Second Derivative Approximation for Origin-Based Algorithm

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Abstract. Origin-based algorithm(OBA) for traffic assignment problem has been demonstrated preferable to the widely accepted and used Frank-Wolfe algorithm and path-based algorithms. Note that OBA can not avoid path enumeration of concerned network, which will lead to two disadvantages. One is the intensive memory requirements and the other is the difficulties in manipulating and storing paths. In order to solve these problems, we first propose the lower and upper bounds of the Hessian matrix, which can be calculated without path enumeration. Then use the lower bound of Hessian matrix to approximate the direction of the origin-based algorithm. According to our computational results, the modified origin-based algorithm(MOBA) improves the convergence performance greatly. The results indicate that MOBA can deliver better and more reliable convergence than OBA and saves much more CPU time especially when large-scale networks are being considered.

Keywords: Traffic Assignment, Origin-based Algorithm, Second Derivative, User Equilibrium.

1 Introduction

Traffic assignment problem (TAP) is the key problem for the long term planning and evaluation of urban transportation network. It is to assign the traffic flows of each OD (Origin-Destination) pair to links of urban transportation network in terms of certain principle, and count out all link traffic flows. There are many principles for TAP, but the most often used principles are the first and second principles of Wardrop, i.e. User Equilibrium principle and System Optimum principle [1]. In order to find a solution satisfying the User Equilibrium principle, Beckmann et al. proposed a convex mathematical programming [2][3] which became the main tool for solving traffic assignment problem. Since the work of Beckmann et al, many algorithms have been suggested to solve it. All of them can be broadly divided into three categories according to the solution space the algorithm resides, i.e., the link- path- or origin-based algorithms.

The link-based algorithms, including Frank-Wolfe (FW) algorithm given by LeBlanc et al. [4] and several modified FW algorithms [5][6][7][8][9], yield link-based solutions. The path-based algorithms, including disaggregated simplicial

decomposition (DSD) algorithm [10], gradient projection (GP) algorithm [11][12] and conjugate gradient projection (CGP) algorithm [13] etc., can determine both an aggregate link-based solution and an individual path-based solution which is not available for link-based algorithms. The origin-based algorithm(OBA)[14] can provide both an aggregate link-based solution and a constructive path-based solution. In the past, path-based algorithms and origin-based algorithm were not considered as a viable solution approach for large-scale network problems because of intensive memory requirements and difficulties in manipulating and storing paths. However, path-based algorithms and origin-based algorithm are now made possible due to the dramatic advances in computing technology.

Recent researches [13][14] have shown and demonstrated that the origin-based algorithm excels not only the FW algorithm but also some path-based algorithms such as DSD, GP and CGP. Although OBA was designed for solutions with high levels of accuracy, it also can not avoid manipulating and storing paths in each iteration of the algorithm. Plentiful computational experiments of OBA also indicate that most of the CPU time of the OBA is to enumerate and manipulate paths of the restricting subnetwork $A_p(\alpha)$ (see [14]).

Motivated by the problems discussed above, we give a second derivative approximation to modify the origin-based algorithm. By using this approximation, the modified origin-based algorithm (MOBA) can avoid enumerating and manipulating paths of the restricting subnetworks and outperforms OBA in the speed of convergence and memory requirements. The remaining sections of this paper are organized as follows. Section 2 describes the traffic assignment problem and introduces the origin-based algorithm. The modified origin-based algorithm by using second derivative approximation is then proposed in section 3. Section 4 describes the computational results generated by OBA and MOBA from two real networks including Sioux Falls and Barcelona network. Finally, a brief conclusion is given in section 5.

2 User-Equilibrium Traffic Assignment and Origin-Based Algorithm

Before introducing the user equilibrium traffic assignment model and the origin-based algorithm, we give the following notations which are used in this paper.

G	$G = (\mathcal{N}, \mathcal{A})$ represent an urban transportation network
$\mathcal{N}$	$\mathcal{N} = (1, 2, \dots, N)$ the set of nodes
$\mathcal{A}$	the set of all directed links
O	the set of origin nodes
D	the set of destination nodes
a	$a = [a_t, a_h]$ the directed link of the network
a_t	the tail node of link a
a_h	the head node of link a
x_a	the traffic flow on link a
c_a	the traffic cost on link a

