Landmark Correspondence Optimization for Coupled Surfaces

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Abstract. Volumetric layers are often encountered in medical images. Unlike solid structures, volumetric layers are characterized by double and nested bounding surfaces. It is expected that better statistical models can be built by utilizing the surface coupleness rather than simply applying the landmarking method on each of them separately. We propose an approach to optimizing the landmark correspondence on the coupled surfaces by minimizing the description length that incorporates local thickness gradient. The evaluations are performed on a set of 2-D synthetic close coupled contours and a set of real-world open surfaces, the skull vaults. Compared with performing landmarking separately on the coupled surfaces, the proposed method constructs models that have better generalization ability and specificity.

1 Introduction

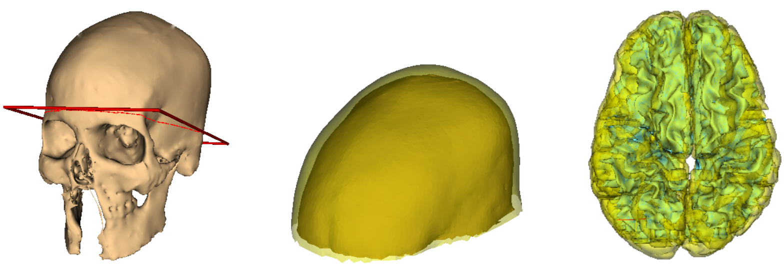
Statistical shape analysis is attracting increasing research interests and efforts because of its wide application in model-based image segmentation and pathological changes detection. Landmark-based shape analysis methods, such as the active shape model, require labeling landmarks with anatomical equivalence. Although manual landmarking can generate acceptable results in 2-D, it is subjective, error-prone, and time-consuming, which limits its application in 3-D.

Bookstein [1] proposed to optimize the positions of corresponding points by minimizing the bending energy between landmarks on two shapes when the landmarks are sliding on the shape boundary. The landmark correspondence problem can actually be solved in a principled way by being interpreted as an optimization problem. Kotcheff et al. [2] proposed to minimize the determinant of the covariance matrix, while Davies et al. [3] designed an objective function based on the minimum description length (MDL) principle that assumes simple descriptions generalize best. Different from landmarking methods that operate

N. Ayache, S. Ourselin, A. Maeder (Eds.): MICCAI 2007, Part II, LNCS 4792, pp. 818–825, 2007.

on an individual base, MDL determines the landmark positions via minimizing the description length of the information needed to transmit the training set, so that a compact description across the whole set can be derived with desired properties. Compared with manual labeling, SPHARM, and DetCov, MDL outperforms as it results in specific, generalized, and compact models [4]. Ericsson et al. [5] used the gradient descent strategy to improve the convergence speed of MDL, which makes MDL more practical in medical applications.

Although MDL is recognized as the "optimal" method in landmark correspondence optimization, applying the MDL principle flexibly and creatively rather than following certain existing algorithm will achieve better results. Our insight is that the shape properties should be well-understood and exploited in designing the landmarking algorithm. For example, to find out the landmarks on shapes with meaningful curvature changes, the curvature information should be considered in the optimization [6]. Richardson et al. [7] deal with the landmarking problem for 2-D open curves by introducing a novel tailor-made method, which achieves better performance than the generic MDL.



(a) (b) (c)

Fig. 1. Examples of coupled-surface structures: (a) the skull; (b) the skull vault, which is the part above the red frame indicated in (a); (c) the cerebral cortex

Volumetric layers are a kind of commonly encountered shapes in medical images, such as the skin of internal organs, myocardium of the left ventricle, and the cerebral cortex. A open coupled-surface structure, i.e., the skull vault, and a closed coupled-surface structure, i.e., the cerebral cortex, are illustrated in Fig. 1. Because they contain double 3-D boundaries, automatic and accurate landmarking is of great importance in analyzing their shapes. However, existing landmarking techniques, including MDL, are designed for single-surface objects. Actually, the local thickness is hidden information, which reflects the correlation of the two surfaces and facilitates human perception of such coupled-surface shapes. Thus it is reasonable that the locations with consistent thickness changes are assigned with landmarks. In this paper, we demonstrate how the coupleness information can be properly incorporated in the description length to solve the automatic landmarking problem in coupled-surface shapes.

2 Automatic Model Building for Coupled Surfaces

2.1 Landmark Initialization

The training surfaces are parameterized for convenient landmarks manipulations. It is desired that when neighbouring parameterized landmarks are adjusted to the same direction, their corresponding points in the training shape move consistently. Thus the conformal mapping that preserves the local angles is preferred.

