Blood Flow Computation in Phase-Contrast MRI by Minimal Paths in Anisotropic Media

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Abstract. In this paper, anisotropic Fast Marching is employed to compute blood flow trajectories as minimal paths in 3D phase-contrast MRI images. Uncertainty in the estimated blood flow vectors is incorporated in a tensor which is used as metric for the anisotropic Fast Marching. A flow connectivity distribution is computed simultaneously to the Fast Marching. Based on the connectivity distribution the most likely flow trajectories can be identified. Results are presented for several PC MRI data sets and the capability of the method to indicate uncertainty of the flow trajectories is shown.

Keywords: blood flow computation, uncertainty, minimal path, anisotropic Fast Marching

1 Introduction

Fast Marching (FM) methods have been widely used for segmentation and analysis of medical images [9]. The FM method is a numerical scheme for solving the so-called Eikonal differential equation, whose solution can be interpreted as the monotone evolution of an interface on a domain with a spatially varying speed function. A main application of FM methods is to find a minimal path that describes the fastest way to travel between two points considering the speed function. The minimal path can for example represent a segmentation contour or the centerline of a vessel [2,4]. A generalization of the original FM method is obtained by considering *anisotropic* speed functions, meaning that the speed at each point is direction dependent. For example, for vessel segmentation, consider a speed function that propagates the interface faster along the vessel direction than orthogonal to it [1]. An anisotropic FM method solves the Eikonal equation when the speed anisotropy is modeled as a metric tensor, i.e., in 3D, the anisotropy has the form of an ellipsoid. Anisotropic FM has been applied in medical imaging for vessel segmentation [1], brain connectivity analysis [8], and tumor growth modeling [7].

In this contribution, a novel application of anisotropic FM is investigated, namely the computation of blood flow trajectories in Phase-Contrast (PC) MRI images. PC MRI is able to measure blood flow and the acquired images can be reconstructed into vector fields in which each voxel contains a vector that describes the local blood flow velocity. The clinical interest in applying PC MRI for assessing stenoses, aneurysms and heart valves, and for surgical planning in congenital heart disease, is increasing. Traditionally, streamline methods such as Runge-Kutta's are used for tracing blood flow trajectories in the PC MRI data [3], but these methods ignore the intrinsic uncertainty due to noise in the measured vector field. Recently, a sequential Monte Carlo method was proposed for investigating the *probability distribution* of flow trajectories [5]. In this work, an alternative approach to incorporate the measurement uncertainty is proposed in the framework of anisotropic minimal paths. Ideas from anisotropic Fast Marching applications in other fields, namely brain connectivity analysis [8] are adapted to the novel application to compute blood flow trajectories. As a result of the modeling of the problem as anisotropic minimal paths, any anisotropic Fast Marching method available can be applied to solve the problem.

2 Methods and Materials

2.1 Minimal Paths

Informally, a minimal path is the path along which one travels between two given points in the shortest time given a spatially varying speed function. Formally, let $\gamma(s) : \mathbb{R}_0^+ \rightarrow \mathbb{R}^3$ be a path connecting two points in 3D space, s be a positive running variable. Furthermore, let $\mathbf{M}(\mathbf{x})$ denote a metric tensor at position $\mathbf{x}$ in 3D space represented by a symmetric positive definite matrix that defines a spatially varying traveling cost, i.e., the inverse of the speed. Hence, the speed along γ is dependent both on the spatial position $\gamma(s)$ as well as the tangent direction $\gamma'(s)$. The local cost of traveling at a certain position $\gamma(s)$ along the path can be written

$$F(\gamma(s), \gamma'(s)) = \sqrt{\gamma'(s)^T \mathbf{M}(\gamma(s)) \gamma'(s)} \quad (1)$$

and the total cost of a path connecting two points $\mathbf{a}$ and $\mathbf{b}$ is

$$J(\gamma) = \int_{s_{\mathbf{a}}=0}^{s_{\mathbf{b}}} F(\gamma(s), \gamma'(s)) \, ds = \int_{s_{\mathbf{a}}=0}^{s_{\mathbf{b}}} \sqrt{\gamma'(s)^T \mathbf{M}(\gamma(s)) \gamma'(s)} \, ds. \quad (2)$$