c'_a	the first-order derivative of the cost of link a
x_a^p	the traffic flow on link a from origin p
α_a^p	the proportion of the traffic flow from origin p that arrived at a_h through link a
μ_a^p	the average cost of link a for origin p
σ_i^p	the average cost from origin p to node i
ν_a^p	the average first-order derivative of the cost of link a for origin p
ρ_i^p	the average first-order derivative of the cost from origin p to node i
g_i^p	the traffic flow arrived at i from origin p
c_r	the cost of path r , which is equal to total costs of all links on the paths
A_p	the restricting subnetwork for origin p
$B(i)$	the set of all directed links whose head node is i
b_i^p	the basic link of the links in $B(i)$ for origin p
$o^p(i)$	the topological order of node i for A_p
lcn_i^p	the last common node of all paths from origin p to node i
R_{ij}	the set of paths connecting i and j
$R_{ij}[A_p]$	the set of paths in A_p connecting i and j
f_{ij}^r	the flow on path r connecting i and j
$\delta_{ij}^{r,a}$	the path-link incidence matrix
q_{ij}	the total traffic demand between i and j , when $i = j$, $q_{ij} = 0$.

2.1 User Equilibrium Formulation

It is well known that TAP can be formulated as an optimization program with a nonlinear objective function and linear constraints. Consider an urban transportation network represented by a directed graph $G = (\mathcal{N}, \mathcal{A})$. For convenience, we suppose $O = D = \mathcal{N}$. Using the above notes, the User Equilibrium (UE) traffic assignment problem can be written as

$$\begin{aligned}
 \min \quad & Z = \sum_{a \in \mathcal{A}} \int_0^{x_a(\alpha)} c_a(\varpi) d\varpi \\
 \text{s.t.} \quad & \sum_{a \in B(i)} \alpha_a^p = 1, & 1 \leq i, p \leq N, i \neq p \\
 & \alpha_a^p \geq 0, & \forall a \in \mathcal{A}, 1 \leq p \leq N
 \end{aligned} \tag{1}$$

where $\alpha = (\alpha^1, \alpha^2, \dots, \alpha^N)$, $\alpha^p = (\dots, \alpha_a^p, \dots)$, $a \in \mathcal{A}$ is the variable of the mathematical programming, and α_a^p is defined as,

$$\alpha_a^p = \frac{x_a^p}{\sum_{b \in B(a_h)} x_b^p} \tag{2}$$

especially when the total traffic flow from origin p that arrived at a_h is zero, we let any one of the proportions of those links with head node a_h be one and the others be zero. $x_a(\alpha)$ is defined as,

$$x_a(\alpha) = \sum_{i=1}^N x_a^i(\alpha) = \sum_{i=1}^N \sum_{j=1}^N \sum_{r \in R_{ij}} f_{ij}^r \cdot \delta_{ij}^{r,a} = \sum_{i=1}^N \sum_{j=1}^N \sum_{r \in R_{ij}} q_{ij} \prod_{b \subseteq r} \alpha_b^i \delta_{ij}^{r,a} \tag{3}$$

For each origin p and every node $j \neq p$ we choose one link $b_j^p \in A_p : (b_j^p)_h = j$ as the basic link of the links with head node j for origin p , call all other links

with head node j in the restricting subnetwork A_p (if there are any) the non-basic links for origin p and denote them as $\text{NB}_j^p = \{a \in A_p : a_h = j; a \neq b_j^p\}$; $\text{NB}^p = \cup_{j \in \mathcal{N}; j \neq p} \text{NB}_j^p$. Then α can be viewed as a function of α^{NB} :

$$\alpha_{b_j^p}^p(\alpha^{\text{NB}}) = 1 - \sum_{a \in \text{NB}_j^p} \alpha_a^{p\text{NB}}, \quad \forall j \in \mathcal{N} \setminus \{p\}, \forall p \in \mathcal{N} \quad (4a)$$

$$\alpha_a^p(\alpha^{\text{NB}}) = \alpha_a^{p\text{NB}}, \quad \forall a \in \text{NB}^p, \forall p \in \mathcal{N} \quad (4b)$$

$$\alpha_a^p(\alpha^{\text{NB}}) = \alpha_a^{p\text{NB}} = 0, \quad \forall a \in \mathcal{A} \setminus A_p, \forall p \in \mathcal{N} \quad (4c)$$

Using (4), (1) can be simply changed into,

$$\begin{aligned} \min Z &= \sum_{a \in \mathcal{A}} \int_0^{x_a(\alpha^{\text{NB}})} c_a(\varpi) d\varpi \\ \text{s.t. } \alpha_a^{p\text{NB}} &\geq 0, \quad \forall a \in \mathcal{A}, 1 \leq p \leq N \end{aligned} \quad (5)$$

It is simply to prove that (1) equals to the famous Beckmann's transformation [2]. Convergence algorithms for solving it have been studied since the 1960s.

