Certainty Measure of Pairwise Line Segment Perceptual Relations Using Fuzzy Logic

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Abstract. Perceptual grouping is an important part of many computer vision systems. When inferring a new grouping from the primitive features there is always an uncertainty degree on this detection, that might be useful in further reasonings. In this paper, we present a fuzzy logic based system for the computation of the certainties assigned to pairwise line segment relations and introduce its application to the detection of continuity, identity, junction, L-junction, incidence, T-junction and parallelism relations. The results presented show that the proposed method might be very promising for future applications.

Keywords: certainty, fuzzy logic, perceptual grouping.

1 Introduction

A major problem in computer vision and image understanding systems is the organisation of the low level features into meaningful higher level features. The bottom-up process of grouping features by means of perceptual principles like proximity, similarity, continuation, closure and symmetry is referred by the computer vision community as perceptual organisation or perceptual grouping [1,2].

The perceptual grouping is present at several stages of the image understanding process. For example, the edges are combined into line segments or curves and these segments are combined into closed contours. In any case, the lower level features are treated as cues for the detection of the higher level groupings that are considered salient for the scene understanding. This process reduces the complexity of the scene and facilitates the operation of higher level processing for object detection.

The target groupings of a perceptual organisation system should fulfil some properties, in order to be considered salient for scene understanding [2]. First, the groupings should represent significant features of the target objects; also, their features should be invariant to projection and illumination changes; finally, the groupings should be non-accidental, i.e. they should not be easily observed by chance. With these properties, if the salient groupings are present in the input image, it is very likely that the target objects have caused them and, also, the object features that the groupings represent could be inferred from the groupings features. However, in most applications, these groupings can not be categorically

detected in base to the cues provided by the lower level features, but with an associated degree of certainty. Dealing with this uncertainty is an important issue as it may affect further reasonings based on the detected groupings. This paper is focused on the detection of pairwise line segment groupings and in the computation of a subjective measure of its certainty.

The detection of pairwise line segment relations has been a main subject in several perceptual organisation applications [3,4,5,6,7]. The analysis of different distance features using threshold based rules, has been used for the categorical detection of pairwise line segment relations in [3]. Other approaches use probabilities as rank for perceptual relation detection [4,6,5,7]. The idea of non-accidentalness probability for the grouping is used in [4]. However, it is necessary to make assumptions about the distribution of the relation evidences in the background of the target images in order to compute these probabilities. Other approaches like those based in Bayesian inference also need to estimate conditional probabilities about the accidental occurrence of evidences in the image, machine learning techniques have been used to address this problem in [5]. Fuzzy sets have been also used for the detection of perceptual relations in [6,7], these approaches use fuzzy predicates for representing the grouping properties and computing a subjective measure of the relations certainty based on the initial definition of their membership functions.

In this paper, we use fuzzy logic for detecting pairwise line segment relations with an assigned uncertainty. Opposite to previous works, we primarily focus our attention on the definition of a general and configurable approach for computing the suitable certainty measures. The general fuzzy inference system proposed is intended to be extensible and tunable based on the needs of further reasonings. Also, an analysis of the suitable properties for the detection of identity, continuity, junction, incidence and parallelism relations between line segments is performed, along with the proposal of new normalisation equations. This paper is structured as follows. Section 2 defines the fuzzy combination of features proposed. Section 3 describes how the method defined in section 2 is applied to the detection of the target relations. Section 4 show the experimental results for the proposed system. Finally, section 5 exposes the conclusions from our work.

2 Perceptual Grouping with Fuzzy Logic

Fuzzy logic [8] is a multi-valued logic that works with predicates which truth values can not be precisely defined as true or false. Instead, their certainty is quantified in the continuous interval $[0, 1]$. Formally, the truth value of a predicate p over the elements $x \in \mathcal{U}$ in the universe of discourse $\mathcal{U}$ is assigned through a membership function $\mu_p : \mathcal{U} \rightarrow [0, 1]$. The membership function μ_p defines the *fuzzy set* of the elements in $x \in \mathcal{U}$ that satisfy p , and its value represents a measure of the certainty on the predicate p given every element in $\mathcal{U}$.