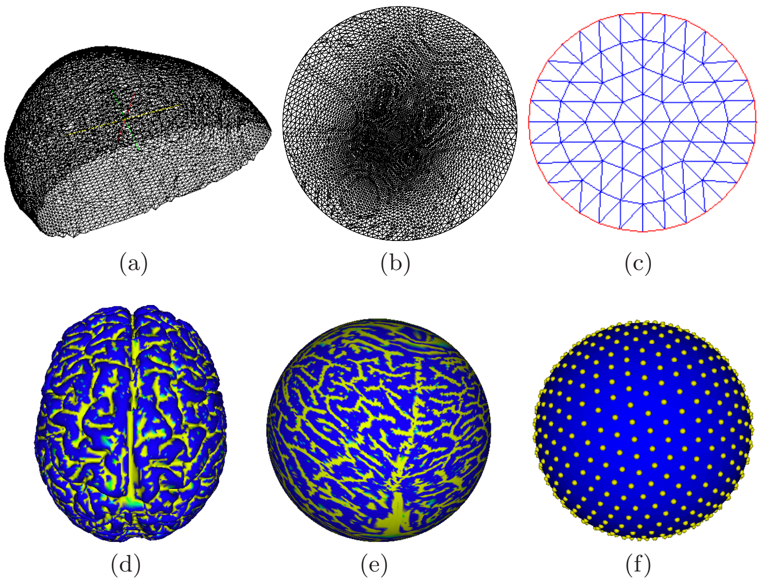


Fig. 2. Conformal mapping of the open and closed surfaces, and landmark initialization: (a) the outer skull vault surface, an open surface; (b) the conformal mapping of (a) to a unit disk; (c) uniform disk subdivision; (d) the GM/CSF interface, a closed surface; (e) the conformal mapping of (d) to a unit sphere; (f) uniform sphere subdivision

Mapping an open surface to a unit disk is achieved by minimizing the string energy of the mesh

$$E(\mathbf{W}, \Omega) = \sum_{[v_1,v_2]\in E} w_{v_1,v_2} \|\Omega(v_1) - \Omega(v_2)\|^2, \qquad (1)$$

where $\Omega(v)$ is the map of vertex v. The weight w_{v_1,v_2} is determined via $w_{v_1,v_2} = (\tan\frac{\alpha}{2} + \tan\frac{\beta}{2})/dist(v_1, v_2)$, where α and β are the adjacent angles in the two triangles sharing the edge $[v_1, v_2]$, and $dist(v_1, v_2)$ is the Euclidean distance between v_1 and v_2. Fig. 2 (a) and (b) give an outer skull vault mesh and its map on a unit disk.

To map a closed mesh to a unit sphere, the string energy in equation (1) is still the objective to be minimized, but the weight is defined as $w_{v_1,v_2} = \frac{1}{2}(\cot\mu + \cot\nu)$, where μ and ν are the opposite angles in the two triangles with the common edge $[v_1, v_2]$. A closed surface, i.e., the brain GM/CSF interface, and its map on a unit sphere are shown in Fig. 2 (d) and (e) respectively.

After the mapping has been determined, we uniformly sample in the parameter domain and map the sample points back to the surface as the initial landmarks. The planar disk is subdivided recursively into small triangles, e.g., Fig. 2(c), and the vertices of those triangles are the sample points. Subdividing the sphere leads to a uniform sampling as Fig. 2(f) shows.

2.2 Landmark Correspondence Optimization Using MDL

Thickness Definition of Volumetric Layer. The thickness of a volumetric layer at a point on the bounding surface is the distance from that point to the opposite surface. There exist several definitions for the layer thickness, such as the closest thickness (T_{close}) and the normal thickness (T_{normal}) [8]. T_{close} is the distance from a point on one surface to the closest point on the other. T_{normal} is the distance from a point on one surface to the point on the other in the direction of the surface normal. To find a generic measure that performs reasonably on every type of layers is impractical. The layer thickness in this study is determined as the distance between each pair of corresponding points on the two surfaces with the same polar coordinate. This measure is named as the radial thickness (T_{radial}). We illustrate the measures of T_{close}, T_{normal}, and T_{radial} on an axial plane of the skull boundary (see Fig.3). Different from T_{close} and T_{normal} that depend on the starting surface, the T_{radial} is unique and landmarks are grouped in pairs through this measurement.

