Formalising and Verifying Reference Attribute Grammars in Coq

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Abstract. Reference attribute grammars are a powerful formalism for concisely specifying and implementing static analyses. While they have proven their merit in practical applications, no attempt has so far been made to rigorously verify correctness properties of the resulting systems. We present a general method for formalising reference attribute grammars in the theorem prover Coq. The formalisation is supported by tools for generating standard definitions from an abstract description and custom proof tactics to help automate verification. As a small but typical application, we show how closure analysis for the untyped lambda calculus can easily be implemented and proved correct with respect to an operational semantics. To evaluate the feasibility of our approach on larger systems, we implement name lookup for a naming core calculus of Java and give a formal correctness proof of the centrepiece of a rename refactoring for this language.

1 Introduction

Verifying program analyses and transformations is hard, even with the assistance of modern proof tools. Previous work [17,18] has followed standard practice in the compilers community by focussing on simple intermediate languages, where issues like complex scoping have been transformed away. For verifying optimisations in a batch compiler this approach is successful and adequate.

However, many modern development tools such as refactoring editors and code checkers operate at source level, and not on an intermediate language. This requires the implementation of analyses and transformations that align with the ‘middle’ phases of a compiler that deal with name binding, type checking and complex control flow structures, which are often the most intricate and error prone [10,6].

This paper presents a framework for verifying analyses and transformations on source, consisting of a specification formalism for source-level analyses and transformations, namely Circular Reference Attribute Grammars, and an embedding of that formalism in Coq. Our embedding is supported by tools that reduce the tedium of working with a complex language definition and allow proofs to be conducted at a high level of abstraction.

The particular variant of attribute grammars we have chosen is the basis of the Jast-Add system [7]. It enhances standard attribute grammars [16] with a notion of node identity (reference attributes), and the ability to express least fixed point computations (circular attributes). Both these extensions are well-known: most industrial attribute

grammar systems support reference attributes [1,23], and the idea of circular attributes goes at least back to Farrow [8].

The success of JastAdd [28] amply demonstrates the power of circular reference attribute grammars and their practical usability. Particularly relevant are its use in the implementation of JastAddJ, a full, highly compliant Java compiler [7], and in the construction of a refactoring framework for Java [24].

While it is well-known that pure attribute grammars can be regarded as non-strict functional programs [15], reference attributes and circular attributes require a more sophisticated translation. We demonstrate that these complications can be handled gracefully by generating boilerplate definitions and judicious use of proof strategies.

The result is a completely general framework for conducting proofs of analyses and transformations that deal with the full complexity of modern source languages. We illustrate its use on two examples: verification of a closure analysis, and the core of the refactoring framework of [24]. The complete formalisation including mechanised proofs is available for download [25]. Major contributions of this paper are:

- The first generic framework for mechanised proofs of the correctness of program analyses and transformations specified at source level.
- Tool support for conducting such proofs in Coq, at a high level of abstraction.
- A substantial case study that illustrates how our framework copes with the complex scoping issues in modern object-oriented languages, an issue that has hitherto not been considered in a formal setting.
- Evidence that proofs in our framework are scalable: non-trivial extensions to the object language require only modest additions to the proofs themselves.

The paper is structured as follows. First we introduce the notion of circular reference attribute grammars in Sec. 2. Next we show in Sec. 3 how this variant of attribute grammars is formalised in Coq. Sections 2 and 3 use a simple closure analysis as a running example. As a more substantial case study, we describe a formalisation of name analysis in an object-oriented language in Sec. 4, and show how it can be used to give a correctness proof of the central component of the Rename refactoring. Section 5 evaluates the development effort for both examples, and investigates scalability issues. Related work is discussed in Sec. 6, and we conclude in Sec. 7.

2 Reference Attribute Grammars

Attribute grammars [16] are a high-level formalism for declaratively specifying and implementing programs working on trees, in particular programming language processors such as compilers. Nodes of the underlying trees are endowed with attributes defined in terms of other attributes on the same node, or attributes on the surrounding nodes.

As originally proposed, attribute grammars make no special provisions for non-local dependencies, i.e. attributes that are defined in terms of other attributes on nodes far away in the tree. This situation commonly occurs in the static analysis of programming languages: For example, to typecheck the use of a variable, information about its type is needed, which has to be obtained from its declaration arbitrarily far away in the tree.

Solutions such as the explicit construction of symbol tables tend to become quite complicated, compromising the conceptual clarity of the underlying approach. Many extensions have been proposed to remedy this and similar problems, a particularly simple one being Reference Attribute Grammars, or RAGs [11], which are closely related to Remote Attribute Grammars [3]. The idea here is to allow attributes to evaluate to node references, on which other attributes can be evaluated in turn, thus superimposing a graph structure on the abstract syntax tree. This alleviates the need for secondary structures such as symbol tables and enables the use of the abstract syntax tree as the only data structure. RAGs become especially useful when combined with circular attributes, i.e. attributes defined by recursive equations; the combined formalism of Circular Reference Attribute Grammars, or CRAGs [20], will be our focus in this paper.

