Measuring Rectangularity Using GR-Signature

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Abstract. Object recognition often operates by making decisions based on the values of several shape properties measured from an image of the object. In this paper, we propose a new exploitation of the Radon Transform using the gradient measurement to generate a new signature (GR-signature) which provides global information of a binary shape regardless its form. We also develop a new method for measuring the rectangularity based on GR-signature. This original approach looks very promising and has several useful properties that keep fundamental geometrical transformations like scale, rotation and translation.

Keywords: Rectangularity, Shape descriptor, Radon Transform, gradient measurement.

1 Introduction

Object recognition is one of the main issues described in most of computer vision applications. This object identification is made in the forms analysis phase which generally occurs after a step of image segmentation [14]. The shape analysis is used in several application areas such as medicine to detect anomalies, security to identify individuals, computer-aided design and computer-aided manufacturing process to compare design parts or mechanical objects design etc. Discrimination of objects is based on their appearance: texture, color and shape. The shape is obviously a powerful tool to describe and differentiate objects since it is a discriminating characteristic of the object. The form according to the mathematician and statistician David George Kendall is defined as [7]: “The shape is the set of geometric information that remains when location, scale and rotational effects are filtered from an object”.

Once the shapes are extracted from the image, they must be simplified before a comparison can be made. The simplified representation of forms is often called the shape descriptor or signature. This is an abstraction of a structured model that captures most of the important information of the form. These simplified

representations are easier to handle, store and compare than the forms directly. The shape may not be entirely reconstructable from the descriptors, but the descriptors for different shapes should be different enough that the shapes can be discriminated [2]. So instead of directly comparing two models, both models are compared by comparing their shape descriptors.

Although some researches has been done in terms of circularity, ellipticity and rectangularity. However many textbooks and surveys do not consider this last as a measure of shape [1, 2]. Moreover, rectangularity can be an advantageous characteristic to extract useful tasks such as filtering of images to find parts potential road in a satellite image.

Beside there are many attempts to measure the rectangularity. The standard method, the Minimum Bounding Rectangle method (MBR), responds unequally to protrusions and indentations, and is sensitive to noise (especially protrusions). Also the research of P.Rosin in [8] develops three methods:

- The agreement method (R_A): breaks down for compact regions and is prone to errors due to inaccuracies in the perimeter estimation. The errors depend on both the region's orientation and resolution.
- The moment-based method (R_M): can respond to other shapes such as rectangles if they have a similar ratio of moments. For compact shapes (e.g. the near square on the bottom row), the orientation estimation is sensitive to noise, which can lead to incorrect rectangularity estimation.
- The discrepancy method (R_D): uses moments to estimate the rectangle fit and is similarly prone to poor orientation estimation for compact shapes.

In his research, Rosin proves that the bounding rectangle (MBR) and discrepancy method (R_D) are the best. Moreover, in [12] the Radon Transform (RT) is used to calculate the R-signature (i.e. the square of the RT) which just characterizes very well the shape of the filled and not emptied object (i.e. object contour only). In this approach, the R-signature of an object is compared to a theoretic R-signature which represents a perfect rectangle and calculates the similarity between them.

In this study a simple but effective method is proposed and it utilizes the RT and the gradient to make a signature (we called the GR-signature). With the help of this signature and a metric of measuring the rectangularity which we propose, we calculate the percentage of the rectangularity of a given object.

This paper is outlined as follows. After we recall the definition and the properties of the RT and the gradient in sections 2 and 3, we describe our GR-signature in section 4. Our new metric is described in section 5 and evaluated on synthetic data in section 6 to determine how well it surmounts the imperfections of previous approaches by making comparison with them result. Finally, we summarize our research and conclude the paper in section 7.

2 Radon Transform

To be useful, a shape recognition framework should allow explicit invariance under the operations of translation, rotation, and scaling. For this reasons, we have decided

to employ the Radon transform. By definition, the RT [9] of an image is determined by a set of projections of the image along lines taken at different angles. For discrete binary image data, each non-zero image point is projected into a Radon matrix. Let $f(x, y)$ be an image. Its Radon transform is defined by [4]:

$$R(\rho, \theta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) \delta(\rho - x \cos \theta - y \sin \theta) dx dy. \quad (1)$$

Where $\delta(\cdot)$ is the Dirac function, $\theta \in [0, \pi]$ and $\rho \in [-\infty, \infty]$.

To represent an image, the Radon Transform takes multiple, parallel-beam projections of the image from different angles by rotating the source around the center of the image. Fig.1 shows a single projection at a specified rotation angle. For example, the line integral of $f(x, y)$ in the vertical direction is the projection of $f(x, y)$ onto the x -axis; the line integral in the horizontal direction is the projection of $f(x, y)$ onto the y -axis [13]. The RT is robust to noise, provided with fast algorithms, and it projects a two-dimensional function into one-dimensional function.