The truth values of the logical expressions containing logical connectives ($\wedge, \vee$) are derived from the truth values of its operands. The membership function of an *and* expression $\mu_{a \wedge b}(x)$ ($x \in \mathcal{U}$) is modelled as a triangular norm, or t-norm,

$\top(\mu_a(x), \mu_b(x))$, while the membership function of an *or* expression $\mu_{a \vee b}(x)$ is modelled as a triangular conorm, or t-conorm, $\perp(\mu_a(x), \mu_b(x))$. The most widely used *t-norm* and *t-conorm* are $\top(a, b) = \min\{a, b\}$ and $\perp(a, b) = \max\{a, b\}$, respectively.

In this work we model pairwise relations between line segments, like parallelism or collinearity, using predicates in fuzzy logic. Our universe of discourse $\mathcal{U}$ is, therefore, the set of all possible segments pairs $x = (s_1, s_2)$ in an image. The membership function $\mu_R : \mathcal{U} \rightarrow [0, 1]$ for a given relation R is modelled as a *logical and* combination:

$$\mu_R(x) = \bigwedge_{f \in \mathcal{F}} \mu_R^f(f(x)) \quad . \quad (1)$$

where $\mathcal{F}$ is a set of features, a feature $f \in \mathcal{F} : \mathcal{U} \rightarrow C_f$ is a distance measured from the input segment pairs, and $\mu_R^f : C_f \rightarrow [0, 1] \ (\forall f \in \mathcal{F})$ is the membership function of a fuzzy relation R , defined over the set of values C_f that f can take.

The previous definition is flexible enough for representing the most usual relations in perceptual organisation. The definition of the relations and the certainty values associated to them can be refined by adding new features or by adjusting the membership functions. This is very important since the certainty values are likely to be used in further reasoning to detect higher level groupings and, thus, the membership functions may depend on the concrete application domain.

Note that the membership functions over the features could have been modelled as logical combinations of fuzzy linguistic terms (i.e. distance is *low*, *very low*, *medium*, etc.). We think that this option would lead to an extremely difficult to adjust system, since the fuzzy linguistic partitions would be shared by different relations that would add dependencies. In return, it would add an easier to understand knowledge representation of the reasonings, but we think that the knowledge behind the relations in perceptual organisation is simple enough to understand using our simple approach.

3 Line Segment Relations

As previously mentioned, this paper is focused on the detection of several perceptual relations between line segment pairs, concretely: junction, L-junction, T-junction, incidence, parallelism, continuity and identity. Most of these relations are of high significance in different kinds of images, including those containing man-made objects. Section 3.2 describes the features used for the detection and section 3.2 defines the membership functions of the target relations.

3.1 Perceptual Features

Given two line segments s_1 and s_2 , the features used for the detection of perceptual relations between them, depicted in figure 1, are: the minimum distance D , the orientation difference α , the overlaps O_{AB} and O_{BA} , and the signed distances from both segments to the intersection of their elongations, I_A and I_B .

Fig. 1. Perceptual features for pairs of line segments

These distances are normalised in order to preserve the salience of the relations regardless of the segment lengths. The normalisation is very usual in perceptual organisation and the distances are commonly normalised by the length of the longest segment [3,6,5,7]. In this work, we propose a new approach for the normalisation, that is described below.

Given the detection of a junction between two segments, a decisive feature would be the Euclidean distance of both segments to their intersection point. If we normalise this distance by the length of the longest segment, then the longer this segment is, the larger this distance can be before a significant loss in the certainty of the relation occurs. For this reason, using this normalisation long segments are more suitable for being related with other segments regardless of their length, which could lead to undesirable high confidence relations with distant and short segments.

Opposite to this, if we normalise this distance by the length of the shortest segment, then weak and blurred edges, that are likely to be detected as short segments, may be involved in meaningful relations that are not detected this way. In many cases, the decision of normalising by one segment length or the other is application dependant. For this reason, the use a weighted average of the segment lengths for normalisation, with parameters that can be independently specified for each feature in base to the final application.