To introduce the key features of attribute grammars in general and reference attribute grammars in particular, we show how to implement closure analysis for the untyped lambda calculus. Closure analysis is a static analysis that predicts the reduction behaviour of terms. Take, for example, the term $(\lambda f. (f I) (f K)) (\lambda x. x)$ where $I = \lambda a. a$ and $K = \lambda b. \lambda c. b$. Closure analysis will be able to predict that during reduction x can only be bound to the terms I and K ; we say that the set of closures of (the single occurrence of) x is $\{I, K\}$. For a discussion of the uses of closure analysis we refer to the literature [26,22], we concentrate on how to compute the set of closures using an attribute grammar.

2.1 Abstract Syntax Trees

Our object language is the simply typed lambda calculus with its reduction relation $t \Downarrow v$, indicating that term t evaluates to value v . The definitions of syntax and reduction relation are standard and we elide them for brevity.

Lambda terms are represented as abstract syntax trees (ASTs): Every node in an AST corresponds to a nonterminal of the grammar (here always `term`). Its children may either be terminals or other nodes, corresponding to a particular production in the grammar. In our example, a `term` node is either a variable node with a single identifier as its only child; or an abstraction node with an identifier (the bound variable) and a `term` node as its children; or an application node with two `term` nodes as its children. A node represents a subterm *at a certain position*, hence in the example the two occurrences of variable f will be represented by different nodes, albeit with the same content (namely the term f).

We want to implement an attribute `closures` that, for each node in the abstract syntax tree of a lambda term, returns the set of possible closures it could evaluate to. We want to prove a coherence result to show that our closure analysis agrees with the reduction relation and correctly predicts reduction outcomes:

Lemma 1 (Coherence). *For any lambda term t and closure v , if $t \Downarrow v$ then there is some node $nd \in \text{closures}(t)$ such that v is the content of nd .*

2.2 The Attributes

To start with, we need to define a parameterised `lookup` attribute which looks up a name from a position within the term. Proceeding outwards, it returns a reference to

the innermost enclosing lambda abstraction binding that name, if there is any. Thus, the result of looking up a name y at a node nd depends on the context of the node:

- if nd is the body of a lambda abstraction binding a variable x , we compare y and x ; if they are the same, nd 's parent node (the lambda abstraction) is returned as result; otherwise, we proceed with the lookup of y from the parent node
- if nd is the left child of an application node, lookup proceeds outwards
- if nd is the right child of an application node, lookup proceeds outwards

These three cases correspond to the clauses of the definition of name lookup as an *inherited attribute*, which is given in Fig. 1 using pseudocode. In these equations, **this** refers to the node lookup is invoked on, whereas **parent** is that node's parent node. The equations are defined by pattern matching on the position of **this** relative to **parent**, which is here symbolically indicated as a term with a bullet showing the position of **this**.

$$\begin{aligned}
 \text{inh lookup} &:: \text{id} \times \text{node} \rightarrow \text{node} \\
 \text{lookup}(y, \lambda x. \bullet) &= \begin{cases} \text{parent} & \text{if } y \equiv x \\ \text{lookup}(y, \text{parent}) & \text{otherwise} \end{cases} \\
 \text{lookup}(y, \bullet t_2) &= \text{lookup}(y, \text{parent}) \\
 \text{lookup}(y, t_1 \bullet) &= \text{lookup}(y, \text{parent})
 \end{aligned}$$

Fig. 1. Informal specification of name lookup

The closure analysis itself is specified as a *synthesised attribute* `closures`, which computes an overapproximation of the closures a term could evaluate to, and a *collection attribute* `args`, which computes an overapproximation of the closures a closure could be applied to. These two attributes are mutually defined as shown in Fig. 2.

$$\begin{aligned}
 \text{syn closures} &:: \text{node} \rightarrow \text{set node} \\
 \text{closures}(x) &= \text{args}(\text{lookup}(x, \text{this})) \\
 \text{closures}(\lambda x. t) &= \{\text{this}\} \\
 \text{closures}(t_1 t_2) &= \bigcup \{\text{closures}(\text{body}(c)) \mid c \in \text{closures}(t_1)\} \\
 \text{coll args} &:: \text{node} \rightarrow \text{set node} \\
 \text{args}(c, t_1 t_2) &\supseteq \text{closures}(t_2) \quad \text{if } c \in \text{closures}(t_1) \\
 \text{args}(c, _) &\supseteq \emptyset \quad \text{otherwise}
 \end{aligned}$$

Fig. 2. Informal specification of closure analysis

The attribute `closures` is defined by case analysis over the *content* of its argument node, not its position as with attribute `lookup` above: this is the crucial difference between synthesised and inherited attributes. The equations are quite straightforward:

- a name can be bound to any value its binding abstraction is applied to
- a lambda abstraction's only possible value is itself
- an application's possible values are those of the body of its first child

Note the use of reference attributes in the third equation: The attribute `closures` returns a set of node references, on which other attributes may in turn be evaluated.