Fig. 1. Parallel-beam projection at rotation angle θ

The Radon transform has several useful properties. Some of them are relevant for shape representation [13]:

- Periodicity $T_{Rf}(\rho, \theta) = T_{Rf}(\rho, \theta + 2k\pi)$, for any integer k . The period is 2π .
- Symmetry $T_{Rf}(\rho, \theta) = T_{Rf}(-\rho, \theta \pm \pi)$.
- Translation of a vector $\vec{u} = (x_0, y_0)$: $T_{Rf}(\rho - x_0 \cos \theta - y_0 \sin \theta, \theta)$ A translation of f results in the shift of its transform in the variable ρ by a distance equal to the projection of the translation vector on the line $\rho = x_0 \cos \theta + y_0 \sin \theta$.
- Rotation by θ_0 : $T_{Rf}(\rho, \theta + \theta_0)$. A rotation of the image by an angle θ_0 implies a shift of the Radon transform in the variable θ .
- Scaling of α : $\frac{1}{\alpha} T_{Rf}(\alpha \rho, \theta)$. A scaling of f results in a scaling of both the ρ coordinates and the amplitude of the transform.

3 Gradient

To exploit the RT and like it contains several peaks (loci of concentration), we choose to use the gradient to locate those peaks. In physics, the gradient is a vector quantity that indicates how a physical quantity varies in space. In image processing, the gradient profile prior is a parametric distribution describing the shape and the sharpness of the gradient profiles in natural image [6]. The definition of the gradient vector $\vec{\nabla}f$ of the fonction $f(x, y)$ is:

$$\vec{\nabla}f = \begin{pmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{pmatrix} \quad (2)$$

The direction of $\vec{\nabla}f$ is the orientation in which the directional derivative has the largest value and $|\vec{\nabla}f|$ is the value of that directional derivative [10]. The gradient profile is a 1-D profile along the gradient direction of the zero-crossing pixel in the image [6]. We use it to find the modes of density in a feature space. The modes are located among the zeros of gradient ($\vec{\nabla}f(x, y) = 0$).

4 GR-Signature

In our research a new exploitation of the RT is proposed. Our method differs from previous 2D RT applications [10, 11]. In these approaches, the encoded information is contour-based allowing only the detection of specific primitives like straight line.

The context of our application is different from previous works. We provide global information of binary shape, whatever its form is, by generating a new signature (GR-signature). In fact, the operating principle of the RT is the summation of the intensity of pixels along the same line for each projection. To obtain an outcome that reflects only the shape, the object must have a unique color. Otherwise the result of RT reflects the brightness of the object in addition to its shape. For that, we will use binary images. Moreover, we do not need any pretreatment like computing the centroid of shapes under consideration when using Fourier descriptors [11].

In the discrete case, fast and accurate algorithms [5] haven been proposed to transform the continuous plane of Radon into an accumulator matrix $R: N_\rho * N_\theta$ cells described by the sinogram in Fig. 2.b. From this 2D accumulator we generate a discrete 1D GR-signature by calculating the gradient.

$$\frac{\partial R}{\partial \theta} \quad (3)$$

The modulus of the gradient vector represents the surface slope Radon point calculation. The local presence of a high modulus indicates a high variation of the coefficients around this point. Where we fix one line of a matrix, the gradient will locate the high coefficients variation in this line. We want to catch the variation in

each θ projection, for that the θ must be the first dimension of the matrix. But in fact θ is the second dimension and the first one is ρ . So we reflect R over its main diagonal (which runs top-left to bottom-right) to obtain R' (the transpose of a matrix R). After the transposing we apply the formula (3) on R' . We obtain the result shown in Fig. 2.c. The graph is very dense and contains an enormous amount of information. Hence, we choose to take only the external shell (i.e. contour) of the gradient result and this is the GR-signature (Fig. 2.d).

In addition, the GR-signature proves to be an excellent measure of shape and it gives very good results with full or empty symbols. This is caused by the fact that the GR-signature is based on the corners of the shape.

Fig. 2. Definition of the GR-signature: a.shape, b.Radon space, c.gradient result calculation, d.GR-signature and e.peaks selection

5 Rectangularity Measure (R_{GR})

In this phase, we will use our GR-signature to define the percentage of rectangularity of every given shape. For this objective and before we come to our metric of rectangularity verification, we study the GR-signature of an ideal rectangle. We find that the two sides (positive and negative) of the GR-signature are symmetric. Also the sum of the absolute values of each opposite peaks is equal to one of the rectangle dimension (i.e. the sum of the two high peaks is equal to the length of rectangle and the sum of the two low peaks is equal to the width). Furthermore, the difference in θ scale between the high and the low peaks of the GR-signature is 90° in each side, which represent the angle between the two perpendicular bisectors of the rectangle. We recall that the two rectangle bisectors are the lines perpendicular to the length and width segments in there middle as shown in Fig. 3.