Given two line segments s_1 and s_2 , the normalisation length $N_f(s_1, s_2)$ for each feature f is defined as:

$$N_f(s_1, s_2) = w_f \max\{L_f^*(s_1), L_f^*(s_2)\} + (1 - w_f) \min\{L_f^*(s_1), L_f^*(s_2)\} \quad (2)$$

$$L_f^*(X) = \min\{\lambda_f^+, \max\{\lambda_f^-, L(X)\}\} \quad (3)$$

where $L(X)$ is the length of the segment X , λ^- and λ^+ are the minimum and maximum segment length normalisation thresholds, and $w_f \in [0, 1]$ is a weighting parameter, used to choose normalisation values varying from the length of the shortest segment when $w_f = 0$, to the length of the longest segment when $w_f = 1$. The parameters λ_f^- and λ_f^+ are chosen in base to the application domain: λ_f^- is the shortest line segment length allowed; λ_f^+ is the maximum segment length considered, with larger lengths truncated to λ_f^+ , since larger line segment lengths do not imply a larger proportional distance for the detection.

The minimum distance $D(s_1, s_2)$, if the segments do not intersect, is computed as the minimum Euclidean distance from each segment's end points to the other segment. If the segments intersect, $D(s_1, s_2)$ is set to 0. In any case, this distance is normalised by $N_D(s_1, s_2)$.

The signed distance to the intersection $I_A(s_1, s_2)$ is computed as the Euclidean distance from the segment s_1 to the intersection point of both segments' elongations. This distance is negative if the intersection point lies inside the s_1 and positive if it lies outside. $I_B(s_1, s_2)$ is computed analogously using s_2 , and both distances are normalised by $N_I(s_1, s_2)$. Note that these distances could be infinite if the orientation difference between line segments was low, for this reason, we truncate this feature to the interval $[-1, 1]$.

The $\alpha(s_1, s_2)$ feature is computed as the orientation difference between segments. No segment length dependant normalisation is applied in this case.

The overlap $O(s_1, s_2)$ is defined as the maximum overlap between line segments. It is computed as $\max\{O_{AB}(s_1, s_2), O_{BA}(s_1, s_2)\}$, where $O_{AB}(s_1, s_2)$ and $O_{BA}(s_1, s_2)$ are the lengths of the projections of s_1 and s_2 over s_2 and s_1 , respectively. Both O_{AB} and O_{BA} are normalised by $\max\{L(s_1), N_O(s_1, s_2)\}$ and $\max\{L(s_2), N_O(s_1, s_2)\}$, respectively, in such a way that large proportional overlaps of short segments over long segments are penalised.

3.2 Membership Functions

As previously stated in eq. 1, the membership function of each pairwise relation is computed as the *logical and* of the membership functions of the corresponding decisive features. In this section, we define the relations analysed in this work and the membership functions associated to them.

Two line segments are identical if they correspond to the same straight edge. In many cases, segment extraction algorithms detect a blurred edge as two or three overlapped line segments, these line segments can be merged in one as they probably represent an unique straight contour. The membership functions used for the detection of identical line segments are depicted in figure 2. The identity relation depends on the distance D , the orientation difference α and the overlap O . Both, distance D and orientation difference α must be very low in identity relations, as reflected in the functions depicted at figures 2(a) and 2(b) respectively. It is also necessary a significant overlap between lines in order to state the identity, the membership function for the identity relation over the overlap O is depicted in figure 2(c).

Opposite to the identical line segments, two line segments are continuous if one is the collinear continuation of the other. These line segments do not necessarily belong to a single contour but to two nearby contours. We think that the decision whether to merge these segments or not depends on further reasoning that might take into account other relations involving these segments. The membership functions for the continuity detection are depicted in figure 3. Similarly to the identity, the continuity depends on the distance D , the orientation difference α and the overlap O . The orientation difference in continuity relations must be very low, as the membership function in figure 3(b) shows. The overlap must be

Fig. 2. Feature member functions for identity detection

Fig. 3. Feature member functions for continuity detection

Fig. 4. Feature member functions for parallelism detection

low enough as depicted in figure 3(c). The distance between line segments D , defined as the distance between endpoints since we enforce a low overlap, can be larger than for the identity, as depicted in figure 3(a).