The attribute `args` is a globally defined collection attribute: Every node in the tree contributes a value, and all these contributions are combined using set theoretic union to yield the attribute's final value. The attribute has type $\text{node} \rightarrow \text{set node}$; for a closure c , $\text{args}(c)$ is the set of all possible arguments c could be applied to. The contribution clauses (written using $\supseteq$) have type $\text{node} \times \text{node} \rightarrow \text{set node}$; for a closure c and a node n , they specify what the node n contributes to the value of $\text{args}(c)$. In particular, an application $t_1 t_2$ with c as a possible value of t_1 contributes all possible values of t_2 .

The attributes `closures` and `args` circularly depend on each other, thus it is not immediately clear that this definition is sensible. The classical solution [5] is to take least fixed points, which can here be constructed by iteration: the definitions are monotonic (as we will prove) and have values over a complete partial order of finite height.

Since the inception of attribute grammars, much research has been done on how to statically detect circularities. It turns out, however, that even the very simple extension of allowing attributes to take additional parameters (like `lookup`) defeats all checks:

Theorem 1 (Undecidability of Circularity). *It is undecidable whether a parameterised attribute is circularly defined.*

Proof. See the technical report for details [25].

3 Formalisation of Reference Attribute Grammars

We now show how to implement reference attribute grammars, such as the example presented in the previous section, in Coq. We choose to use a shallow embedding of reference attribute grammars into Coq: Abstract syntax trees are represented as terms, with attributes as functions on them. This means that attribute equations can use the full power of the Coq language, in particular they can exploit its powerful type system.

3.1 Formalising Abstract Syntax

For the formalisation of the object language's abstract syntax, we make use of Ott [27], a tool for specifying the syntax and semantics of programming languages and calculi. Given an abstract grammar, Ott generates one Coq datatype per nonterminal, with one constructor per production. In our case this yields

```
Inductive term : Set := Var : string → term
      | Abs : string → term → term
      | App : term → term → term.
```

This representation, however, is not sufficient to model reference attributes. The attribute `lookup`, for example, should return a reference to a lambda abstraction, which is not only characterised by its content (given by term of type `term`), but also its position in the syntax tree to distinguish it from other abstractions that might happen to have the same body. This leads to our first implementation challenge:

Challenge 1 (Node Identity) *We need a datatype representing nodes with an identity such that they can be distinguished from other nodes, even if they have identical content.*

To achieve this, we use Huet’s zipper [12]. The basic idea of the zipper is to describe a node’s position as a “path” from the node to the root of the tree. Let n be the node under consideration. If it is the root node, its path is empty. Otherwise, n must be the i th child of some node p and has left siblings $n_1, \dots, n_{i-1}$ and right siblings $n_{i+1}, \dots, n_m$, where m is the total number of children of p . Then the path of n consists of a first “step” containing all the siblings $n_1, \dots, n_{i-1}, n_{i+1}, n_m$ (represented by their subtrees), and a path for p .

The possible steps are prescribed by the grammar: since every abstract syntax tree must be derivable from the start symbol, a step can only go from the occurrence of a nonterminal t' on the right hand side of some production p to the nonterminal t on its left hand side. In our grammar, there are only three possible steps and only one nonterminal, hence the definition of the `step` datatype is very simple:

```
Inductive step : Set := Term1InApp : term → step
                        | Term2InApp : term → step
                        | TermInAbs   : string → step.
```

The three constructors correspond exactly to the position patterns used in our informal specification of the `lookupattribute` in the previous section: A `step Term1InApp t2` describes (the position of) a node which is the first child of a node representing an application, and whose right sibling is t_2 – in the pseudocode, this was written as $(\bullet t_2)$. Likewise, `Term2InApp tt1` corresponds to $(t_1 \bullet)$ and `TermInAbs x` is $(\lambda x. \bullet)$.

In order to fully describe a node’s position, we need to give the complete sequence of steps all the way to the root of the syntax tree. Thus the datatype of positions is

```
Inductive pos : Set := Root : pos
                        | Step : step → pos → pos.
```

Finally, nodes are nothing but pairs of positions and content (i.e., terms), with convenience functions `node_pos` and `node_content` to access these two parts.

The example term from Sec. 2 is represented in Coq as

```
App (Abs "f" (App (App (Var "f") I) (App (Var "f") K)))
    (Abs "x" (Var "x"))
```

where I and K are Coq representations of the terms I and K , respectively. The position of the only occurrence of x is

```
Step (TermInAbs "x")
(Step (Term2InApp (Abs "f" (App (App (Var "f") I)
                                (App (Var "f") K)))) Root)
```

If we “plug” the content of the node into the first step on its path, we obtain the tree representation of the parent node’s content. If we continue plugging all the way up to the root node, we get back the representation of the whole tree. These operations can be implemented systematically for every nonterminal as two functions `plug_step` and `root` which can be understood as tree navigation primitives for going “upwards”.

The definitions of the zipper datatypes `step`, `pos`, `node`, and the helper functions `plug_step` and `root` can clearly be automatically derived from the abstract grammar. Indeed, we have extended Ott to provide this facility, but since the general case of grammars with multiple nonterminals requires somewhat more involved definitions than the ones given above, we defer discussion of this feature until Sec. 4.

