A New Constitutive Equation of Reynolds Stress

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Abstract

According to the conditions proposed by Donaldson which should be satisfied for the numerical turbulence models, a non-linear composition of vorticity tensor which is symmetrical and whose trace is zero is added to the constitutive relation between Reynolds stresses and time-averaged quantities of flow field. Then, a new constitutive equation of Reynolds stress is given by

$$-\rho \overline{u_i u_j} = -A \rho \frac{k^2}{\varepsilon} S_{ij} - B \rho \frac{k^3}{\varepsilon^2} (\omega_{im} \omega_{mj} - \frac{1}{3} \{\omega_{lm} \omega_{mk}\} \delta_{ij}) - \frac{2}{3} \rho k \delta_{ij}$$

Here $-\rho \overline{u_i u_j}$ is the Reynolds stress tensor, S_{ij} is the rate of deformation tensor, ω_{ij} is vorticity tensor, k is the turbulence kinetic energy and ε rate of dissipation. Note that the transport equations of k and ε take the same forms as that in standard $k - \varepsilon$ model respectively. The values of coefficients A and B must be found by experiment.

For the turbulent flows in a straight pipe with square cross section and a straight pipe with circular cross section, the theoretical calculation results indicate that the new constitutive equation of Reynolds stresses can properly predict the existence of secondary flow and significantly improve the predictions in terms of the Reynolds normal stresses and their anisotropy. While the standard $k - \varepsilon$ model in which the Reynolds stresses constitutive equation only include the rate of deformation tensor is used to calculate the same flow, neither the secondary flow can be predicted nor the anisotropy of Reynolds normal stresses can be described. From this point, the new constitutive equation of Reynolds stresses has an advantage over that of standard $k - \varepsilon$ model. Some further analyses about the coefficients in the new Reynolds stress equation and its applicability to considerable velocity gradient flow are carried out. The new model is different from some other modified $k - \varepsilon$ models in which the effects of vorticity are added to turbulence kinetic energy equation and/or to its dissipation rate equation. It can be applied to establishing a new numerical turbulence model or to modifying the existing turbulence calculation models and provides useful information about practical approaches to the modeling of Reynolds stresses.

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