Towards Automatic Generation of Conceptual Interpretation of Clustering^{*}

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Abstract. In this paper the *Methodology of conceptual characterization by embedded conditioning CCEC*, oriented to the automatic generation of conceptual descriptions of classifications that can support later decision-making is presented, as well as its application to the interpretation of previously identified classes characterizing the different situations on a WasteWater Treatment Plant (WWTP). The particularity of the method is that it provides an interpretation of a partition previously obtained on an *ill-structured domain*, starting from a hierarchical clustering. The methodology uses some statistical tools (as the *boxplot multiple*, introduced by Tukey, which in our context behave as a powerful tool for numeric variables) together with some machine learning methods, to learn the structure of the data; this allows extracting useful information (using the concept of *characterizing variable*) for the automatic generation of a set of useful rules for later identification of classes. In this paper the usefulness of *CCEC* for building domain theories as models supporting later decision-making is addressed.

Keywords: Hierarchical clustering, class interpretation, Knowledge Discovery and Data Mining.

1 Introduction

In automatic classification where the classes composing a certain domain are to be discovered, one of the most important required processes and one of the less standardized one, is the interpretation of the classes, closely related with *validation*[1] and critical in the later usefulness of the discovered knowledge. The interpretation of the classes, so important to understand the meaning of the obtained classification as well as the structure of the domain, use to be done in an artistic-like way. But this process becomes more and more complicate as the number of classes grows. This work tries to face the problem of the automatic generation of useful interpretations of classes in such a way that decisions about the treatment of a new object can be modelled, to develop, in the long term, a software tool which supports decision-making.

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Conceptual characterization by embedded conditioning (CCEC)[2] is the proposal presented in this paper to cover these goals. Given a clustering, *CCEC* provides an automatic interpretation for it that supports the later construction of intelligent decision support systems for assisting decision-making. A particular application to WWTP is addressed. The presented proposal integrates different findings from a series of previous works [3][4] in a single methodological tool which takes advantage of the hierarchical structure of the clustering to overcome some of the limitations observed in [3], [5]. The application is presented in next section in order to be used for illustration in §3.

This paper is organized as follows: After the introduction, WWTP are presented in §2 as well as, the description of the specific data base that has been analyzed, the basic concepts are in §3. The §4 presents the proposed methodology *CCEC* and the results of applying *CCEC* to the data described in §5. Finally in §6 the conclusions and the future work are addressed.

2 Application

In this paper a real WWTP is used to validate the proposed methodology [4]. First, a very brief description of the general WWTP process is briefly introduced the water flows sequentially through three or four processes (see [6] for a detailed description of the process): (i): In the *pretreatment*, an initial separation of solids from wastewater is performed. (ii) *Primary* treatment consists of leaving the wastewater in a settler for some hours. Solids will deposit down the settler and could be sent out. (iii) *Secondary* treatment occurs inside a biological reactor; a population of microorganisms (biomass) degrades the organic matter solved in the wastewater. (iv) In the *advanced* treatment another settler is used to separate the water from the biomass. The settler output (solids or biomass) produces a kind of mud which is the input of another process called *sludge line*. Next the plant used in this research described is a WWTP is located in Catalunya (Spain). Data analyzed in this work is a sample of 396 observations taken from September the first of 1995 to September the 30th of 1996. Each observation refers to a daily mean, and it is identified by the date itself.

The state of the Plant is described through a set of 25 variables, considered the more relevant upon expert's opinions. They can be grouped as: (i) Input (measures taken at the entrance of the plant): Q-E: Inflow wastewater (daily m^3 of water); FE-E Iron pre-treatment (g/l); pH-E; SS-E: Suspended Solids (mg/l); SSV-E: Volatile suspended solids (mg/l); COD-E: Chemical organic matter (mg/l); BOD-E: Biodegradable organic matter (m/l). (ii) After Settler (measures taken when the wastewater comes out of the first settler): PH-D: pH; SS-D: Suspended Solids (mg/l); SSV-D: Volatile suspended solids (mg/l); COD-D: Chemical organic matter (mg/l); BOD-D: Biodegradable organic matter (m/l). (iii) Biological treatment (measures taken in the biological reactor): Q-B: Biological reactor-flow; V30: Index at the biological reactor (measures the sedimentation quality of the mixed liquor, in ml/l); MLSS-B: Mixed liquor suspended solids at the biological reactor; MLVSS-B: Mixed liquor volatile

suspended solids: MCRT-B: Mean cell residence time at the biological reactor. (iv) Output (when the water is meeting the river): PH-S: pH ; SS-S: Suspended Solids (mg/l); SSV-S: Volatile suspended solids (mg/l); COD-S: Chemical organic matter (mg/l); BOD-S: Biodegradable organic matter (m/l). (v) Other variables: QR-G: Recirculated Flow ; QP-G: Purged flow; QA-G: Air inflow.