The membership functions used for the parallelism detection are depicted in figure 4. The most important feature for parallelism detection is the orientation difference α , which should not be higher than the transformation caused by the perspective projection. The distance D between parallel segments should not be very low, in order to differentiate this relation from the continuity or the identity. If the distance between segments is too high, the certainty of the relation is lowered, as these lines are less likely to be related. But, in any case, a high distance is not discriminative enough for refusing the parallelism relation, so the membership function for the distance feature is the one depicted in figure 4(b). The high overlap between line segments is also indicative of parallelism but, again, a low overlap is not discriminative enough for refusing the relation, this membership function is depicted in figure 4(c).

Fig. 5. Feature member functions for junction and l-junction detection

Fig. 6. Feature member functions for incidence and t-junction detection

In this work, we differentiate between regular junctions and L-junctions, being L-junctions those where the orientation difference is close to $\frac{\pi}{2}$. These L-junctions are very likely to be present in images containing man-made objects, since square corners are one of their characteristic features. The membership functions used for the detection of junctions and L-junctions are depicted in figure 5. The distance to the intersection point is the most discriminative feature used for junction detection, line segment pairs with I_A and I_B low positive or very low negative are detected as junctions through the membership functions depicted in 5(a) and 5(c), respectively. The membership function of the orientation difference α , however, is different in junctions and L-junctions. For L-junctions it must be close to $\frac{\pi}{2}$, with some variations allowed due to projection effects, while for junction can be any that is not very low, as depicted in figure 5(d) and 5(c), respectively.

Incidences and T-junctions are the only non-symmetric relations in this paper, with a line segment being incident in a supporting line segment if its endpoints meets the second segment in some location between its endpoints. The membership functions used for the detection of incidences and T-junctions are depicted in figure 6, the only different is the orientation difference, depicted in figure 6(b) and figure 6(d) respectively, that for T-junctions must be close to $\frac{\pi}{2}$. Similarly to junctions, the most decisive feature for the detection of incidences and T-junctions are the distances to the intersection point I_A and I_B . If s_1 is the incident line segment and s_2 is the supporting line segment, the distance to the intersection point I_A , depicted in figure 6(a), must be low and positive or very low and negative, as we penalise more the incidences past the intersection point. Opposite to this, the distance to the intersection point I_B from the supporting line segment s_2 in an incidence relation must be negative and high enough to establish the intersection point lies inside the segment s_2 , as the membership function depicted in figure 6(d) represents.

4 Results

We have tested our system with a set of artificially generated segment pairs. Figure 7 shows the associated certainties to the different relations analysed in this paper. We have used the parameters $w_D = 0.5$, $w_I = 0.2$ and $w_O = 0.2$ for the example, but these parameters would not affect the results if they were different, as the line segment lengths of each segment in the pair are very similar in all the examples. Also, the threshold parameters have been set as neutral values as they are of no use for this examples, concretely, we have selected $\lambda_f^- = 0$ and $\lambda_f^+ = \infty$.

	(a)	(b)	(c)	(d)	(e)
Junction	0.0	0.851	0.297	0.919	0.328
Incidence	1.0	0.0	0.0	0.003	0.833
L-junction	0.0	0.210	0.210	0.681	0.328
T-junction	0.210	0.0	0.0	0.003	0.833
Continuity	0.0	0.0	0.0	0.0	0.0
Identity	0.0	0.0	0.0	0.0	0.0
Parallelism	0.0	0.0	0.0	0.0	0.0