Considering all paths $\Gamma_{\mathbf{a},\mathbf{b}}$ between $\mathbf{a}$ and $\mathbf{b}$, the minimal path is the one with the minimum cost. Minimal paths from all points $\forall \mathbf{x} \in \Omega \subseteq \mathbb{R}^3$ to a target region $\partial\Omega$ can be described using the so-called value function $U(\mathbf{x})$

$$U(\mathbf{x}) = \begin{cases} \min_{\gamma \in \Gamma_{\mathbf{x}, \partial\Omega}} J(\gamma) & \mathbf{x} \in \Omega \\ 0 & \mathbf{x} \in \partial\Omega. \end{cases} \quad (3)$$

Along a minimal path, the values of U are strictly decreasing as we move closer in time to the target region, i.e., for a point $\mathbf{g} = \gamma(s_g)$ on the minimal path between the target region and a point $\mathbf{x} = \gamma(s_x)$, it holds that $U(\mathbf{g}) < U(\mathbf{x})$. More precisely,

$$U(\mathbf{x}) = U(\mathbf{g}) + \int_{s_g}^{s_x} F(\gamma(s), \gamma'(s)) ds, \quad (4)$$

and forms the basis for *Dijkstra's shortest path* and FM algorithms. Furthermore, it can be shown that the value function U satisfies the static Hamilton-Jacobi equation [6]

$$\nabla U(\mathbf{x})^T \mathbf{M}(\mathbf{x})^{-1} \nabla U(\mathbf{x}) = 1, \quad (5)$$

also known as the anisotropic Eikonal equation, and that the tangents of the minimal paths satisfy $\gamma' \propto \mathbf{M}^{-1} \nabla U$. Thus, we are able to solve the problem of computing the minimal paths by first solving the partial differential equation in Eq. 5 with boundary condition $U(\mathbf{x}) = 0$ on the target region $\partial\Omega$ to find $U(\mathbf{x})$, and then reconstructing the minimal paths by backtracking using standard numerical methods like Euler's, Heun's or Runge-Kutta's. It can further be noted that for an isotropic metric $\mathbf{M}(\mathbf{x}) = f(\mathbf{x})\mathbf{I}$, Eq. 5 reduces to the well known Eikonal equation $\|\nabla U(\mathbf{x})\| = \frac{1}{\sqrt{f(\mathbf{x})}}$. The numerical scheme to compute the solution of Eq. 5 with given boundary conditions $U(\mathbf{x}) = 0$ is a novel anisotropic Fast Marching solver that we developed. The description of the solver is beyond the scope of this contribution and will be done in a future publication. Generally, any anisotropic Fast Marching solver available is applicable to solve the stated problem.

2.2 Modeling Blood Flow Trajectories as Minimal Paths

Anisotropic FM has previously been applied for computing brain connectivity maps in Diffusion Tensor Imaging [8]. In this work, blood flow computation and flow connectivity mapping in PC MRI images using anisotropic FM is proposed. From the PC MRI images, a 3D vector field of blood flow velocities can be reconstructed, see Fig. 2(a). Traditionally, vector field visualization methods such as streamlines computed using Euler, Heun or Runge-Kutta schemes are used to compute blood flow trajectories [3]. However, these methods do not consider the uncertainty in the measured flow vectors due to image noise and the streamlines may therefore give a false impression of precision. In [5], it was shown that each component $\tilde{v}$ of a measured flow vector is well approximated by a Gaussian distribution around a true velocity v :

$$\tilde{v} \in N(v, \sigma) \text{ with } \sigma = \frac{v_{enc}}{\pi} \frac{\sqrt{2}}{\text{SNR}}, \quad (6)$$

