Spectral Estimation of Digital Signals by the Orthogonal Kravchenko Wavelets $\{\widetilde{h_a(t)}\}$

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Abstract. In this article, the approach based on the orthogonal Kravchenko wavelets $\{\widetilde{h_a(t)}\}$ is proposed. There is shown that obtained structures have some advantages in comparison with spectral wave analysis of ultra wideband (UWB) signals that are widely used in the remote sensing. This approach based on application of wavelets as spectral kernels is considered in the problems of digital UWB signal processing. In communication theory, the signals are represented in the form of linear combination of elementary functions. Application of spectral analysis of UWB signals in basis of digital functions in comparison with the spectral harmonious analysis gives certain advantages which are defined by the physical nature of the signal representation. Optimal processing in spectral area in comparison with the time possesses has advantages on application of numerical algorithms. The physical characteristics and analysis of numerical experiment confirm efficiency of new wavelets in the spectral estimation and digital UWB signal processing.

Keywords: Atomic functions, Wavelets, Remote sensing, Digital ultra wide-band signal processing.

1 Introduction

Application of the spectral analysis of signals in basis of digital functions in comparison with the spectral harmonious offers certain advantages which are defined by the physical nature of representation of the signals [1-11] in the remote sensing problems [1, 2]. Signals can be set in the form of some linear combination of elementary functions [3-5]

$$s(t) = \sum_{k=0}^{N-1} C(k) \varphi(k, t), \quad (1)$$

where $\varphi(k, t)$ is an elementary function of number k , and N is quantity of the functions used in the decomposition. At approximation the generalized Fourier transformation of a kind

$$C(k) = \int_0^T s(t) \varphi(k, t) dt \quad (2)$$

provides the minimum value of mean-square error. Thus, there is signal decomposition on some basis, which in many problems of digital signal processing can not be orthogonal. The work purpose is consisted of justify the advantages of the new Kravchenko wavelets.

2 Atomic Functions $h_a(t)$ and Their Properties

Atomic functions [5-8], [12-17] $h_a(t)$ ($a > 1$) are finite decisions of the functional-differential equation

$$y'(t) = \frac{a^2}{2} (y(at+1) - y(at-1)), \quad (3)$$

where a is any real number. The basic properties of $h_a(t)$ are the following.

1°. Compact support: $\left[-\frac{1}{a-1}; \frac{1}{a-1}\right]$. 2°. $h_a(t) = \frac{a}{2}$ for $t \in \left[-\frac{a-2}{a(a-1)}; \frac{a-2}{a(a-1)}\right]$,

$a \geq 2$. 3°. Position of inflection $t = \left[\mp \frac{1}{a}, \frac{a}{4}\right]$. 4°. The Fourier transform of $h_a(t)$ looks like

$$\hat{h}_a(\omega) = \prod_{k=1}^{\infty} \text{sinc}\left(\frac{\omega}{a^k}\right). \quad (4)$$

In practical calculations, it is enough to limit the product (4) to a small number of terms, as they quickly aspire to unit with growth of k .

5°. Derivatives of $h_a(t)$ are expressed through shifts-compression of the function recurrently by means of a parity (3).

3 Constructing of the Orthogonal Kravchenko Wavelets

The orthogonal Kravchenko wavelets that is based on AF $h_a(t)$ and have smooth Fourier transform are proposed. It allows providing the best time localization in comparison with Kotelnikov-Shannon wavelet. Their construction [7-11] is carried out by means of the quadrature mirror filters $\hat{m}_0(\omega)$. For maintenance of orthogonality performance, the following conditions in transitive area are necessary

$$\left|\hat{m}_0(\omega)\right|^2 + \left|\hat{m}_0(\omega - 2\pi)\right|^2 = 2. \quad (5)$$

The Fourier transform of scaling function [7-11] is defined from the equation

$$\hat{\varphi}(\omega) = \frac{1}{\sqrt{2}} \hat{m}_0\left(\frac{\omega}{2}\right) \hat{\varphi}\left(\frac{\omega}{2}\right) \Leftrightarrow \hat{\varphi}(\omega) = \prod_{k=1}^{\infty} \frac{1}{\sqrt{2}} \hat{m}_0\left(\frac{\omega}{2^k}\right). \quad (6)$$