2.2 Origin-Based Algorithm

The origin-based algorithm operates directly on the space of the traffic flow proportion. It makes successive moves as an approximate Newton direction at each iteration. The main body of the algorithm can be found in [14].

3 Modified Origin-Based Algorithm

Our plentiful computational experiments and investigation of OBA indicate that there is no efficient and quick methods to find the last common nodes of the restricting subnetwork $A_p(\alpha)$ and we cannot but enumerate all paths of the restricting subnetwork in the algorithm. We also found the last common nodes were only used to calculate the approximation of the Hessian matrix. Hence if we can propose a new Hessian matrix approximation without using the last common nodes to replace that of the OBA, we will avoid path enumeration. Below the lower bound and upper bound of the Hessian matrix will be calculated firstly, then a novel Hessian matrix approximation is determined by these bounds. The novel approximation will be used to modify the origin-based algorithm.

Before giving the bounds of the Hessian matrix, we need the following definitions and propositions.

Definition 1. For any origin p , and any two nodes i and j , we define the following formal expression as the proportion of the flow from origin p that arrives at node j through node i .

$$\chi_{i \rightarrow j}^p = \begin{cases} \sum_{r \in R_{ij}[A_p]} \left(\prod_{a \subseteq r} \alpha_a^p \right), & \forall p \in \mathcal{N}, i \neq j \\ 1, & i = j \end{cases} \quad (6)$$

Definition 2. For any origin p and node j , the following equation is defined as the average cost from origin p to node j .

$$\sigma_j^p = \sum_{r \in R_{pj}[A_p]: a \subseteq r} c_r \cdot \prod_{a \subseteq r} \alpha_a^p \quad (7)$$

Definition 3. For any origin p and link a , the following equation is defined as and the average cost of link a for origin p .

$$\mu_a^p = \sum_{r \in R_{pa_h}[A_p]: a \subseteq r} c_r \cdot \prod_{a' \subseteq r: a' \neq a} \alpha_{a'}^p = c_a + \sigma_{a_t}^p \quad (8)$$

Definition 4. For any origin p , the topological order of all nodes in A_p is defined as a one-to-one function $o : \mathcal{N} \rightarrow \{1, 2, \dots, N\}$ such that, $\forall a = [i, j] \in A_p \Rightarrow o^p(i) < o^p(j)$, and particularly $o^p(p) = 1$.

Definition 5. The total flow arrived at node j from origin p is defined as,

$$g_j^p = \sum_{i=1}^N \sum_{r \in R_{pi}[A_p]; j \in r} q_{pi} \prod_{a \subseteq r} \alpha_a^p = \sum_{i=1}^N q_{pi} \cdot \chi_{p \rightarrow j}^p \cdot \chi_{j \rightarrow i}^p = \sum_{i=1}^N q_{pi} \cdot \chi_{j \rightarrow i}^p, \quad \forall p \in \mathcal{N} \quad (9)$$

Using these definitions, the following equations can be easily gotten,

$$\sigma_j^p = \sum_{a \in A_p: a_h = j} \alpha_a^p \cdot \sum_{r \in R_{pj}[A_p]: a \subseteq r} c_r \cdot \prod_{a' \subseteq r: a' \neq a} \alpha_{a'}^p = \sum_{a \in A_p: a_h = j} \alpha_a^p \cdot \mu_a^p \quad (10)$$