3.2 Formalising Attributes

A pleasant side effect of using a zipper-based representation of nodes is that it yields a nice characterisation of the difference between synthesised and inherited attributes: A synthesised attribute works on the first component of a node (its children), whereas an inherited attribute works on its second component (its siblings and ancestors).

We start out by rephrasing our informal specification of `lookup` from Sec. 2 in Coq. First, we need to remedy a loophole of that specification: if we try to lookup an unbound variable, the inherited attribute described in Fig. 1 will reach the root of the syntax tree without having found any binding abstraction, and the specification does not say what is supposed to happen in this case. Our implementation of the attribute as a Coq function will have an option type as its return type, with a designated value `None` indicating that no binder was found.

If a binder *was* found, what should the function return? In the specification, it returns the binder's node. However, given our current node implementation, all nodes have type `node`, so we cannot by its type distinguish a node representing an abstraction from a node representing, say, an application. Hence we introduce a type that contains the same information as a node containing an abstraction, but in more explicit form¹:

Definition `closure := pos * string * term.`

The type of the `lookup` attribute is now `string → node → option closure`. When evaluated to lookup an unbound variable, it will return `None`, otherwise `Some c` where `c` is the closure representing the binding abstraction:

Definition `lookup (y:string) :=`
`inh (fun _ => option closure) (fun _ => None)`
`(fun s p t up =>`
`match s with`
`| TermInAbs x =>`
`if string_eq x y then Some (p, x, t) else up`
`| _ => up`
`end).`

The implementation makes use of the `inh` combinator which takes three arguments:

1. The result type of the attribute. .
2. The default value of the inherited attribute at the root node.
3. A function to compute the attribute value on a non-root node. The four parameters of this function are

¹ A perhaps more elegant alternative would be to have a grammar with a separate nonterminal for abstractions; our syntax tree would then be more strongly typed, and we could directly return a node from `lookup`. This is the path followed in Sec. 4.

Next, we implement the synthesised attribute `closures`. According to its specification, it should return a set of nodes representing closures; instead of sets, our implementation uses lists (without duplicate elements). The implementation of the attribute is shown in Fig. 3, where the underlined code should be ignored for the moment. It makes use of some user-defined syntactic sugar for handling sets to improve legibility.

3.3 Circularity

Challenge 2 (Circularity) *Many attributes are most easily given (mutually) circular definitions. We need a way to formalise such definitions in Coq.*

In our example, the attributes take values in the poset of (finite) lists of closures². The iterative fixed point computation is formalised by giving all circularly defined attributes an extra “time to live” (TTL) parameter ttl , which is a natural number that is decreased on every unsafe recursive invocation. If it ever reaches 0, the recursion stops and $\perp$ (the bottom element of the poset) is returned. To achieve this, we need to introduce extra code (underlined in Fig. 3), which makes use of a convenience function to effect the “cut-off” once the counter drops to zero:

This solution is quite lightweight and requires pleasantly little extra machinery.

² This partial order is a priori infinite (and of infinite height), but for any given term t the attribute closures will only return closures from t , which form a finite set.


```

Fixpoint closures (nd:node) ttl := TTLcut (fun ttl =>
  match node_content nd with
  | Var x => match lookup x nd with
    | Some c => coll union [] (args c ttl)
    | None => []
    end
  | Abs x t => [nd]
  | App t1 t2 =>
    let f := Node (Step (Term1InApp t2) (node_pos nd)) t1 in
    ∪ { { closures (closure_body c) ttl | c ← closures f ttl } }
  end ttl
with args c ttl (nd:node) := TTLcut (fun ttl =>
  let p := node_pos nd in
  match node_content nd with
  | App t1 t2 =>
    let n1 := Node (Step (Term1InApp t2) p) t1 in
    let n2 := Node (Step (Term2InApp t1) p) t2 in
    if contains (closures n1 ttl) c then closures n2 ttl
    else []
  | _ => []
  end ttl.

```

Fig. 3. Definition of closures and args; circularity handling code underlined

Our first and most urgent task is to prove monotonicity of the definitions to ensure that we can sensibly speak about their least fixed points. Since the two attributes are mutually defined, the Coq proof simultaneously establishes their monotonicity by induction on ttl.

We can now proceed to the formalisation of the coherence lemma. The reduction relation on terms can be abstractly specified in Ott and is extracted to Coq as a binary predicate `reduce`. Lemma 1 then reads like this:

```

Lemma closures_sound : ∀ t v, reduce t v →
  ∃ ttl, In v (closures (Node Root t) ttl).

```

In words, this asserts that for any term t and any value v such that $t \Downarrow v$ there is a sufficiently large TTL value such that the analysis predicts a closure c containing v . The proof proceeds by induction on the reduction of t to v .

To summarise, we have shown how the implementation of closure analysis as a CRAG from Sec. 2 can be translated to Coq. The two features of reference attributes and circular attributes pose the two challenges of how to represent nodes with identity and how to accommodate non-wellfounded mutual recursion. These challenges are overcome via automatically generated datatype definitions and combinators that make it easy to translate the informal attribute specifications into Coq code, on which we can then prove the desired coherence property.