The Fourier transform of wavelet function can be written as

$$\hat{\psi}(\omega) = \frac{1}{\sqrt{2}} \hat{g}\left(\frac{\omega}{2}\right) \hat{\varphi}\left(\frac{\omega}{2}\right), \quad (7)$$

where $\hat{g}(\omega) = e^{-i\omega} \hat{m}_0(\omega + \pi)$. Considering the properties of $h_a(t)$ it is possible to simplify a construction of function $\hat{m}_0(\omega)$. For this purpose, we modify $h_a(t)$ according to following additional conditions.

6°. The support of function $t \in \left[-\frac{2}{3}\pi; \frac{2}{3}\pi\right]$. 7°. $\tilde{h}_a(t) = \sqrt{2}$ for $t \in \left[-\frac{1}{3}\pi; \frac{1}{3}\pi\right]$.

8°. For interface of the Fourier transforms $\tilde{h}_a\left(\frac{1}{2}\pi\right) = 1$.

Thus, as $\hat{m}_0(\omega)$ we take $\tilde{h}_a(t)$ with formal replacement of argument $t = \omega$. Properties 1st and 7th allow rewriting (7) in such a form

$$\hat{\varphi}(\omega) = \tilde{h}_a\left(\frac{\omega}{2}\right), \quad (8)$$

so long as $\tilde{h}_a\left(\frac{\omega}{2}\right) \neq 0$ only for $t \in \left[-\frac{4}{3}\pi; \frac{4}{3}\pi\right]$, where $\tilde{h}_a\left(\frac{\omega}{4}\right) = \sqrt{2}$. From 5th property, it follows that in transitive area $h_a(t)$ is symmetric relatively of inflection. Therefore, if as function $\tilde{h}_a(\omega)$ we take a square root of the $h_a(t)$, then the condition (4) is carried out. Hence, the function $\tilde{h}_a(\omega)$ is constructed as follows:

(a) replacement of variable $t = \frac{3}{2\pi}\omega$, (b) to satisfy the property 6th it should be $a \geq 4$,

(c) scaling of function and argument, and also taking a square root.

Thus, we obtain the function for the scaling equation with various velocities of increase and recession depending of parameter a

$$\tilde{h}_a(\omega) = \frac{2}{\sqrt{a}} \sqrt{h_a\left(\frac{2}{a\pi}\omega\right)}. \quad (9)$$

Finally, we can write the Fourier transforms for scaling and wavelet functions as follows:

$$\hat{\varphi}(\omega) = \frac{2}{\sqrt{a}} \sqrt{h_a\left(\frac{\omega}{a\pi}\right)}, \quad \hat{\psi}(\omega) = e^{-i\omega} \frac{2\sqrt{2}}{a} \sqrt{h_a\left(\frac{2}{a\pi}(\omega + \pi)\right) h_a\left(\frac{\omega}{a\pi}\right)}. \quad (10)$$

Fig. 1. The Kravchenko $\{\widehat{h_a(t)}\}$ scaling (dark line) and wavelet (light line) functions for $a = 4$ (a) and $a = 6$ (b)

As an example, the Kravchenko wavelets $\{\widehat{h_a(t)}\}$ and their spectrums are exposed in Fig.1 for $a = 4, 6$.

Energy of scaling and wavelet functions on N^{th} level is equal to energy of scaling function of $(N+1)^{\text{th}}$ level because

$$\left| \widehat{\varphi}(\omega) \right|^2 + \left| \widehat{\psi}(\omega) \right|^2 = \left| \widehat{m}_0 \left(\frac{\omega}{4} \right) \right|^2 = \left| \widehat{\varphi} \left(\frac{\omega}{2} \right) \right|^2. \quad (11)$$

Constructed functions are satisfying with all wavelet properties [6-10] and also they are orthogonal ones.