Fig. 1. Left: Hierarchical tree $[\tau_{Gi2,R1}^{En,G}][2]$; Right: Notation

The data base previously presented, was classified in a previous work [4] by using clustering based on rules [3], producing the hierarchical tree of Figure 1(left).

The Figure 2 contains the *class panel graph* [7] of the 25 variables regarding the top partition $\mathcal{P}_2 = \{C_{393}, C_{392}\}$ where the *multiple boxplot*[8] of variables for each class are displayed. As usual in hierarchical clustering, the final partition is the horizontal cut of the tree that maximizes the ratio between *heterogeneity* between classes with respect to *homogeneity* within classes, what guaranties the *distinguishability* of classes. The result is a 4-classes partition $\mathcal{P}_4 = \{C_{392}, C_{389}, C_{390}, C_{383}\}$.

3 Basic Concepts

Four main concepts are in the kernel of this work (basic notation is introduced in Figure 1(right)):

- Variable X_k is *Totally characterizing* of class $C \in \mathcal{P}$, if all the values taken by X_k in class C are *characteristic* of C (objects of other classes take other values). As an example, see variables Q-E or QR-G in Figure 2.
- Variable X_k is *Partially characterizing* of class $C \in \mathcal{P}$ if there is at least one characteristic value of C , although the class can share some other values

Fig. 2. class panel graph of $\mathcal{P}_2$

with other classes. See PH-E in Figure 2, which is partially characterizing the class C_{392} if it takes very low or high values.

- Covering (Cov): Given a rule $r : x_{ik} \in I_s^k \rightarrow C$ the covering of r is the number of objects in $\mathcal{I}$ that satisfy the rule. $Cov(r) = \text{card}\{i \in C \text{ tq } x_{ik} \in I_s^k\}$
- Relative covering ($CovR$): Given a rule, the relative covering is the proportion of of class C that satisfy the rule. $CovR(r) = \frac{\text{card}\{i \in C \text{ tq } x_{ik} \in I_s^k\}}{\text{card}\{C\}} * 100$

Boxplot based discretization (BbD) is presented in [2], [9] and revised in [10], as an efficient way of transforming a numerical variable into a qualitative one in such a way that the cut points for discretizing identify where the subsets of intersecting classes between classes change and, given $X_k, \mathcal{P}$, it consists of the following steps:

1. Calculate the *minimum* (m_C^k) and *maximum* (M_C^k) of X_k inside any class. Built $\mathcal{M}^k = \{m_{C_1}^k, \dots, m_{C_\xi}^k, M_{C_1}^k, \dots, M_{C_\xi}^k\}$, where $\text{card}(\mathcal{M}^k) = 2\xi$
2. Built the *set of cutpoints* $\mathcal{Z}^k$ by sorting $\mathcal{M}^k$ in increasing way into $\mathcal{Z}^k = \{z_i^k ; i = 1 : 2\xi\}$. At every z_i^k the intersection between classes changes arity.
3. Built the *set of intervals* I_s^k induced by $\mathcal{P}$ on X_k by defining an interval I_s^k between every pair of consecutive values of $\mathcal{Z}^k$. $\mathcal{I}^k = \{I_1^k, \dots, I_{2\xi-1}^k\}$ is the boxplot based discretization of X_k . The I_s^k , $s \in \{1, 2\xi - 1\}$, intervals are variable length and the intersection among classes changes from one to another.

As an example, Table 1(left) contains descriptive statistics of Q-E versus $\mathcal{P}_2$. In the right hand side of Table 1 the set of minimums and maximums of every class

Table 1. (left): Summary statistics for Q-E vs $\mathcal{P}_2$; (right): $\mathcal{M}^{Q-E}$, Set of extreme values of Q-E| $C \in \mathcal{P}_2$ and $\mathcal{Z}^{Q-E}$, corresponding ascendant sorting

	Class	C393	C392
Var.	N = 396	$n_c = 390$	$n_c = 6$
Q-E	$\bar{X}$	42,112.9453	22,563.7988
	S	4,559.2437	1,168.8481
	min	29,920	20,500
	max	54,088.6016	23,662.9004
	N*	1	0

$\mathcal{M}^{Q-E}$	$\mathcal{Z}^{Q-E}$
29,920	20,500
20,500	23,662.9004
54,088.6016	29,920
23,662.9004	54,088.6016

