Geometry and Topology from Point Cloud Data

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Many applications in science and engineering require modeling and analyzing a geometric shape or a function defined on it for scientific simulations, rapid prototyping, visualizations, and various other purposes. Often the geometry in question is only available through a set of sample points called a point cloud data. These point clouds can either be generated by scanning a physical object, or can be the output of a procedure that implicitly probes a hidden geometry, often a low dimensional manifold embedded in a relatively high dimensional Euclidean space.

In recent years, researchers in computational geometry and computational topology have studied the questions of extracting geometric and topological information from a hidden shape that an input point cloud presumably sample. This include, for example, (i) extracting a piecewise linear approximation (triangulation) of the shape, (ii) finding ranks of homology groups, (iii) finding homology cycles that are optimal with respect to the geometry, and (iv) various structures such as persistent homology and Reeb graphs related to a function defined on the sampled space.

In this talk we will present recent results in addressing the above problems. In all cases we will state sampling conditions under which an algorithm extracts the relevant information with a theoretical guarantee. For example, if the input point cloud samples a smooth surface with an inter-point distance sufficiently smaller than their distances to the medial axis, a subcomplex of the Delaunay triangulation of the input point set reconstructs the original surface both in terms of topology and geometry. This answers the question in (i) for three dimensions. Similarly, to address (iii) we show how one may compute a set of loops from an input point data which approximate a shortest basis of the one dimensional homology group of the sampled manifold embedded in an Euclidean space.

The talk will introduce the concepts from differential geometry, algebraic topology, computational geometry and computational topology which are necessary for explaining the results. The general tone of the talk will be of survey with occasional in depth analysis. Some of the algorithms which we have implemented will be presented with practical results.