only qualitative results (Fig. 3), but also give quantitative results in the form of the DSC to illustrate the viability of the proposed method in the context of prostate segmentation (see Table. 1).

Table 1. Quantitative evaluation of the segmentation

Patient no	DSC of FAC	DSC of FAC TV
Patient no 01	80,02%	85,63%
Patient no 02	79,1%	82,69%
Patient no 03	75,06%	79,23%
Patient no 04	77,21%	79,75%
Patient no 05	75,7%	79,30%

To quantify the accuracy of the segmentation, we measured the overlap between the segmented prostate areas defined by manual segmentation and our segmentation method.

Fig. 3. Segmentation by TV FAC. In blue color traditional FAC, red color segmentation results of our method and in yellow color manual segmentation.

6 Conclusion

We developed a fast globally segmentation based convex Finsler active contours model for binary segmentation in TV framework incorporating statistical and shape prior knowledge. The position and orientation are dependent on prior for the boundary segmentation in Finsler metrics. Finsler active contours provide an alternative approach to integrating image-based priors on the location and orientation of the traditional boundary descriptor. Future work will address extending other classes of energies [9] that can be optimized in TV framework.

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