$$\begin{aligned} \mu_a^p &= c_a + \sum_{r \in R_{pa_t}[A_p]: a \subseteq r} c_r \cdot \prod_{a' \subseteq r} \alpha_{a'}^p \\ &= c_a + \sum_{r \in R_{pa_t}[A_p]: a \subseteq r} \prod_{a' \subseteq r} \alpha_{a'}^p \cdot \sum_{e \subseteq r} c_e \\ &= c_a + \sum_{a' \in A_p} c_{a'} \cdot \chi_{p \rightarrow a'_t}^p \cdot \alpha_{a'}^p \cdot \chi_{a'_h \rightarrow a_t}^p \\ &= c_a + \sum_{a' \in A_p: o^p(a'_h) < o^p(a_h)} c_{a'} \cdot \alpha_{a'}^p \cdot \chi_{a'_h \rightarrow a_t}^p \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{\partial x_{a'}}{\partial \alpha_a^p} &= \sum_{i=1}^N \sum_{r \in R_{pi}[A_p]; a' \subseteq r; a \subseteq r} q_{pi} \cdot \prod_{e \subseteq r; e \neq a} \alpha_e^p \\ &= \begin{cases} \sum_{i=1}^N q_{pi} \sum_{r \in R_{pi}[A_p]; a \subseteq r; a' = a} \prod_{e \subseteq r; e \neq a} \alpha_e^p = g_{a_h}^p, & a' = a \\ 0, & a'_h = a_h, a' \neq a \\ \alpha_{a'}^p \cdot \chi_{a'_h \rightarrow a_t}^p \cdot g_{a_h}^p, & o^p(a'_h) < o^p(a_h) \\ \chi_{a_h \rightarrow a'_t}^p \cdot \alpha_{a'}^p \cdot g_{a'_h}^p, & o^p(a'_h) > o^p(a_h) \end{cases} \quad (12) \end{aligned}$$

$$\frac{\partial x_{a'}}{\partial \alpha_a^{p\text{NB}}} = \begin{cases} g_{a_h}^p, & a' = a \\ -g_{a_h}^p, & a' = b_{a_h}^p \\ 0, & a'_h = a_h, a' \neq a, a' \neq b_{a_h}^p \\ \alpha_{a'}^p \cdot g_{a_h}^p \cdot (\chi_{a'_h \rightarrow a_t}^p - \chi_{a'_h \rightarrow b_{a_h}^p}^p), & o^p(a'_h) < o^p(a_h) \\ 0, & o^p(a'_h) > o^p(a_h) \end{cases} \quad (13)$$

Hence the first-order and second-order derivatives of the variables α^{NB} can be describe as,

$$\begin{aligned} \frac{\partial Z}{\partial \alpha_a^{p\text{NB}}} &= \sum_{a' \in \mathcal{A}} \frac{\partial Z}{\partial x_{a'}} \cdot \frac{\partial x_{a'}}{\partial \alpha_a^{p\text{NB}}} = \sum_{a' \in \mathcal{A}} c_{a'} \cdot \frac{\partial x_{a'}}{\partial \alpha_a^{p\text{NB}}} \\ &= c_a \cdot g_{a_h}^p - c_{b_{a_h}^p} \cdot g_{a_h}^p + \sum_{\substack{a' \in A_p; \\ o^p(a'_h) < o^p(a_h)}} c_{a'} \cdot \alpha_{a'}^p \cdot g_{a_h}^p \cdot (\chi_{a'_h \rightarrow a_t}^p - \chi_{a'_h \rightarrow b_{a_h}^p}^p) \\ &= g_{a_h}^p (\mu_a^p - \mu_{b_{a_h}^p}^p) \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{\partial^2 Z}{\partial \alpha_a^{p\text{NB}^2}} &= \sum_{a' \in \mathcal{A}} \left[\frac{\partial^2 Z}{\partial x_{a'}^2} \cdot \left(\frac{\partial x_{a'}}{\partial \alpha_a^{p\text{NB}}} \right)^2 + \frac{\partial Z}{\partial x_{a'}} \cdot \left(\frac{\partial^2 x_{a'}}{\partial \alpha_a^{p\text{NB}^2}} \right) \right] = \sum_{a' \in \mathcal{A}} c'_{a'} \cdot \left(\frac{\partial x_{a'}}{\partial \alpha_a^{p\text{NB}}} \right)^2 \\ &= (c'_a + c'_{b_{a_h}^p}) \cdot g_{a_h}^{p^2} + \sum_{\substack{a' \in A_p; \\ o^p(a'_h) < o^p(a_h)}} c'_{a'} \cdot \alpha_{a'}^{p^2} \cdot g_{a_h}^{p^2} \cdot (\chi_{a'_h \rightarrow a_t}^p - \chi_{a'_h \rightarrow b_{a_h}^p}^p)^2 \end{aligned} \quad (15)$$

Notice that,

$$\chi_{p \rightarrow i}^p \cdot \chi_{i \rightarrow j}^p = \sum_{\substack{r \in R_{pj}[A_p]; \\ i \in r}} \prod_{a \subseteq r} \alpha_a^p \leq \sum_{r \in R_{pj}[A_p]} \prod_{a \subseteq r} \alpha_a^p = \chi_{p \rightarrow j}^p$$

and for any node j and any origin p , it is easy to prove $\chi_{p \rightarrow j}^p = 1$ by Lemma 4 of [14], then,

$$0 \leq \chi_{i \rightarrow j}^p \leq 1, \quad \forall p \in \mathcal{N}, 1 \leq i, j, \leq N$$