$\mathcal{M}^{Q-E}$ is sorted into $\mathcal{Z}^{Q-E}$ and its values can be considered as the cut points of a discretization of Q-E in 3 intervals: $I^{Q-E} = \{I_1^{Q-E}, I_2^{Q-E}, I_3^{Q-E}\}$, with $I_1^{Q-E} = [20500.0, 23662.9]$, $I_2^{Q-E} = (23662.9, 29920.0)$, $I_3^{Q-E} = [29920.0, 54088.6]$. Thus, I^{Q-E} is a discretization of X_k which takes 3 values. Only elements of C_{392} can take value I_1^{Q-E} . Only elements of C_{393} take value I_3^{Q-E} and I_2^{Q-E} is empty. For variable $I^{SS-S} = \{I_1^{SS-S}, I_2^{SS-S}, I_3^{SS-S}\}$, with $I_1^{SS-S} = [2.80, 3.200]$, $I_2^{SS-S} = [3.20, 20.00]$, $I_3^{SS-S} = (20.0, 174.8]$. Only elements of C_{391} can take value I_1^{SS-S} ; along I_2^{SS-S} , C_{391} is intersecting with C_{389} and along I_3^{SS-S} does not intersect. The values of $\mathcal{M}^k$ are identifying the points where intersections among classes change.

The methodology *boxplot based induction rules (BbIR)* is presented in [9]. It is a method for generating probabilistic concepts with a minimum number of attributes in the antecedent, using of the boxplot based discretization of X_k .

1. Use the *boxplot based discretization* to build $r_s : \mathcal{I}^k = \{I_1^k, I_2^k, I_3^k, \dots, I_{2\xi-1}^k\}$.
2. Build table $I^k \times P$ where rows are indexed by $s \in \{1, 2\xi - 1\}$, and columns by $C \in \{1, \xi\}$.
3. Built the table $I^k | \mathcal{P}$ by dividing the cells of $I^k \times \mathcal{P}$ by the row totals. Cells, $p_{sc} = P(C|I^k = I_s^k) = P(i \in C | x_{ik} \in I_s^k) = \frac{\text{card}\{i : x_{ik} \in I_s^k \wedge i \in C\}}{\text{card}\{i \in \mathcal{I} : x_{ik} \in I_s^k\}} = \frac{n_{sc}}{\sum_{\forall s} n_{sc}}$
If $\exists C : p_{sc} = 1$ & $\text{Cov}(r_s) = 100\%$, then X_k is a *totally characterizing* variable of C .
4. For every cell in table $I^k | \mathcal{P}$ produce the rule: If $x_{ik} \in I_s^k \xrightarrow{p_{sc}} i \in C$
For example, see Table 2(left) $I^{Q-E} \times P_2$, since $\text{card}\{i \in \mathcal{I} : x_{ik} \in I_1^{Q-E}\} = \text{card}\{i \in \mathcal{I} : i \in C_{392}\} = \text{card}\{i \in \mathcal{I} : x_{ik} \in I_1^{Q-E} \wedge i \in C_{392}\} = 6$. Then $p_{1C_{392}} = 1$ and the rule: $x_{Q-E,i} \in (23662.9, 29920.0] \xrightarrow{1.0} C_{392}$ is generated.

From $I^{Q-E} | \mathcal{P}_2$ (see Table 2(center)) the following system of probabilistic rules, can be induced for the variable Q-E:

$$\begin{aligned}
 r_{1, \text{classer}393}^{Q-E} : x_{Q-E,i} \in [20500.0, 23662.9] &\xrightarrow{0.0} \text{classer}393 \\
 r_{1, \text{classer}392}^{Q-E} : x_{Q-E,i} \in [20500.0, 23662.9] &\xrightarrow{1.0} \text{classer}392 \\
 r_{3, \text{classer}393}^{Q-E} : x_{Q-E,i} \in [29920.0, 54088.6] &\xrightarrow{1.0} \text{classer}393 \\
 r_{3, \text{classer}392}^{Q-E} : x_{Q-E,i} \in [29920.0, 54088.6] &\xrightarrow{0.0} \text{classer}392
 \end{aligned}$$

Finally too more concepts are used in this work.

Table 2. (left): $I^{Q-E} \times P_2$; (center): $I^{Q-E} \mid P_2$ table; (right): $P_2 \times P_3$

	C_{392}^2	C_{393}^2
I_1^{Q-E}	6	0
I_2^{Q-E}	0	0
I_3^{Q-E}	0	389

	C_{392}^2	C_{393}^2
I_1^{Q-E}	1	0
I_2^{Q-E}	0	0
I_3^{Q-E}	0	1

P_3 vs P_2	C_{393}^2	C_{392}^2
C_{392}^3	0	6
C_{391}^3	320	0
C_{389}^3	70	0

4 The Metodology

CCEC takes advantage of the existence of τ , and uses the property of any binary hierarchical structure that $\mathcal{P}_{\xi+1}$ has the same classes of $\mathcal{P}_\xi$ except one, which splits in two subclasses in $\mathcal{P}_{\xi+1}$. Binary hierarchical structure will be used by *CCEC* to discover particularities of the final classes step by step also in hierarchical way. The *CCEC* [2] allows generation of automatic conceptual interpretations of a given partition $\mathcal{P} \in \tau$. The steps to be followed are described bellow. The application to the WWTP is illustrated in section §5:

1. Cut the tree at highest level (make $\xi = 2$ and consider $\mathcal{P}_2 = \{C_1, C_2\}$).
2. Use the *boxplot based discretization* presented in [9] and revised in [10], to find (total or partial) characteristic values for numerical variables [5].
3. Use *boxplot based induction rules (BbIR)*, to induce a the knowledge Base describing both classes.
4. For classes in $\mathcal{P}_2$, determine concepts $A_1^{\xi, X_k} : "[X_k \in I_s^k]"$, $A_2^{\xi, X_k} : \neg A_1^{\xi, X_k}$ associated to C_1, C_2 , by taking the intervals provided by a totally characteristic variable or the partial one with greater relative covering and $p_{sc} = 1$.
5. Go down one level in the tree, by making $\xi = \xi + 1$ and so considering $\mathcal{P}^{\xi+1}$. As said before $\mathcal{P}^{\xi+1}$ is *embedded* in $\mathcal{P}^\xi$ in such a way that there is a class of $\mathcal{P}^\xi$ split in two new classes of $\mathcal{P}^{\xi+1}$, namely $C_i^{\xi+1}$ and $C_j^{\xi+1}$ and all other classes are common to both partitions. Since in the previous step $C_i^{\xi+1} \cup C_j^{\xi+1}$ were conceptually separated from the rest, at this point it is only required to find the variables which separate (or distinguishes) $C_i^{\xi+1, X_k}$ from $C_j^{\xi+1, X_k}$, by repeating steps 2-4. Suppose $B_i^{\xi+1, X_k}$ and $B_j^{\xi+1, X_k}$ the concepts induced from $C_i^{\xi+1, X_k}$ and $C_j^{\xi+1, X_k}$, in the step $\xi + 1$.
6. Integrate the extracted knowledge of the iteration $\xi + 1$ with that of the iteration ξ , by determining the compound concepts finally associated to the elements of $\mathcal{P}_{\xi+1}$. The concepts for the classes of $\mathcal{P}_{\xi+1}$ will be: $A_q^{\xi+1, X_k} = A_q^{\xi, X_k}$, $A_i^{\xi+1, X_k} = \neg A_q^{\xi, X_k} \wedge B_i^{\xi+1, X_k}$ and $A_j^{\xi+1, X_k} = \neg A_q^{\xi, X_k} \wedge B_j^{\xi+1, X_k}$.
7. Make $\xi = \xi + 1$, and return to the step 2) repeating until $\mathcal{P}_\xi = \mathcal{P}$.

5 Results

The 2-class Partition. For the presented data, $\mathcal{P}_2 = \{C_{393}, C_{392}\}$. Figure 2 shows the *class-panel graph* of $\mathcal{P}_2$ [7]. As stated in §3, I^k is built using *Boxplot*

based discretization (*BbD*) for all variables. Table 2(left) shows the crossing of I^{Q-E} and $\mathcal{P}_2$. Then the $I^k \mid \mathcal{P}_2$ table is built and totally characterizing variables are identified as those with $I^k \mid \mathcal{P}$ table having one 1 per row. Next is to use *BbIR* to generate all the rules induced for $\mathcal{P}_2$. Only rules with $p_{sc} = 1$ are considered. Totally characterizing variables are those with $p_{sc} = 1$ and $CovR = 100\%$ (Q-E, QB-B, QR-G, QP-G, MCRT-B). In this case any of them is useful for conceptualization. Here Q-E is choosen. The Table 3 shown the knowledge Base whit certain rules ($p_{sc} = 1$) for *classer392*.

Since Q-E is a totally characterizing variable the following concept association can be done:

- $A_{C_{392}}^{2,Q-E} = "x_{Q-E,i} \in [20500.0, 23662.9]"$ is associated to C_{392}
- $A_{C_{393}}^{2,Q-E} = "x_{Q-E,i} \in [29920.0, 54088.6]"$ is associated to C_{393}

Or, in other words:

- Class C_{392} , "Low values of Inflow wastewater".
- Class C_{393} , "High values of Inflow wastewater".

Being *low* and *high* defined according to the above numeric limits. Any totally characterizing variable could be used instead. The next step is to go one level down the tree.