Fig. 3. The two bisectors of a rectangle

After this pretreatment phase, we create our metric of rectangularity which is a combination of two different measures: Angle measurement and Amplitude measurement. To proceed to the two measurements, and since they depend of the number of shape's corners, a phase of detecting peaks in the GR-signature is needed. We treat the signature side by side identically. First, we extract its extrema (maxima in the positive side and minima in the negative) and sort them according to its amplitude in an ascend order. Depending on the number of the rectangular shape's corners, we choose four extrema. We locate the highest peak in the GR-signature. The second extremum is located in the same side taking into account that it is far from the first by 90° with margin of tolerance of $\pm 5^\circ$. The third and fourth peaks are located in the same way symmetrically.

The Angle measurement is described by formula (4):

$$\text{Angle_measurement} = \frac{|90 - (\theta_{\text{low}} + \theta_{\text{high}})|}{90} \quad (4)$$

Where θ_{low} is the difference between the two low peaks and θ_{high} is the difference between the two high peaks. The sum of these two differences represents the angle rate error. The expression $(90 - |\theta_{\text{max}} + \theta_{\text{min}}|)$ represents the angle between the two bisectors of the rectangle. To unify it, we devise it by 90. A value of one is produced for an exact rectangle, while decreasing values correspond to less rectangular figures.

The Amplitude measurement is calculated with the help of the amplitudes of each selected peaks. At first we normalize these amplitudes to be in the range of [0 1] using formula (5):

$$A_i = \frac{A_i - A_{\min}}{A_{\max} - A_{\min}} \quad (5)$$

Where $A_{\max}$ is the greatest amplitude and $A_{\min}$ is the smallest one. A_i ($i \in [1..4]$) is the amplitude of each one of the four peaks. After that we sort these amplitudes in an ascend order and then we calculate the difference of the first two devised by the difference of the last two. This measurement is described by formula (6) which peaks at one for perfect rectangles:

$$\text{Amplitude_measurement} = 1 - \left| \frac{A_2 - A_1}{A_4 - A_3} \right| \quad (6)$$

Where A_i ($i \in [1..4]$) is the amplitudes of the four peaks.

After the calculation of the two measurements (Angle measurement and Amplitude measurement) and since percentage of rectangularity depends equally on both of them, we define the rectangularity measure (R_{GR}) as the average of the two measurements which peaks at one for perfect rectangles:

$$R_{GR} = \frac{\text{Angle_measurement} + \text{Amplitude_measurement}}{2} \tag{7}$$

6 Evaluation

We evaluate the R_{GR} measure by applying it to some synthetic shapes. This enables us to track the rectangularity values, as we continuously change the shapes, to prove that the GR-signature conserves the several useful properties of Radon and well behaves with noised figures. These evaluations are illustrated in Table 1.

Table 1. Properties of the GR-signature

Property	Full shape	Empty shape	Translation	Rotation
Shape
GR-signature
R_{GR}	1.0000	1.0000	1.0000	0.9868

Property	Scaling	Gaussian noise	Protrusions and Indentations	Boundary noise
Shape
GR-signature
R_{GR}	0.9994	0.9773	0.9926	0.9775

We conclude that full or empty shape don't affect the rectangularity measurement. This is of crucial importance in object recognition, because each object must have a unique representation either is full or not. Our descriptor is invariant under geometrical transformations (translation, rotation and scaling). When we applied our rectangularity measurement on geometric transformed shapes we obtain very good results (the means of R_{GR} over 0.98) which provide the stability of our metric. This R_{GR} measurement is robust to the noise also. We applied Gaussian noise, Boundary noise and protrusions and

indentations on a shape but the measurement still have good values of measurement (the means of R_{GR} over 0.97). This refers to the ability of the representation to express the basic features of a shape and to abstract from detail. So the R_{GR} appears a good rectangularity measure and the GR-signature looks a crucial descriptor.

We evaluate our descriptor by applying it to an images' database and compare its figures classification result to classification [5] proposed by Paul Rosin for one hand and that based on the R-signature as looks in figures Fig. 4, Fig. 5 and Fig. 6.

Fig. 4. The classification of the images database using the rectangularity measurement proposed by Paul Rosin

Fig. 5. The classification of the images database using the rectangularity measurement based on the R-signature

Fig. 6. The classification of the images database using the rectangularity measurement based on our GR-signature

The analysis of the GR-signature arrangement of the images' database reveals that on viewpoint discrimination of the rectangular shape; our descriptor looks well since all the 18th first figures have a rectangular form. A small comparison between the results of classification according to Rosin, R-signature and GR-signature is illustrated in Table 2.