where v_{enc} is a known velocity encoding sequence parameter and SNR the signal-to-noise-ratio of the PC MRI images within vessels. Typical values for the parameters are $v_{enc} = 1500$ mm/s and an SNR of 10, resulting in a standard deviation of about $\sigma = 70$ mm/s. As a reference, normal flow velocities range

between 0 mm/s and 1500 mm/s at peak systole in the aorta, and in smaller vessels, the peak flow rate is lower. Each PC MRI flow vector consists of an v_x , v_y and v_z component, $\mathbf{v} = [v_x, v_y, v_z]^T$, so that a flow vector can be considered as drawn from a multivariate Gaussian distribution with independent components identically distributed according to the variance parameter σ .

To capture uncertainty in flow trajectories traced in PC MRI data, instead of considering deterministic vectors, in this work we view the data as a field of 3-dimensional multivariate Gaussian distributions. We incorporate the uncertainty of the measured velocities into a metric tensor and then solve the minimal path problem in the ensuing metric space to find flow trajectories. The inverse of the metric tensor is constructed as

$$\mathbf{M}^{-1} = \mathbf{v}\mathbf{v}^T + \sigma\mathbf{I}, \quad (7)$$

see Fig. 1 for an schematic illustration and Fig. 2(b) for a tensor field constructed based on a PC MRI data set of an aorta. Figure 2(c) shows the value function

Fig. 1. Construction of a metric tensor from a given PC-MRI velocity vector

U obtained with anisotropic FM from a start point in the aortic arch. From every point in the aorta, a minimal path that represents the flow trajectory to that point can be found via a backtracking on U . To differentiate more likely paths from less likely paths, an approach similar to the one proposed in [8] in the context of DTI is adopted, in which the alignment of the minimal path tangent with the major eigenvector of the metric tensor $\mathbf{M}$ is evaluated. In the current case, the alignment can be measured by $\mathcal{C}(\gamma(s), \gamma'(s)) = \|\mathbf{v}(\gamma(s))^T \gamma'(s)\|$ and the mean alignment $\mu(\mathbf{x})$ along the minimal path to the point $\mathbf{x}$ is

$$\mu(\mathbf{x}) = \frac{1}{U(\mathbf{x})} \int_0^{s_{\mathbf{x}}=U(\mathbf{x})} \mathcal{C}(\gamma(s), \gamma'(s)) ds. \quad (8)$$

Figure 2(d) shows $\mu(\mathbf{x})$, which can be interpreted as a flow connectivity map. Figure 2(e) and 2(f) show the minimal paths with the highest connectivities, representing the most likely paths under the influence of noise in the PC MRI images.

Note that the PC MRI flow vectors have a direction whereas the constructed metric tensor in Eq. 7 has not. In a flow application, it may be of interest to trace trajectories either forwards or backwards. To this end, the direction of the FM propagation needs to be restricted as follows: After computation of the minimal path tangent during the FM update at a point, the propagation is restricted to points for which the dot product with the flow direction is strictly positive or negative respectively.

2.3 Data

Three *PC MRI data* sets are used to demonstrate the blood flow computation using anisotropic FM: a data set covering the aorta of a healthy subject (P1), see Fig. 2(a), a patient with an aortic aneurysm (P2), see Fig. 4, and a patient with an aneurysm in the carotid artery (P3), see Fig. 5. The aorta data sets have a spatial resolution of about $1.7 \times 1.7 \times 3\text{mm}^3$, and the carotid artery data set has a resolution of $0.86 \times 0.86 \times 1.4\text{mm}^3$.