The 3-class partition. Take $\mathcal{P}_3 = \{C_{392}, C_{391}, C_{389}\}$ and first identify correspondences between the classes of $\mathcal{P}_2 = \{C_{393}, C_{392}\}$ and $\mathcal{P}_3 = \{C_{392}, C_{391}, C_{389}\}$. Table 2(right) shows the crossing and elicits that class C_{393} splits into C_{391} (referred as C_i^3 in methodology), C_{389} (referred as C_j^3 in methodology) while C_{392} remains in both $\mathcal{P}_2$ and $\mathcal{P}_3$ (C_{392} is referred as C_i^2 in methodology). From the previous iteration it is already known that there is a common characteristic of both C_{391} and C_{389} (Q-E is greater) which distinguishes them from C_{392} (with lower Q-E). Thus, it is only required to find the separation between C_{391} and C_{389} . A similar procedure as the used in previous step for separating C_{392} and C_{393} was used with all the variables regarding C_{391} and C_{389} .

Totally characterizing variables do not exists here. So partially characterizing variables with greater relative covering are considered, in this case SS-D and SSV-D, using variable SS-D for this example. The following system of probabilistic rules than can be induced for the variable SS-D in $\mathcal{P}_3 \setminus \{C_{392}\}$:

$$\begin{aligned} r_{2, \text{classer391}}^{SS-D} : x_{SS-D,i} \in [63, 136] &\xrightarrow{0.8547} \text{classer391} \\ r_{3, \text{classer391}}^{SS-D} : x_{SS-D,i} \in (136, 192] &\xrightarrow{1.0} \text{classer391, being } B_{391}^{3,SS-D} = x_{SS-D,i} \in (136, 192] \\ r_{1, \text{classer389}}^{SS-D} : x_{SS-D,i} \in [48.0, 63] &\xrightarrow{1.0} \text{classer389, being } B_{389}^{3,SS-D} = x_{SS-D,i} \in [48, 63] \end{aligned}$$

Combining with the results of previous iteration of *CCEC* leads on the following interpretation of $\mathcal{P}_3$:

- Class C_{392} is such that "Q-E is in $[20500.0, 23662.9]"$ $(A_{392}^{2,Q-E})$
- Class C_{391} is such that "Q-E is in $[29920.0, 54088.6]"$ and "SS-D is in $(136, 192]"$ $(\neg A_{392}^{2,Q-E} \wedge B_{391}^{3,SS-D})$
- Class C_{389} is such that "Q-E is in $[29920.0, 54088.6]"$ and "SS-D is in $[48, 63]"$ $(\neg A_{392}^{2,Q-E} \wedge B_{389}^{3,SS-D})$

Table 3. Knowledge Base for classer392

Concep.	Rules	Cov	CovR
$A_{392}^{2,Q-E}$	$r_{1,classer392}^{Q-E} : x_{Q-E,i} \in [20500.0, 23662.9] \xrightarrow{1.0} classer392$	6	100%
$A_{392}^{2,QB-B}$	$r_{1,classer392}^{QB-B} : x_{QB-B,i} \in [19883.0, 22891.0] \xrightarrow{1.0} classer392$	6	100%
$A_{392}^{2,QR-G}$	$r_{1,classer392}^{QR-G} : x_{QR-G,i} \in [17932.6, 18343.5] \xrightarrow{1.0} classer392$	6	100%
$A_{392}^{2,QP-G}$	$r_{1,classer392}^{QP-G} : x_{QP-G,i} \in [0.0, 0.0] \xrightarrow{1.0} classer392$	6	100%
$A_{392}^{2,QA-G}$	$r_{1,classer392}^{QA-G} : x_{QA-G,i} \in [96451.0, 124120.0] \xrightarrow{1.0} classer392$	3	50%
$A_{392}^{2,SSV-E}$	$r_{1,classer392}^{SSV-E} : x_{SSV-E,i} \in [19.0, 30.0] \xrightarrow{1.0} classer392$	1	16,7%
$A_{392}^{2,DQO-E}$	$r_{1,classer392}^{DQO-E} : x_{DQO-E,i} \in [27.0, 100.0] \xrightarrow{1.0} classer392$	3	50%
$A_{392}^{2,DBO-E}$	$r_{2,classer392}^{DBO-E} : x_{DBO-E,i} \in [73.0, 73.0] \xrightarrow{1.0} classer392$	1	16,7%
$A_{392}^{2,SS-D}$	$r_{1,classer392}^{SS-D} : x_{SS-D,i} \in [40.0, 48.0] \xrightarrow{1.0} classer392$	2	33,33%
$A_{392}^{2,SSV-D}$	$r_{1,classer392}^{SSV-D} : x_{SSV-D,i} \in [13.0, 30.0] \xrightarrow{1.0} classer392$	3	50%
$A_{392}^{2,DQO-D}$	$r_{1,classer392}^{DQO-D} : x_{DQO-D,i} \in [27.0, 90.0] \xrightarrow{1.0} classer392$	3	50%
$A_{392}^{2,DBO-D}$	$r_{2,classer392}^{DBO-D} : x_{DBO-D,i} \in [54.0, 54.0] \xrightarrow{1.0} classer392$	1	16,7%
$A_{392}^{2,MLSS-B}$	$r_{3,classer392}^{MLSS-B} : x_{MLSS-B,i} \in [2978.0, 3294.0] \xrightarrow{1.0} classer392$	2	33,33%
$A_{392}^{2,MLVSS-B}$	$r_{3,classer392}^{MLVSS-B} : x_{MLVSS-B,i} \in [2054.0, 2100.0] \xrightarrow{1.0} classer392$	1	16,7%
$A_{392}^{2,MCRT-B}$	$r_{3,classer392}^{MCRT-B} : x_{MCRT-B,i} \in [179.8, 341.99] \xrightarrow{1.0} classer392$	6	100%