4 Case Study: Name Analysis

The example in the previous section is quite simple and could easily have been written without using CRAGs. As a case study in using our framework to formalise a more substantial attribute grammar, we will now briefly discuss the formalisation of name analysis for a subset of Java. Although this is a much more complicated object language than the untyped lambda calculus, the same techniques are applicable; in fact, name analysis can be implemented using only inherited and synthesised attributes, there is no need for collection attributes. Tool support for automatic generation of definitions for the abstract syntax and the zipper datatypes is much more important now, as are carefully chosen tactics to simplify proofs. These two facilities ensure easy extensibility of the language: adding a non-naming related feature to the language usually can be done by changing only the high-level specification.

4.1 The Grammar

We define NameJava, a subset of Java that concentrates on the language’s naming features for fields and classes, to the exclusion of almost all “operational” aspects. In particular it supports compilation units and packages, nested classes, initialisers with nested blocks, member fields and local variables, qualified names for variables and classes, and the keywords **this** and **super**.

The name lookup rules for NameJava are the same as for Java [9]; an example program that highlights some of the salient points appears in Fig. 4. The five local variables `a1` to `a5` are assigned as follows: `a1` is bound to the local variable `a`, `a2` to the field `a` of class `D`, `a3` to the field `a` of class `B`, `a4` to the field `a` of class `B` (since the local variable `A` obscures the class of the same name), and `a5` to the field `a` of class `A`.

```
package p;      class B {
                  A a;
class A {        class D extends A {
    A a;          C c; // I
    class C { }   A a;
}                 { A a = a; B A = A; A a1 = a; A a2 = this.a;
                  A a3 = B.this.a; A a4 = A.a; A a5 = p.A.a; }
                  }
                  }
```

Fig. 4. Example NameJava program

The grammar of NameJava is, as for the untyped lambda calculus, specified in Ott, so datatype definitions for the abstract syntax and the zipper are automatically generated. For example, the inductive datatype for programs is

```
Inductive program : Set := CProgram : list compunit → program.
```

expressing that a program is a list of compilation units.

For the zipper datatypes, our automatic extraction actually generates a more strongly typed version than the one shown in Section 2: there is not a single datatype node of nodes, but rather a family of datatypes indexed by the nonterminal they represent. The index type is an enumeration type with one constructor per nonterminal of the grammar:

```
Inductive nt : Set := Program | Compunit | Classdecl | Extclause
  | Bodydecl | Block | Stmt | Vardecl | Expr | Access | ...
```

The datatype of steps is doubly indexed over `nt`. A `step t' t` describes the position of a subtree derived from nonterminal `t'` within its parent, which is derived from nonterminal `t`, i.e. it is a “step from `t'` to `t`”. We cannot give the definition of `step` for NameJava in full due to space constraints, but a fairly typical constructor is

```
BodydeclInClassdecl : string → extclause →
  list bodydecl → list bodydecl → step Bodydecl Classdecl
```

describing the position of a body declaration within a class declaration: To reconstruct the complete class declaration given one of its body declarations `b`, we need to know the class name (a `string`), the **extends** clause, the body declarations to the left of `b`, and the body declarations to the right of `b`—exactly the data stored in the `step` constructor.

As before, the `pos` datatype is a sequence of steps. It is, however, likewise indexed over a nonterminal (thus `pos t` describes the type of positions of subtrees derived from `t`), and it ensures that the steps it is composed of “fit together”:

```
Inductive pos : nt → Type :=
  Root : pos Program
| Step : ∀ t t', step t t' → pos t' → pos t.
```

Finally, to define the type of nodes, we need to ensure that the content of a node `t` actually is of the right type, i.e., a term of the inductive datatype corresponding to the index `t`. To this end we make use of an (automatically generated) function `content`, that maps indices to datatypes—for example, `content Program` is `program`:

```
Inductive node : nodetype → Type :=
  Node : ∀ t, pos t → content t → node t.
```

The upshot of using this more complicated, indexed representation of zippers is that it rules out many positions and nodes that could never occur in a well-formed program. For example, with the given NameJava grammar we can prove (in Coq) that the parent of a node `Stmt` will always be a node `Block`. This is crucial for the definition of `plug_step` and `root`, which both map nodes to trees; if the nodes were improperly nested, no corresponding trees could be constructed.

4.2 Attributes for Name Analysis

Name lookup in NameJava proceeds basically in an “outwards” movement, searching through enclosing lexical scopes until a matching declaration is found. Member classes and fields, however, can also be inherited from an ancestor class, hence an “upwards” movement (up the inheritance chain) is superimposed on the underlying outwards movement. Qualified names, finally, contribute a third type of movement: first, the declaration of the qualifier (resp. its type if it is a variable) is looked up, then the

qualified name is looked up as a member within this declaration (again possibly following the inheritance chain upwards, but never going outwards through enclosing scopes).