Fig. 2. Flow trajectories as minimal paths: (a) PC MRI vector field inside the aorta, (b) the constructed tensor field, (c) level-sets of $U(\mathbf{x})$, (d) the connectivity distribution $\mu(\mathbf{x})$, (e) the 20% most likely trajectories superimposed on the connectivity distribution, and (f) the 20% most likely trajectories inside the flow field

3 Results

The experiments were carried out on a PC with 2.4 GHz CPU and 2 GB memory. The different steps for calculating flow connectivity maps and flow trajectories as minimal paths based on PC MRI data are shown in Fig. 2 for the normal aorta data set P1. From a start point, the value function $U(\mathbf{x})$ and connectivity map $\mu(\mathbf{x})$ as described in Sec. 2.2 are calculated. The flow connectivity map may be interpreted as a distribution of possible flow trajectories under the influence of image noise. Figure 3 shows a comparison of the FM-based connectivity map with the probabilistic connectivity map computed with the method described in [5] and for which the authors kindly provided an implementation. The flow field is generated based on a sinus curve that is superimposed in the figures. The probabilistic connectivity map counts the number of pathlines passing through

Fig. 3. Visual comparison of (a) the probabilistic connectivity map [5] and (b) the FM-based connectivity map. Superimposed in both images is the sinus curve, based on which the artificial flow field was generated, and the flow field itself. The source point is located at the bottom of the field and is marked by a sphere.

Fig. 4. Blood particle trajectories in an aorta with an aortic aneurysm: (a) Flow field and connectivity distribution, (b) blood flow trajectories, and (c) a close-up. The trajectories are color coded based on the connectivity: (green/brightest) 0.8-1.0, (yellow) 0.7-0.8, and (red/darkest) 0.6-0.7.

Fig. 5. Blood flow trajectories in a carotid artery with an aneurysm: (a) flow field and connectivity distribution and (b) blood flow trajectories. Trajectories are color coded based on the connectivity: (green/brightest) 0.8-1.0, (yellow) 0.7-0.8, and (red/darkest) 0.6-0.7.

each point of the map. Both methods show a widening effect going further away from the source point that reflects uncertainty due to image noise. The major difference between both results is that the FM-based connectivity map gives the mean value of connectivity along a path, thus resulting in an almost constant value along the path, while the probabilistic connectivity map has decreasing values along a path. Figure 4 shows a visualization of the trajectories computed for data set P2 with an aortic aneurysm. The trajectories are color coded according to how likely they are: green (brightest) means a connectivity between 0.8 and 1.0, yellow between 0.7 and 0.8, and red (darkest) between 0.6 to 0.7. Similar flow paths for the carotid artery data set P3 are shown in Fig. 5. The computations for the above examples took about 10 seconds using our developed FM method.

4 Discussion and Future Work

In this contribution, a novel modeling of blood flow trajectories as minimal paths is presented. Knowing the measurement uncertainty is important in all science and decision making, including medical imaging. A visualization of blood flow as minimal paths and flow connectivity maps gives a notion of the inherent uncertainty in the PC MRI data which is not provided by traditional streamlining techniques. The focus in this contribution is to present the methodological basis for blood flow estimation based on minimal paths. Currently, the main application is uncertainty visualization for flow patterns, e.g., for surgical planning in congenital heart disease. There are several possible extensions for future work, the most immediate one being to implement the method for 4D data as the PC MRI images frequently also have a temporal dimension. Furthermore, more experimental work, using for example physical flow phantoms or flow simulations, to further validate the method is required. However, as the flow connectivity maps obtained with the anisotropic FM method do not correspond to a physical

quantity, a strict validation may prove difficult. Similar problems are encountered in other applications, such as in the reconstruction of white matter fibers based on Diffusion Tensor Imaging, where both stochastic tracking and anisotropic FM approaches are utilized, and in which validation of the corresponding connectivity maps is still an open issue. A different avenue of work is to investigate if the flow connectivity maps can be used as an imaging biomarker for disease, for example, to investigate via group studies if connectivity-based biomarkers have any discriminative power to separate patients from healthy controls. Example applications for this include stroke and embolization pathways, flow patterns related to aneurysm rupture, and connectivities through leaking heart valves.

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