Description Length Minimization for Coupled-Surface Structures. The MDL is recognized as the "optimal" method for generating corresponding landmarks, since it is based on the philosophy that the simplest description generalizes best. Our point is that landmarks could have properties other than spatial locations, and these properties can also be considered to minimize the description length. Therefore, in our method, the coupled surfaces are treated as a master

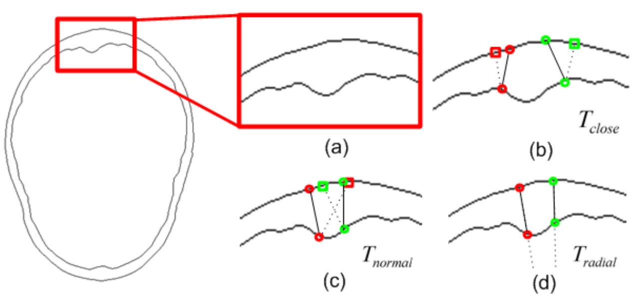


Fig. 3. Different thickness definitions: (a) the coupled surfaces; (b) the closest thickness measure; (c) the normal thickness measure; (d) the proposed radial thickness measure

surface and a supplementary surface. The information at each landmark in the master surface consists of both the spatial position and the thickness gradient at that landmark, i.e., $[x, y, z, \xi t']$, where ξ is the parameter controlling the importance of the thickness gradient t'. And the landmarks in the inner surface are obtained naturally through the thickness measurement. For the skull vault, we take the outer surface as the master surface because it is more dominant in determining the shape of the volumetric layer. In the cases that the inner surface is more important, the master surface can be switched to the inner surface. Actually, the shape of the supplementary surface is not discarded, as it is embeded in the "thickness" information. Once the landmark position is adjusted, the thickness at that particular landmark will be recomputed. We adopt a simplified version of the description length [6],

$$F = \sum_m \mathcal{L}_m \quad \text{with} \quad \mathcal{L}_m = \begin{cases} 1 + \log(\lambda_m/\lambda_{cut}) & \text{if } \lambda_m \geqslant \lambda_{cut} \\ \lambda_m/\lambda_{cut} & \text{if } \lambda_m < \lambda_{cut}. \end{cases} \tag{2}$$

Note that λ_m are the eigenvalues derived from the landmarks in the master surface. λ_{cut} can be determined by $\lambda_{cut} = (\sigma/\overline{r})^2$, where σ is the standard deviation of noise in the training data and $\overline{r}$ depends on the resolution of the images from which the training shapes are extracted.

The landmark positions are adjusted by locally warping the parameterization inside Gaussian kernel regions. The magnitude of the adjustment is proportional to the distance to each kernel center. The optimization is implemented by the gradient descent strategy. Suppose matrix $\mathbf{L}$ contains landmarks on the training shapes as columns, k is the number of landmarks in each shape, and s is the number of shapes in the training set. Since each vertex on the mesh contains both spatial position and thickness gradient value at that point, the dimension of the matrix $\mathbf{L}$ is $4k \times s$. Let $\mathbf{A} = \frac{1}{\sqrt{s-1}}(\mathbf{L} - \overline{\mathbf{L}})$, where $\overline{\mathbf{L}}$ is the matrix with all columns set to the mean shape $\overline{\mathbf{x}}$. Using the singular value decomposition (SVD), the matrix $\mathbf{A}$ can be written as $\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^T$. $\mathbf{U}$ and $\mathbf{V}$ are column-orthogonal matrices, and $\mathbf{D}$ is a diagonal matrix. Since the mesh to be analyzed in this study is 2-manifold, two variables (θ, ϕ) are involved in the disk or sphere parameter domain. Take the n^{th} landmark of the j^{th} sample for instance, we have the landmark movement $(\Delta\theta, \Delta\phi)$ as follows,

$$\Delta\theta = \frac{\partial F}{\partial \theta_{nj}} = \sum_{i=4n-3}^{4n} \left(\sum_m \frac{\partial \mathcal{L}_m}{\partial a_{ij}}\right) \cdot \frac{\partial a_{ij}}{\partial \theta_{nj}} \tag{3}$$

$$\Delta\phi = \frac{\partial F}{\partial \phi_{nj}} = \sum_{i=4n-3}^{4n} \left(\sum_m \frac{\partial \mathcal{L}_m}{\partial a_{ij}}\right) \cdot \frac{\partial a_{ij}}{\partial \phi_{nj}}, \tag{4}$$