Name lookup for classes is implemented using five attributes, modelled after the implementation in the JastAddJ Java compiler [7]:

- `toplevel_class` looks up a `toplevel` class within a compilation unit
- `canonical_class` looks up a class by name and package
- `local_class` looks up a local member class within a class
- `member_class` looks up a member class in a class or one of its ancestors
- `lookup_class` looks up a class, starting from an arbitrary node and searching through enclosing scopes

The first three of these just iterate over a list of declarations, looking for the first one with the right name. The behaviour of `lookup_class` and `member_class` and their interaction, is more interesting, since they circularly depend on each other: Suppose that `lookup_class` is used to lookup the type name "C" from the line marked with "1" in Fig. 4. First, class D is searched for a `member_class` named C. Member classes include both locally defined classes and inherited member classes. Since no `local_class` C exists, we need to invoke `member_class` on D's super class.

To resolve the super class, we need to lookup the name "A" by `lookup_class`, causing `member_class` to depend on `lookup_class`. That lookup will search for a `member_class` named "A" in the enclosing class B, causing `lookup_class` to depend on `member_class` in turn. Since A is not a member class of B, it will be found as a `toplevel_class`. Once we have a reference to A, class C is successfully looked up as a `member_class` of A.

Observe that this last step makes crucial use of reference attributes: The node of class A is retrieved as the value of attribute `lookup_class`, and then attribute `member_class` is in turn evaluated on it. The example also shows that the two attributes recursively invoke each other, and thus are circularly defined.

4.3 Circularity in Name Analysis

Nevertheless, this circularity is quite different from the one encountered when implementing closure analysis: Whereas the specification of closure analysis is genuinely circular and the actual result of the analysis is the least fixed point of the given equations, the circularity of the definitions of `lookup_class` and `member_class` is somewhat accidental. We would, in fact, hope that the mutual recursive invocations of these two attributes are well-founded so that the attribute evaluation eventually terminates.

However, the language described so far allows classes that extend themselves, i.e. definitions like `class A extends A { }`. Looking up a member in A will lead the two attributes to repeatedly invoking each other on the same arguments, and thus to nontermination. The Java Language Specification [9] forbids programs like the above: Well-formed Java programs must not contain classes that are their own descendants.

It is clear that lookup on classes with acyclic inheritance hierarchy always terminates: When `member_class` recursively invokes itself on the superclass (as determined by `lookup_class`), we have moved up one level of inheritance, so eventually we will arrive at a class that extends no other class, and the member lookup will terminate. So

in order to implement a version of `lookup_class` that is guaranteed to terminate, we need to require that the node it is invoked on belongs to a program with acyclic inheritance hierarchy. However, in order to define the concept of an acyclic inheritance hierarchy we need a definition of inheritance, and for this we need `lookup_class`!

There does not seem to be an easy solution to this chicken-and-egg problem, so we accept the presence of a possible circularity, and treat it in the same way as for closure analysis: The circularly defined attributes take an additional TTL parameter and produce values over the flat poset of optional values, defined by the Coq type `option`, with `None` standing for nontermination.

Once we have implemented class lookup as a partial function, we can define what it means for a class to have an acyclic inheritance hierarchy, and we can give a (constructive) proof that when looking up a member class of such a class, there is a sufficiently large number n for which lookup will always return a value when invoked with n as its TTL counter. In other words, we can now prove that member lookup on classes with acyclic inheritance hierarchy always terminates.

Note that the need for circularly defined class lookup arises from the presence of nested classes. In a language without nested classes, all classes are toplevel classes within compilation units, so class lookup could be implemented by the two attributes `toplevel_class` and `canonical_class` alone, without any circularity.

4.4 Towards the Verification of Rename Refactorings

We cannot prove the correctness of our lookup implementation, since there is no formal specification to validate it against, so we apply our framework to a different problem: In [24], we describe how the well-known *Rename* refactoring, i.e. changing the name of a declaration and all its uses, can be implemented using right inverses to the lookup functions. We start from the observation that a reasonable correctness criterion for a renaming would be that the “binding structure” of the program does not change, i.e. all names in the program still refer to the same declarations as before.

We therefore define an *access computation* function that is a right inverse to lookup: Whereas `lookup` computes the declaration d a name n binds to at a program location p , `access` computes a name n that a declaration d can be accessed under from a location p , where n might be qualified to avoid capture. The inversion property means that if `access` gives a name n to access d from p , looking up n from p will yield d .

Given an implementation of access computation with this property, it is, in principle, easy to implement a rename refactoring: Given an input program, we first determine which declaration every individual name refers to. Then we perform the requested name change. Now we go over the program again. For every name, we know the declaration it ought to refer to, and if it does not do so anymore we can use access computation to determine an alternative name that does, and replace the old name with it.

In our current case study we concentrate on proving the inversion property. Given the implementation of the attributes for class and variable lookup, we can write corresponding access computation attributes. For `member_class`, which looks up a member class, we have an attribute `access_member`, which constructs an access to a member class; for `lookup_class` we have `access_class`, and so on. The inversion

of an attribute definition is quite systematic, and it is easily verified that all the access computation attributes are indeed right inverse to their corresponding lookup attributes.