4 Physical Characteristics

To study the scaling and wavelet functions, we use the modified physical characteristics [5]: *wideband index* μ ; *central frequency of spectral density function (SDF)* f_0 ; *relative position of SDF maximum* (defined as $\gamma_1 = f_m / f_0$, where f_m is frequency of the main maximum); *relative position of SDF first zero* γ_2 ; *relative SDF width on level -3 dB* γ_3 ; *relative SDF width on level -6 dB* γ_4 ; *coherent amplification* defined

as $\gamma_7 = \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} |\varphi(t)| dt$; *equivalent noise band* γ_8 ; *maximum level of sidelobes* (in dB)

γ_9 . Mentioned physical characteristics of the Kravchenko wavelets and known ones for comparison are exposed in Table 1 below.

Table 1. Physical characteristics of the Kravchenko wavelets in comparison with known ones

a	γ_1	γ_2	γ_3	γ_4	γ_7	γ_8	γ_9	
Kravchenko wavelets $\left\{h_a(t)\right\}$								
$\varphi(t)$	4	0	0,676	0,563	0,608	0,704	4,158	-19,65
$\psi(t)$		0,850	0,665	0,688	0,836	0,810	4,032	-19,77
$\varphi(t)$	5	0	0,637	0,546	0,585	0,655	3,744	-17,36
$\psi(t)$		0,874	0,577	0,642	0,762	0,713	3,442	-17,30
$\varphi(t)$	6	0	0,619	0,534	0,568	0,629	3,505	-16,73
$\psi(t)$		0,876	0,528	0,614	0,710	0,671	3,230	-16,55
Kotelnikov-Shannon wavelet								
$\varphi(t)$	-	0	0,540	0,494	0,512	0,500	2,429	-11,73
$\psi(t)$	-	0,996	0,290	0,489	0,523	0,458	2,037	-10,56
Meyer wavelet								
$\varphi(t)$	-	0	0,654	0,523	0,557	0,586	3,430	-29,11
$\psi(t)$	-	0,863	0,648	0,574	0,671	0,565	3,196	-27,13
Daubechies 4 wavelet								
$\varphi(t)$	-	0	0,500	0,284	0,329	0,352	2,485	-10,92
$\psi(t)$	-	0,847	0,637	0,375	0,506	0,343	2,362	-10,63

5 Models of Ultra Wideband Signals

We consider of following models of ultra wideband (UWB) signals [3, 4], [5-7] in respect of remote sensing problems [1, 2]:

$$1. \ y_1(t) = -\left(H\left(\frac{t+0,5}{\tau}\right) - H\left(\frac{t-0,5}{\tau}\right)\right) \cdot \operatorname{sgn}(t),$$

$$2. \ y_2(t) = (-1)^n \cdot \sin\left(\pi n \frac{t}{\tau}\right) \cdot \exp\left(-\left|\frac{t}{\tau}\right|\right) \cdot \left(H\left(\frac{t}{\tau}+1\right) - H\left(\frac{t}{\tau}-1\right)\right),$$

$$3. \ y_3(t) = -\frac{2t}{\tau^2} \cdot \exp\left(-\left(\frac{t}{\tau}\right)^2\right), \quad 4. \ y_4(t) = -\frac{2}{\tau^2} \left(1 - \frac{2t^2}{\tau^2}\right) \cdot \exp\left(-\left(\frac{t}{\tau}\right)^2\right),$$

$$5. \ y_5(t) = \exp\left(-\left(\frac{t}{2\tau}\right)^2\right) n! \sum_{k=0}^{[n/2]} \left(-\frac{1}{2}\right)^k \frac{(t/\tau)^{n-2k}}{(n-2k)!k!},$$

where $H(t) = \begin{cases} 0, & t < 0, \\ 1, & t \geq 0. \end{cases}$ is the Heaviside function and $\text{sgn}(t)$ is the sign function.

Their physical characteristics are presented in Table 2.