Hence the upper bound of the Hessian matrix will be,

$$\frac{\partial^2 Z}{\partial \alpha_a^{p\text{NB}^2}} \leq c'_a \cdot g_{a_h}^{p^2} + c'_{b_{a_h}^p} \cdot g_{a_h}^{p^2} + \sum_{\substack{a' \in A_p; \\ o^p(a'_h) < o^p(a_h)}} c'_{a'} \cdot \alpha_{a'}^{p^2} \cdot g_{a_h}^{p^2} \quad (16)$$

Because,

$$\begin{aligned} \frac{\partial^2 Z}{\partial \alpha_a^{p\text{NB}^2}} &= c'_a \cdot g_{a_h}^{p^2} + c'_{b_{a_h}^p} \cdot g_{a_h}^{p^2} \\ &\quad + \sum_{a' \in A_p; o^p(a'_h) < o^p(a_h)} c'_{a'} \cdot \alpha_{a'}^{p^2} \cdot g_{a_h}^{p^2} \cdot (\chi_{a'_h \rightarrow a_t}^p - \chi_{a'_h \rightarrow b_{a_h}^p}^p)^2 \\ &\geq c'_a \cdot g_{a_h}^{p^2} + \sum_{a' \in A_p; a' \in B(a_t)} c'_{a'} \cdot \alpha_{a'}^{p^2} \cdot g_{a_h}^{p^2} \cdot (1 - \chi_{a_t \rightarrow b_{a_h}^p}^p)^2 \\ &\quad + c'_{b_{a_h}^p} \cdot g_{a_h}^{p^2} + \sum_{a' \in A_p; a' \in B(b_{a_h}^p)} c'_{a'} \cdot \alpha_{a'}^{p^2} \cdot g_{a_h}^{p^2} \cdot (\chi_{b_{a_h}^p \rightarrow a_t}^p - 1)^2 \end{aligned} \quad (17)$$

and there is no loop for any restricting network A_p , hence

$$\begin{aligned}\chi_{a_t \rightarrow b_{a_h t}}^p &= \alpha_{[a_t, b_{a_h t}]}^p \\ \chi_{b_{a_h t} \rightarrow a_t}^p &= \alpha_{[b_{a_h t}, a_t]}^p\end{aligned}$$

Therefore the lower bound of the Hessian matrix is

$$\begin{aligned}\frac{\partial^2 Z}{\partial \alpha_a^{p_{NB}} 2} &\geq c'_a \cdot g_{a_h}^{p_{NB}} + \sum_{\substack{a' \in A_p; \\ a' \in B(a_t)}} c'_{a'} \cdot \alpha_{a'}^{p_{NB}} \cdot g_{a_h}^{p_{NB}} \cdot (1 - \alpha_{[a_t, b_{a_h t}]}^p)^2 \\ &\quad + c'_{b_{a_h t}} \cdot g_{a_h}^{p_{NB}} + \sum_{\substack{a' \in A_p; \\ a' \in B(b_{a_h t})}} c'_{a'} \cdot \alpha_{a'}^{p_{NB}} \cdot g_{a_h}^{p_{NB}} \cdot (\alpha_{[b_{a_h t}, a_t]}^p - 1)^2 \\ &\geq c'_a \cdot g_{a_h}^{p_{NB}} + c'_{b_{a_h t}} \cdot g_{a_h}^{p_{NB}}\end{aligned}\quad (18)$$

Using the bounds of the Hessian matrix, we can get the following approximation of Newton direction of the variables α^{NB} ,

$$\Delta d_a^n = \frac{\mu_{b_{a_h t}}^n - \mu_a^n}{(c'_a + c'_{b_{a_h t}}) \cdot g_{a_t}^n}, \forall a \in NB^n, n \in D.$$

Thus for any origin p , we have,

$$\Delta \alpha_a^{p_{NB}} = \begin{cases} \max \left\{ -\alpha_a^{p_{NB}}, \lambda \cdot \frac{\mu_{b_{a_h t}}^p - \mu_a^p}{(c'_a + c'_{b_{a_h t}}) \cdot g_j^p(\alpha^{NB})} \right\}, & g_{a_h}^p(\alpha^{NB}) > 0 \\ -\alpha_a^{p_{NB}}, & g_{a_h}^p(\alpha^{NB}) = 0 \end{cases}, \quad \forall a \in NB^p \quad (19a)$$

$$\Delta \alpha_a^{p_{NB}} = 0, \quad \forall a \in A \setminus A_p \quad (19b)$$

where λ is a step size which will be determined by the following algorithm.