Many of the proofs proceed by induction on either trees or positions, and hence have to handle (at least) one case per language construct to establish the induction step. Only a limited number of cases is actually interesting for every given proof, however: When proving properties of name lookup, for example, only constructs corresponding to naming scopes need special attention, in all the other cases the induction hypothesis can be applied directly. This situation can be handled gracefully by the Coq tactic language: When doing an induction, we specify a default tactic to be tried on all new subgoals; usually, Coq's `auto` tactic is sufficient to discharge all the trivial cases, if its hint database has been populated with enough domain specific lemmas.

5 Evaluation

5.1 Statistics

Figure 5 gives some statistics about the size of the complete Coq development for both our examples: the implementation of closure analysis in Sec. 3 and the name analysis for NameJava in Sec. 4. For both we give the line count for the Ott specification and for the Coq code automatically extracted from that specification, written as a sum: The first summand represents the size of the code extracted by Ott, and the second the size of the code extracted by our extension. Furthermore, we give the line count for the handwritten Coq specification, and for the proof scripts, as well as the total line count.

	Closure Analysis	NameJava
Ott specification (LOC)	80	322
extracted Coq code (LOC)	118+163	403+443
Coq specification (LOC)	301	2219
proof scripts (LOC)	233	1694
total LOC	614	4235

Fig. 5. Size of development

As expected, the name analysis is far bigger than the closure analysis, at almost seven times the size. In both cases, we have profited clearly from writing a high-level specification in Ott and automatically extracting the definitions of tree and zipper datatypes: the Ott specification is only about a third of the size of the extracted code. It is worth noting, however, that we use Ott merely as a convenient language for describing the object language syntax. All the proofs are carried out inside Coq, on the generated abstract syntax, and are fully certified by the Coq kernel.

It is interesting to see that the proof scripts are smaller than the specification they verify, only comprising about 40% of the total line count on both projects. This number is similar to other formalisation projects, such as Leroy's CompCert [19].

5.2 Extensibility

Although NameJava has all the major naming related features of full Java, it is a much smaller language. We have shown how the same implementation techniques can be used to specify closure analysis on the lambda calculus and to prove the inversion property of access computation and lookup in NameJava. But what about extending to a larger subset of Java, or even the full language? Will all the proofs have to be redone?

Fortunately, the answer is no. Adding a non-naming related feature to the language is possible without breaking any of the proofs: Introducing an `if` statement to the language only needs five additional lines of Ott. Neither lookup nor access computation have to be changed, and all the proofs still go through unchanged. For language features such as the `for` loop, which do influence the lookup without introducing any new naming related concepts, the effort is still moderate (40 lines of specification, 50 lines of proof). Adding, e.g., interfaces would certainly require more effort, but the experience of JastAddJ shows that CRAGs are powerful enough to handle the full Java language.

6 Related Work

Proofs of analyses and transformations. Cachera et al. [4] derive a dataflow analysis in Coq by abstract interpretation, independent of an implementation language. On the other end of the spectrum, Lerner et al. [18] introduce a domain specific language to encode analyses, which are then automatically proved correct. Our work is positioned in the middle: Using CRAGs offers a convenient implementation language, which is nevertheless fully integrated with Coq; the price we pay is less automation.

Formalising attribute grammars. Traditionally, attribute grammars have been encoded in lazy functional languages as families of functions, one per nonterminal, that take inherited attributes as parameters and return synthesised attributes as results [15]. Our method represents different attributes by different functions, thus staying closer to the original attribute grammar. It is not clear if the traditional encoding could accommodate reference attributes. The zipper has been used before to implement inherited attributes [29,2], but without making the connection to reference attributes.

Formalisations of Java. There has been much interest in the formalisation of (subsets of) the Java language [14,21], but most formalisations focus on the semantics of the language and mostly ignore more syntactic aspects like lookup rules. Only FJI [13] makes an effort to model nested classes, but the formalisation is geared towards minimality and does not give a satisfying solution to the name lookup circularity problem.

7 Conclusions

We have presented a novel framework for formalisation and verification of analyses and transformations that are expressed at source level, rather than on a simplified intermediate language. The key to this framework are circular reference attribute grammars, which have proven their practical worth in compiler generation tools [1,23,8,7].

We have presented a general method for formalising them in Coq, enabling the expression of specifications and proofs at a high level of abstraction. We provide tools

that automatically generate datatype definitions and utility functions from a high-level description of the object language, which are complemented by domain-specific tactics to simplify proofs. We have validated these tools and tactics with a non-trivial case study, namely renaming in the presence of the complex binding rules of modern object-oriented languages. The verification effort is manageable, and the proofs are scalable, so that they do not have to be redone from scratch when the object language changes.

As future work, we would like to explore the implementation of a domain specific language for attribute grammars on top of Coq. This would give additional possibilities for automation, and it might be possible to support automatic proofs of common lemmas such as the monotonicity of circularly defined attributes. Still, we feel that it is essential to provide close integration of the attribute implementations with the Coq language in general, so as not to lock the user into the DSL, since the more complex proofs can likely not be fully automated and certainly benefit from the power of the full system.

