Piercing Quasi-Rectangles: On a Problem of Danzer and Rogers

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Abstract. It is an old problem of Danzer and Rogers to decide whether it is possible arrange $O(\frac{1}{\varepsilon})$ points in the unit square so that every rectangle of area ε contains at least one of them. We show that the answer to this question is in the negative if we slightly relax the notion of rectangles, as follows. Let δ be a fixed small positive number. A *quasi-rectangle* is a region swept out by a continuously moving segment s , with no rotation, so that throughout the motion the angle between the trajectory of the center of s and its normal vector remains at most δ . We show that the smallest number of points needed to pierce all quasi-rectangles of area ε is $\Theta(\frac{1}{\varepsilon} \log \frac{1}{\varepsilon})$.

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