Modified Origin-based Algorithm

Initialization

For p from 1 to N

A_p = tree of minimum cost paths from origin p

x^p = all-or-nothing assignment using A_p

Calculate α^p by using (2)

End for

Main loop

While x does not satisfy the convergence condition do

For p from 1 to N

Update restricting subnetwork A_p

Update origin-based link flows for origin p

End for

End while

Update restricting subnetwork for origin p

Remove unused links from A_p

Compute maximum cost u_i from p to i for all $i \in \mathcal{N}$

For $a = [a_t, a_h]$ in $\mathcal{A}$

 If $u_{a_t} < u_{a_h}$ add link a to A_p

End for

Find topological order for new A_p

Update data structures

Update origin-based link flows for origin p

Using (14) and (18) to compute average costs and Hessian approximations

 For step size $\lambda = 2^{-k}$, $k = 0, 1, 2, 3, \dots$

 Using (19) to compute flow shifts $\Delta\alpha^p$ for λ

 Projection and aggregate flow shifts

 If new value of objective function is less than the old then stop

 End for

Apply flow shifts

Update total link flows and link costs.

4 Numerical Examples

Below we will illustrate both OBA and MOBA by the following networks whose basic characteristics are presented in Table 1. All programs are coded in C# and executed on a microcomputer with P4 2.0G, 512M. For fair comparison, common data structures for storing network topology and links and same convergence criterion are used in both OBA and the modified algorithm(MOBA). The following steps are used for the comparison of algorithm performance.

Table 1. Basic Characteristics of Test Urban Networks

Netork	Origin	Destination	Node	Link	OD Pair
Sioux Falls	24	24	24	76	528
Barcelona	97	108	930	2,522	7,922

- Step 1. Solve TAP by LCFW (a modified Frank Wolfe algorithm)[8], with very large number of iteration and highly strict convergence criterion. Tag its final solution as Ideal Optimal Solution(IOS) and denote it with $x^I = (\dots, x_a^I, \dots)$. The intuitive reason for choosing LCFW is that it outperforms the other algorithms(including FW, GP etc.[8]) and it can avoid path enumerations.
- Step 2. Considering that there is only one flow pattern that minimizes program (1)[3], we use the following convergence criterion in both OBA and MOBA.

$$\varepsilon = \frac{\sum_{a \in \mathcal{A}} (x_a(n) - x_a^I)^2}{\sum_{a \in \mathcal{A}} x_a^{I^2}} \quad (20)$$

where $x_a(n)$ is the link flows of the n -th iteration of the algorithms. When ε is less than a given small positive constant such as 1.0×10^{-n} , $n = 0, 1, 2, \dots$, the algorithms will stop.

Fig. 1. OFV-CPU Time for Sioux Falls

Fig. 2. OFV-CPU Time for Barcelona

Fig. 3. $\log \varepsilon$ -CPU Time for Sioux Falls

Fig. 4. $\log \varepsilon$ -CPU Time for Barcelona

Figure 1 and 2 show the relationship between the objective function value (OFV) and CPU time of the two algorithms for Sioux Falls network and Barcelona network. Figure 3 and 4 show the convergence of the two algorithms as measured by $\log \varepsilon$ vs. CPU time for those two real networks. From those four figures, we know the performance of the modified origin based algorithm is better than that of origin-based algorithm. The modified algorithm is superior to the origin-based algorithm in convergence and calculation time. At the same precision, the CPU time of the modified algorithm is only one third of the origin-based algorithm, even less than that. And the modified algorithm can solve the traffic assignment problem for large-scale network in an ideal time.

5 Conclusion

In this paper we provide a modified origin-based algorithm based on estimation of the lower and upper bounds of the Hessian matrix for, which can be calculated without path enumeration. In this algorithm we use the lower bound of Hessian matrix to approximate the direction of the origin-based algorithm. According

to our computational results, the modified origin-based algorithm(MOBA) improves the convergence performance greatly. The results indicate that MOBA can deliver better and more reliable convergence than OBA and saves much more CPU time especially when large-scale networks are being considered.

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