Table 4. Knowledge Base for classer389

Concep.	Rules	Cov	CovR
$B_{389}^{3,Q-E}$	$r_{3,classer389}^{Q-E} : x_{Q-E,i} \in (52255.8, 54088.6] \xrightarrow{1.0} classer389$	3	4,28%
$B_{389}^{3,QB-B}$	$r_{3,classer389}^{QB-B} : x_{QB-B,i} \in (49695.8, 52244.6] \xrightarrow{1.0} classer389$	3	4,28%
$B_{389}^{3,QR-G}$	$r_{1,classer389}^{QR-G} : x_{QR-G,i} \in [26218.0, 27351.0] \xrightarrow{1.0} classer389$	2	2,85%
$B_{389}^{3,QP-G}$	$r_{1,classer389}^{QP-G} : x_{QP-G,i} \in [188.0, 327.6] \xrightarrow{1.0} classer389$	2	2,85%
$B_{389}^{3,QA-G}$	$r_{1,classer389}^{QA-G} : x_{QA-G,i} \in [124120.0, 136371.0] \xrightarrow{1.0} classer389$	2	2,85%
$B_{389}^{3,PH-E}$	$r_{3,classer389}^{PH-E} : x_{PH-E,i} \in (7.9, 8.0] \xrightarrow{1.0} classer389$	1	1,42%
$B_{389}^{3,SS-E}$	$r_{1,classer389}^{SS-E} : x_{SS-E,i} \in [62.0, 82.0] \xrightarrow{1.0} classer389$	1	1,42%
$B_{389}^{3,SSV-E}$	$r_{1,classer389}^{SSV-E} : x_{SSV-E,i} \in [30.0, 60.0] \xrightarrow{1.0} classer389$	2	2,85%
$B_{389}^{3,DQO-E}$	$r_{1,classer389}^{DQO-E} : x_{DQO-E,i} \in [100.0, 158.0] \xrightarrow{1.0} classer389$	1	1,42%
$B_{389}^{3,DBO-E}$	$r_{1,classer389}^{DBO-E} : x_{DBO-E,i} \in [69.0, 90.0] \xrightarrow{1.0} classer389$	5	7,14%
$B_{389}^{3,SS-D}$	$r_{1,classer389}^{SS-D} : x_{SS-D,i} \in [48.0, 63.0] \xrightarrow{1.0} classer389$	24	34,28%
$B_{389}^{3,SSV-D}$	$r_{1,classer389}^{SSV-D} : x_{SSV-D,i} \in [30.0, 47.0] \xrightarrow{1.0} classer389$	25	35,71%
$B_{389}^{3,DBO-D}$	$r_{1,classer389}^{DBO-D} : x_{DBO-D,i} \in [36.0, 56.0] \xrightarrow{1.0} classer389$	5	7,14%
$B_{389}^{3,DBO-S}$	$r_{1,classer389}^{DBO-S} : x_{DBO-S,i} \in [2.0, 4.0] \xrightarrow{1.0} classer389$	2	2,85%
$B_{389}^{3,MLSS-B}$	$r_{3,classer389}^{MLSS-B} : x_{MLSS-B,i} \in (2696.0, 2978.0] \xrightarrow{1.0} classer389$	1	1,42%
$B_{389}^{3,MCRT-B}$	$r_{3,classer389}^{MCRT-B} : x_{MCRT-B,i} \in (28.8, 34.4] \xrightarrow{1.0} classer389$	1	1,42%

Since anyone of these interpretations is based on non totally characterizing variables, some degree of certainty should be associated depending on the probabilities of the corresponding generated concepts. The interpretation with the highest global degree of certainty is the one that intends to be refined in the following iteration. Table 4 shows the Knowledge Base with certain rules ($p_{sc} = 1$) induced for *classer389*.