Table 2. Model UWB signals and their physical characteristics

No.	Model UWB signals	μ	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	γ_8	γ_9
1	$y_1(t), \tau=1 \text{ s}$	1,54	0,91	1,04	0,85	1,11	-1,58	-1,41	1,00	1,00	-5,29
2	$y_1(t), \tau=1,5 \text{ s}$	1,50	0,92	1,00	0,83	1,11	-1,49	-1,34	0,67	0,67	-5,28
3	$y_2(t), n=1$	1,46	0,91	1,06	0,81	1,13	-11,47	-7,81	0,81	0,63	-12,99
4	$y_3(t), \tau=0,3 \text{ s}$	1,59	0,87	2,26	0,81	1,13	-37,58	-9,17	1,43	1,04	-49,15
5	$y_3(t), \tau=0,5 \text{ s}$	1,48	0,87	1,82	0,88	1,18	-8,80	-5,88	0,87	6,49	-18,87
6	$y_4(t), \tau=0,3 \text{ s}$	1,22	0,90	2,19	0,84	1,16	-32,05	-8,43	1,94	1,07	-43,44
7	$y_4(t), \tau=0,5 \text{ s}$	1,20	1,00	1,11	0,83	1,17	-6,20	-4,60	1,22	0,69	-14,08
8	$y_5(t), n=1, \tau=0,15 \text{ s}$	1,59	0,87	2,26	0,81	1,13	-37,58	-9,17	1,43	1,04	-49,15
9	$y_5(t), n=3, \tau=0,15 \text{ s}$	1,77	0,85	1,39	0,89	1,09	-25,77	-10,44	1,15	0,81	-34,63
10	$y_5(t), n=5, \tau=0,15 \text{ s}$	1,81	0,84	1,08	0,93	1,07	-17,01	-10,40	1,02	0,70	-26,63
11	$y_5(t), n=1, \tau=0,1 \text{ s}$	1,59	0,86	3,28	0,85	1,15	-96,77	-8,86	2,14	1,57	-108,3
12	$y_5(t), n=3, \tau=0,1 \text{ s}$	1,75	0,82	2,21	0,90	1,10	-81,50	-10,37	1,73	1,22	-92,79
13	$y_5(t), n=5, \tau=0,1 \text{ s}$	1,83	0,85	1,69	0,93	1,06	-68,61	-11,34	1,53	1,04	-77,97
14	$y_5(t), n=7, \tau=0,1 \text{ s}$	1,85	0,87	1,42	0,95	1,06	-57,50	-11,65	1,40	0,93	-67,70

6 Quality Functional of Wavelet-Basis Choice for Analysis of UWB Signals

It is proposed to apply the quality functional in the analysis of the UWB signals allowing optimal choice of basic wavelets functions in such a form

$$J(\psi, y) = \sum_{k=0}^N \left| \frac{\gamma_k^{\psi} - \gamma_k^y}{\gamma_k^y} \right|^2, \tag{12}$$

where $\psi(t)$ is wavelet function, $y(t)$ is analyzed signal, γ_k^{ψ} and γ_k^y are their physical characteristics, and N is quantity of compared parameters. Here, $\gamma_0 = \mu$ and $N=4$. Below, in Table 3, the values of quality functional of model UWB signal processing for the Kravchenko $\{\widetilde{h_2(t)}\}$ and Meyer wavelets are presented.

Table 3. The values of quality functional of model UWB signal processing for the Kravchenko $\{\widetilde{h_2(t)}\}$ and Meyer wavelets

No. of UWB signals realization	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Kravchenko wavelet	29,63	29,92	29,77	68,43	49,55	43,77	12,02	68,43	106,7	132,1	88,58	131,1	143,9	158,9
Meyer wavelet	35,15	35,63	35,57	71,48	52,28	45,20	15,60	71,48	113,6	142,3	92,44	137,7	151,6	167,9

Fig. 2. Discrete wavelet coefficients of 1st (a), 3th (b), 4th (c), and 7th (d) UWB signals realization for the Kravchenko $\{\widetilde{h_2(t)}\}$ wavelet

7 Conclusions

In this article, the application of the orthogonal Kravchenko wavelets $\{\widetilde{h_a(t)}\}$ for the digital UWB signal processing is proposed and justified. A number of the test signals and their wavelet transform examples are presented. A numerical experiment carried out and also an analysis of physical results have shown the advantages of novel wave-

let class in its applications for areas of remote sensing, radar, computer vision, radio physics, etc.

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