We hope this paper has enthused others to venture into the formalisation and verification of source level analyses. Modern development tools need to stay at the source level to interact with the user, but bugs are common even in production quality environments. Our work shows that rigorous verification of the underlying analyses is feasible, which can help in improving tools used by developers on a daily basis.

References

1. Augustijn, L.: Definition of the programming language Elegant, 2000. Release 7.1, Philips Research Laboratories, Eindhoven, the Netherlands (February 2000)
2. Badouel, É., Fotsing, B., Tchougong, R.: Yet Another Implementation of Attribute Evaluation. Technical Report 6315, INRIA Rennes (2007)
3. Boyland, J.T.: Remote attribute grammars. *J. ACM* 52(4), 627–687 (2005)
4. Cachera, D., Jensen, T., Pichardie, D., Rusu, V.: Extracting a data flow analyser in constructive logic. *Theor. Comput. Sci.* 342(1), 56–78 (2005)
5. Chirica, L.M., Martin, D.F.: An Order-Algebraic Definition of Knuthian Semantics. *Mathematical Systems Theory* 13, 1–27 (1979)
6. Ekman, T., Ettinger, R., Schäfer, M., Verbaere, M.: Refactoring bugs (2008), <http://progtools.comlab.ox.ac.uk/refactoring/bugreports>
7. Ekman, T., Hedin, G.: The JastAdd Extensible Java Compiler. In: OOPSLA (2007)
8. Farrow, R.: Automatic generation of fixed-point-finding evaluators for circular, but well-defined, attribute grammars. In: CC, pp. 85–98 (1986)
9. Gosling, J., Joy, B., Steele, G., Bracha, G.: The Java Language Specification (2005)
10. Hanson, D., Proebsting, T.: A research C# compiler. *SPE* 34(13) (2004)
11. Hedin, G.: Reference Attributed Grammars. *Informatica* (24), 301–317 (2000)
12. Huet, G.P.: The Zipper. *J. Funct. Program.* 7(5), 549–554 (1997)
13. Igarashi, A., Pierce, B.C.: On inner classes. *Inf. and Comp.* 177(1), 56–89 (2002)
14. Igarashi, A., Pierce, B.C., Wadler, P.: Featherweight Java: A Minimal Core Calculus for Java and GJ. In: OOPSLA, pp. 132–146 (1999)
15. Johnsson, T.: Attribute grammars as a functional programming paradigm. In: Kahn, G. (ed.) *FPCA 1987. LNCS*, vol. 274, pp. 154–173. Springer, Heidelberg (1987)
16. Knuth, D.E.: Semantics of context-free languages. *Mathematical Systems Theory* 2, 127–146 (1968); correction: *Mathematical Systems Theory* 5, 95–96 (1971)
17. Lacey, D., Jones, N.D., Van Wyk, E., Frederiksen, C.C.: Proving Correctness of Compiler Optimizations by Temporal Logic. In: POPL, pp. 283–294. ACM Press, New York (2002)

18. Lerner, S., Millstein, T., Rice, E., Chambers, C.: Automated soundness proofs for dataflow analyses and transformations via local rules. *SIGPLAN Not.* 40(1), 364–377 (2005)
19. Leroy, X.: Formal certification of a compiler back-end. In: *POPL*, pp. 42–54 (2006)
20. Magnusson, E., Hedin, G.: Circular Reference Attributed Grammars. *Sci. Comput. Program.* 68(1), 21–37 (2007)
21. von Oheimb, D., Nipkow, T.: Machine-checking the java specification: Proving type-safety. In: Alves-Foss, J. (ed.) *Formal Syntax and Semantics of Java*. LNCS, vol. 1523, p. 119. Springer, Heidelberg (1999)
22. Palsberg, J.: Closure Analysis in Constraint Form. *TOPLAS* 17(1), 47–62 (1995)
23. Reps, T.W., Teitelbaum, T.: The Synthesizer Generator: A system for constructing language-based editors. In: *Texts and Monographs in Computer Science*. Springer, Heidelberg (1989)
24. Schäfer, M., Ekman, T., de Moor, O.: Sound and Extensible Renaming for Java. In: Kiczales, G. (ed.) *OOPSLA*. ACM Press, New York (2008)
25. Schäfer, M., Ekman, T., de Moor, O.: Formalising and Verifying Reference Attribute Grammars in Coq (Technical Report and Implementation) (2009), <http://progttools.comlab.ox.ac.uk/projects/refactoring/formalising-rags>
26. Sestoft, P.: Analysis and Efficient Implementation of Funct. Prog. Ph.D. thesis (1991)
27. Sewell, P., Nardelli, F.Z., Owens, S., Peskine, G., Ridge, T., Sarkar, S., Strnisa, R.: Ott: effective tool support for the working semanticist. In: *ICFP*, pp. 1–12 (2007)
28. Ekman, T.: JastAdd (2008), <http://www.jastadd.org>
29. Uustalu, T., Vene, V.: Comonadic functional attribute evaluation. In: *TFP* (2005)