Table 5. Knowledge Base for classer383

Concep.	Rules	Cov	CovR
$\neg C_{390}^{4,SS-E}$	$r_{3,classer383}^{SS-E} : x_{SS-E,i} \in (480.0, 655.0] \xrightarrow{1.0} classer383$	5	14,70%
$\neg C_{390}^{4,SSV-E}$	$r_{3,classer383}^{SSV-E} : x_{SSV-E,i} \in (336.0, 593.0] \xrightarrow{1.0} classer383$	4	11,76%
$\neg C_{390}^{4,DQO-E}$	$r_{3,classer383}^{DQO-E} : x_{DQO-E,i} \in (1279.0, 1579.0] \xrightarrow{1.0} classer383$	1	2,94%
$\neg C_{390}^{4,DBO-E}$	$r_{3,classer383}^{DBO-E} : x_{DBO-E,i} \in (382.0, 987.0] \xrightarrow{1.0} classer383$	7	20,58%
$\neg C_{390}^{4,MLVSS-B}$	$r_{1,classer383}^{MLVSS-B} : x_{MLVSS-B,i} \in [185.0, 611.0) \xrightarrow{1.0} classer383$	2	5,88%
$\neg C_{390}^{4,MCRT-B}$	$r_{1,classer383}^{MCRT-B} : x_{MCRT-B,i} \in [1.8, 6.2) \xrightarrow{1.0} classer383$	1	2,94%

The final partition. The process would continue separating the classes C_{390} and C_{383} of the following partition $\mathcal{P}_4$, see figure 1(left), which are the subdivision of C_{391} . Similarly, the following interpretation of $\mathcal{P}_4$, which is the final partition [4] is obtained with the following Knowledge Base with certain rules for classer383(Table 5) and classer390 (Table 6):

And de final interpretation is:

- Class C_{392} is such that $A_{392}^{2,Q-E}$
- Class C_{389} is such that $\neg A_{392}^{2,Q-E} \wedge B_{391}^{3,SS-D}$
- Class C_{390} is such that $\neg A_{392}^{2,Q-E} \wedge \neg B_{391}^{3,SS-D} \wedge C_{390}^{4,DBO-E}$
- Class C_{383} is such that $\neg A_{392}^{2,Q-E} \wedge \neg B_{391}^{3,SS-D} \wedge \neg C_{390}^{4,DBO-E}$. Or, in other words:
- Class C_{392} , “Low values for Inflow wastewater”
- Class C_{389} , “High values for Inflow and few Suspended solids at the settler”
- Class C_{390} , “High values of Inflow wastewater, high Suspended solids at the exit and few Biodegradable organic matter at the input”
- Class C_{383} , “Medium-high values of Inflow wastewater, high Suspended solids at the settler and high Biodegradable organic matter at the input.”

This set of rules can, in fact, be considered as a domain model which can support later decision on the treatment to be applied to a new day, provided that a standard treatment is previously associated to every class by experts. In this association the possibility of easily interpreting the classes is critical as well as to provide the experts means for easily understanding the meaning of the classes. In this sense the proposed method provides simple and short rules which use to be easier to handle than those provided by other induction rules algorithms. This final interpretation is consistent with the one provided by the experts and presented in [4].

6 Conclusions and Future Work

In this paper the first approach of a methodology to generate automatic interpretations from a group of classes is presented. Concepts associated to classes are built taking advantage of hierarchical structure of the underlying clustering. The *Conceptual characterization by embedded conditioning CCEC* [2] is a quick and effective method that generates a conceptual model of the domain, which will be from great support to the later decision making based on a combination