Table 2. Comparison between the results of classification presented in fig. 4-6

Images' database	Rosin rank	R-signature rank	GR-signature rank
Face 1	9	10	44
Oval shape	12	9	49
Face 2	13	21	56
Tree	14	18	27
Guitar	16	26	45
Snow crystal	18	19	30
Maple leaf	21	33	50
Africa map	26	20	47
Sword	30	23	4
Noised rectangle 1	25	16	9
Noised rectangle 2	55	53	10
Noised rectangle 3	23	17	12
Noised rectangle 4	24	27	13
Noised rectangle 5	40	48	21
Noised rectangle 6	35	36	22
Noised rectangle 7	41	37	24

Table 2 reflects that our descriptor is able to discriminate the rectangular shapes from others forms since it improves the rank of rectangular shapes and disapproves that of other forms compared to other classifications.

7 Conclusions

Our paper shows that the GR-signature can be of great interest to differentiate between graphical symbols and also in the measure of rectangularity. The computation of such a feature is fast (low complexity). Moreover, it overcomes the problems of other approaches. A weakness of using the MBR is that it is very sensitive to protrusions from the region [8]. But using our metric, protrusions and indentations have no considered effect on the rectangularity measurement. The rectangularity value of the rectangular shape with protrusions and indentations illustrate in Table 1 using our metric is 0.9926. It is a very good value despite the protrusions and indentations. Its signature clearly shows the rectangular form and picks are well chosen (Table 1).

What makes this metric better in comparison with Rosin methods and the standard method (MBR) is the fact that the proprieties of GR-signature inherited from the Radon transform overcame the problems of geometrical transformations. As regards the mismatch problem that appears in R_M , it is solved by the similarity pretreatment preceding our rectangularity measurements. And what differentiate our method from

the R-signature is that we found a better exploitation of the Radon space which allowed revealing useful properties (Angle, Amplitude measurements) and not only matching two signatures. This gives more accurate result in the rectangularity measurements.

Of course, the results presented in this paper must still be considered as preliminary. We need to process much larger databases of graphical symbols to assess the discriminating power and the robustness of the method.

References

1. Ballard, D.H., Brown, C.M.: Computer Vision. Prentice Hall, Englewood Cliffs (1982)
2. Morse, B.S.: Lecture 9: Shape Description (Regions). Brigham Young University (1998–2000)
3. Campbell, L., MacKinlay: The Econometrics of Financial Markets. Princeton University Press, NJ (1996)
4. Deans, S.R.: Applications of the Radon Transform. Wiley Inter-science Publications, New York (1983)
5. Girardeau-Montaut Daniel, application de la transformé de Radon à l'identification de symboles graphiques, DEA de Institut National Polytechnique de Lorraine (2002).
6. Sun, J., Sun, J., Xu, Z., Shum, H.-Y.: Image Super-Resolution using Gradient Profile Prior. In: IEEE Conference on Computer Vision and Pattern Recognition, Xi'an Jiaotong University, Microsoft Research Asia Beijing, P. R. China (2008)
7. Kendall, D.G.: Shape Manifolds, Procrustean Metrics, and Complex Projective Spaces. Bulletin of the London Mathematical Society (1984)
8. Rosin, P.: Measuring rectangularity. Machine Vision and Applications 11, 191–196 (1999)
9. Radon, J.: Ufiber die Bestimmung von Funktionen durch ihre Integral-werte langs gewisser Mannigfaltigkeiten, Berichte Sa-chsische Akademie der Wissenschaften, Leipzig. Math.-Phys. Kl. 69, 262–267 (1917)
10. Schey, H.M.: Div, Grad, Curl, and All That: An Informal Text on Vector Calculus, 3rd edn. W. W. Norton, New York (1997)
11. Tabbone, Ramos Terrades, O., Barrat, S.: Histogram of Radon Transform. In: A useful descriptor for shape retrieval of 19th International Conference on Pattern Recognition - ICPR, S University of Nancy, LORIA (2008)
12. Tabbone, S., Wendling, L., Girardeau-Montaut, D.: Mesures de rectangularité et d'ellipticité à partir de la transformée de Radon. C.I.F.E.D. Hammamet, Tunisie (2002)
13. Jia-wen, W., Yang-jun, L.: MATLAB 7.0 Image Processing, pp. 190–191. National Defence Industry Publishing, Beijing (2006)
14. Naouai, M., Hamouda, A., Weber, C., Melki, N.: linear structure recognition based on image vectorization. In: International Conference on Imaging Theory and Applications, Algarve, Portugal, March 5-7 (2011)