	(f)	(g)	(h)	(i)	(j)
Junction	0.595	0.0	0.0	0.0	0.0
Incidence	0.0	0.0	0.0	0.0	0.0
L-junction	0.595	0.0	0.0	0.0	0.0
T-junction	0.0	0.0	0.0	0.0	0.0
Continuity	0.0	0.0	0.0	0.0	0.0
Identity	0.0	0.0	0.0	0.0	0.0
Parallelism	0.0	1.0	0.627	0.699	1.0

	(k)	(l)	(m)	(n)	(o)
Junction	0.0	0.0	0.0	0.0	0.0
Incidence	0.0	0.0	0.0	0.0	0.0
L-junction	0.0	0.0	0.0	0.0	0.0
T-junction	0.0	0.0	0.0	0.0	0.0
Continuity	0.0	0.604	0.853	0.707	0.0
Identity	0.714	0.0	0.0	0.0	0.0
Parallelism	0.065	0.061	0.061	0.14	0.14

Fig. 7. Resulting relation certainties for 15 testing line segment pairs

The results obtained are coherent with the perceptual interpretation of the relations between the segment pairs. For example, in figure 7(b) the junction certainty is very high (0.851), but the L-junction certainty is lower (0.210) since α is not close to $\frac{\pi}{2}$. As the α gets closer to $\frac{\pi}{2}$, the L-junction certainty increases closely to the junction's, e.g. figures 7(d) and 7(f). Note also that in figures 7(a), 7(c) and 7(f), the minimum distance between segment endpoints are identical, but the certainties for the junction and incidence relations are significantly different, but coherent with how the relations between these segment pairs are subjectively perceived. This is due to the use of the signed distances to the intersection point proposed in this paper for junction, incidence, L-junction and T-junction detection. Parallelisms are also detected with an adequate certainty. For example, in figure 7(h) the parallelism certainty is significantly lower than the one in figure 7(g) due to the higher orientation difference. Also, the low overlapping causes the reduction of this certainty in figure 7(i). Similarly, the continuity certainty is lower for figure 7(l) than for figure 7(m) due to the higher overlap.

5 Conclusions

In this paper, we have introduced a fuzzy logic based method for computing the certainty associated to pairwise line segment relations. The features employed by the method and the effect of normalisation on the final certainty measure has also been widely discussed in the text.

The method proposed is general enough to be suitable for the detection of a wide set of relations: identity, continuity, junction, L-junction, incidence, T-junction and parallelism, among others that could have also been analysed. Also, the method is highly parametrisable, so the certainty associated to each target relations can be tuned in base to the application domain, whether it uses the certainty as a discriminant measure or as a base for further uncertain reasonings. Section 4 show the promising results of this approach that could be of use in environments where subjective scores for perceptual relations are necessary.

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References

1. Lowe, D.G.: Perceptual Organization and Visual Recognition. Kluwer Academic Publishers, Dordrecht (1985)
2. Nevatia, R.: Perceptual organization for generic object descriptions. In: Boyer, K.L., Sarkar, S. (eds.) Perceptual Organization for Artificial Vision Systems, pp. 173–189. Kluwer Academic, Dordrecht (2000)

3. Iqbal, Q., Aggarwal, J.: Retrieval by classification of images containing large man-made objects using perceptual grouping. *Pattern Recognition* 35, 1463–1479 (2002)
4. Lowe, D.G.: Three-dimensional object recognition from single two-dimensional images. *Artificial Intelligence* 31(3), 355–395 (1987)
5. Sarkar, S., Soundararajan, P.: Supervised learning of large perceptual organization: Graph spectral partitioning and learning automata. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 22(5), 504–517 (2000)
6. Ralescu, A.L., Shanahan, J.G.: Perceptual organization for inferring object boundaries in an image. *Pattern Recognition* 32, 1923–1933 (1999)
7. Kang, H.B., Walker, E.L.: Perceptual grouping based on fuzzy sets. In: *IEEE International Conference on Fuzzy Systems*, pp. 651–659. IEEE, New York, USA (1992)
8. Zadeh, L.A.: Fuzzy sets. *Information and Control* 8(3), 338–353 (1965)