Table 6. Knowledge Base for classer390

Concept.	Rules	Cov	CovR
$C^4_{390}, Q-E$	$r^Q-E_{1,classer390} : x_{Q-E,i} \in [29920.0, 34284.4] \xrightarrow{1.0} classer390$	9	3,15%
$C^4_{390}, QB-B$	$r^{QB-B}_{3,classer390} : x_{QB-B,i} \in (39000.0, 49695.8] \xrightarrow{1.0} classer390$	60	21,06%
$C^4_{390}, QR-G$	$r^{QR-G}_{3,classer390} : x_{QR-G,i} \in (44568.6, 49527.0] \xrightarrow{1.0} classer390$	13	4,56%
$C^4_{390}, QP-G$	$r^{QP-G}_{3,classer390} : x_{QP-G,i} \in (831.1, 1080.0] \xrightarrow{1.0} classer390$	56	19,64%
$C^4_{390}, QA-G$	$r^{QA-G}_{1,classer390} : x_{QA-G,i} \in [136371.0, 156320.0] \xrightarrow{1.0} classer390$	7	2,45%
$C^4_{390}, FE-E$	$r^{FE-E}_{3,classer390} : x_{FE-E,i} \in (63.3, 89.8] \xrightarrow{1.0} classer390$	7	2,45%
$C^4_{390}, PH-E$	$r^{PH-E}_{3,classer390} : x_{PH-E,i} \in (7.8, 7.9] \xrightarrow{1.0} classer390$	8	2,80%
$C^4_{390}, SS-E$	$r^{SS-E}_{1,classer390} : x_{SS-E,i} \in [82.0, 114.0] \xrightarrow{1.0} classer390$	4	1,40%
$C^4_{390}, SSV-E$	$r^{SSV-E}_{1,classer390} : x_{SSV-E,i} \in [60.0, 92.0] \xrightarrow{1.0} classer390$	7	2,45%
$C^4_{390}, DQO-E$	$r^{DQO-E}_{1,classer390} : x_{DQO-E,i} \in [158.0, 414.0] \xrightarrow{1.0} classer390$	113	39,64%
$C^4_{390}, DBO-E$	$r^{DBO-E}_{1,classer390} : x_{DBO-E,i} \in [90.0, 220.0] \xrightarrow{1.0} classer390$	126	44,21%
$C^4_{390}, PH-D$	$r^{PH-D}_{1,classer390} : x_{PH-D,i} \in [7.2, 7.3] \xrightarrow{1.0} classer390$	3	1,05%
$C^4_{390}, SS-D$	$r^{SS-D}_{3,classer390} : x_{SS-D,i} \in (112.0, 192.0] \xrightarrow{1.0} classer390$	30	10,52%
$C^4_{390}, SSV-D$	$r^{SSV-D}_{3,classer390} : x_{SSV-D,i} \in (92.0, 134.0] \xrightarrow{1.0} classer390$	11	3,85%
$C^4_{390}, DQO-D$	$r^{DQO-D}_{3,classer390} : x_{DQO-D,i} \in (329.0, 538.0] \xrightarrow{1.0} classer390$	22	7,71%
$C^4_{390}, DBO-D$	$r^{DBO-D}_{3,classer390} : x_{DBO-D,i} \in (224.0, 274.0] \xrightarrow{1.0} classer390$	4	1,40%
$C^4_{390}, PH-S$	$r^{PH-S}_{3,classer390} : x_{PH-S,i} \in (7.8, 8.0] \xrightarrow{1.0} classer390$	16	5,61%
$C^4_{390}, SS-S$	$r^{SS-S}_{3,classer390} : x_{SS-S,i} \in (29.0, 174.8] \xrightarrow{1.0} classer390$	25	8,77%
$C^4_{390}, SSV-S$	$r^{SSV-S}_{3,classer390} : x_{SSV-S,i} \in (19.0, 134.8] \xrightarrow{1.0} classer390$	38	13,3%
$C^4_{390}, DQO-S$	$r^{DQO-S}_{3,classer390} : x_{DQO-S,i} \in (95.0, 163.0] \xrightarrow{1.0} classer390$	24	8,4%
$C^4_{390}, DBO-S$	$r^{DBO-S}_{3,classer390} : x_{DBO-S,i} \in (35.0, 84.0] \xrightarrow{1.0} classer390$	4	1,40%
$C^4_{390}, V30-B$	$r^{V30-B}_{1,classer390} : x_{V30-B,i} \in [77.0, 140.0] \xrightarrow{1.0} classer390$	26	9,12%
$C^4_{390}, MLSS-B$	$r^{MLSS-B}_{3,classer390} : x_{MLSS-B,i} \in (2248.0, 2696.0] \xrightarrow{1.0} classer390$	17	5,96%
$C^4_{390}, MLVSS-B$	$r^{MLVSS-B}_{3,classer390} : x_{MLVSS-B,i} \in (1726.0, 2054.0] \xrightarrow{1.0} classer390$	18	6,31%
$C^4_{390}, MCRT-B$	$r^{MCRT-B}_{3,classer390} : x_{MCRT-B,i} \in (16.0, 28.8] \xrightarrow{1.0} classer390$	8	2,81%

of boxplot based discretization and an interactive combination of concepts upon hierarchical subdivisions of the domain. This is a preliminary proposal that has been applied with success to real data coming from a WWTP. Benefits of those proposal are specially interesting in the interpretation of partitions with great number of classes. Automatic generation of interpretations cover the important goal of KD of describing the domain [11]. However, in this proposal a direct connection between the generated concepts and the automatic rules generation allows direct construction of a decision model for the domain oriented to later class prediction. As a matter of fact, automatic production of probabilistic or fuzzy classification rules regarding concepts provided by *CCEC* is direct, as discussed in [12]. By associating an appropriate standard treatment to every class a model for deciding the appropriate treatment of a concrete day upon a reduced number variables is obtained together with an estimation of the risk associated to that decision (which is related with the certainty of the rule). At present other automatic criteria for deciding which variable is kept at every iteration (here the covering of the all concepts was used) and how to propagate uncertainty from

one iteration to the next is in progress (here rules with $p_{sc} = 1$ are used). The idea is to use an approach which avoids the explicit construction of all the concepts to evaluate their coverage. Comparison of rules produced with *CCEC* and other inductive methods ID3-like is also in progress, as well as validation with a second real WWTP from Slovenia.

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