where

$$\frac{\partial \mathcal{L}_m}{\partial a_{ij}} = \begin{cases} 2u_{im}v_{jm}/d_m & \text{if } \lambda_m \geqslant \lambda_{cut} \\ 2d_m u_{im} v_{jm}/\lambda_{cut} & \text{if } \lambda_m < \lambda_{cut} \end{cases}. \tag{5}$$

u_{im} and v_{jm} are the elements of the matrices $\mathbf{U}$ and $\mathbf{V}$ respectively. d_m is the element of the diagonal matrix $\mathbf{D}$, and it is equal to $\sqrt{\lambda_m}$. The surface gradients $(\frac{\partial a_{ij}}{\partial \theta_{nj}}, \frac{\partial a_{ij}}{\partial \phi_{nj}})$ are estimated by the finite difference.

3 Experimental Evaluation and Comparison

The quality of landmark correspondence is evaluated by the performance of the resultant model. Given different number of modes M, the generalization ability is the ability to describe the object that is not included in the training set, because a good shape model should not be overfitted by training samples. Its error $G(M)$ is usually calculated as the averaged leave-one-out error. The specificity reflects if a model only generates samples similar to training shapes. Its error $S(M)$ can be estimated by the averaged distances between samples newly generated with the model and the closest training shape. In our experiment, 10,000 test samples are generated. The parameter σ is set to 0.3, $\overline{r}$ is 100, and ξ is 1.0.

3.1 Results on the Synthetic Dataset

A set of 50 samples of a simple 2-D shape with varying thickness values at different positions are generated. Fig. 4 shows the landmarking results of the proposed method (MDL-thickness) and the MDL performed on two boundaries separately (MDL-separate) on three of them. For each training shape, the number of landmarks on either the inner or the outer surface is set to 14.

The results show that landmarks obtained by MDL-thickness are in pairs and are located in the regions where the thickness values reach the extrema. However, landmarks obtained by MDL-separate are equally spaced in the outer contour, while in the inner contour, they are located at positions with small spatial group changes. The MDL considering local curvature [6] can place the landmarks onto the "peaks" and "valleys" in the inner contour because large curvatures are

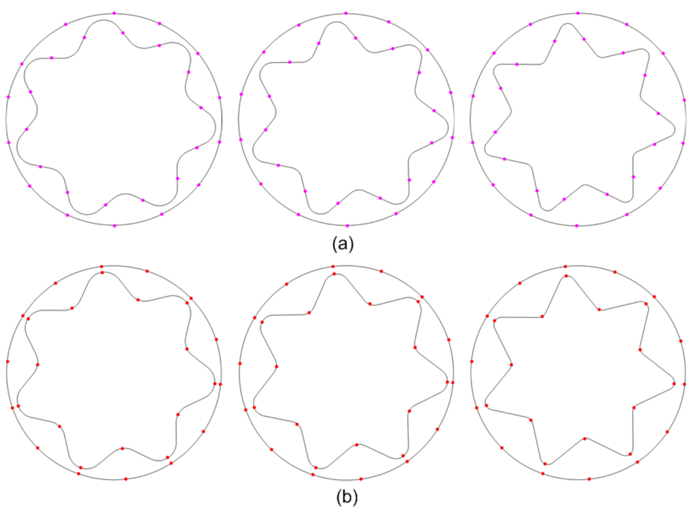


Fig. 4. Landmarking results on three training shapes in the synthetic dataset: (a) result of MDL-separate; (b) result of MDL-thickness

detected there. Since the outer contour does not have any curvature change, the result on the outer contour will be the same as that from MDL-separate. Generalization errors and specificity errors of MDL-separate and MDL-thickness using different numbers of modes are plotted in Fig. 5 (a) and (b) respectively. It can be observed that both $G(M)$ and $S(M)$ of MDL-thickness are smaller than those of MDL-separate when various numbers of modes are chosen.

3.2 Evaluation on the Real Skull Vault Dataset

The skull volumes of 18 subjects were segmented from the head CT data collected in the Prince of Wales Hospital, Hong Kong. The field of view of the CT data is 512×512 and the voxel size is $0.49mm \times 0.49mm \times 0.63mm$. The skull vault is the upper part of the skull and is an open coupled-surface structure. A total of 578 corresponding landmarks are determined using MDL-thickness and MDL-separate respectively. We plot the quality measures $G(M)$ and $S(M)$ of the models built with MDL-separate and MDL-thickness under different numbers of modes in Fig. 5 (c) and (d). It can be observed that both $G(M)$ and $S(M)$ of the model built with MDL-thickness are smaller than those built with the MDL-separate.

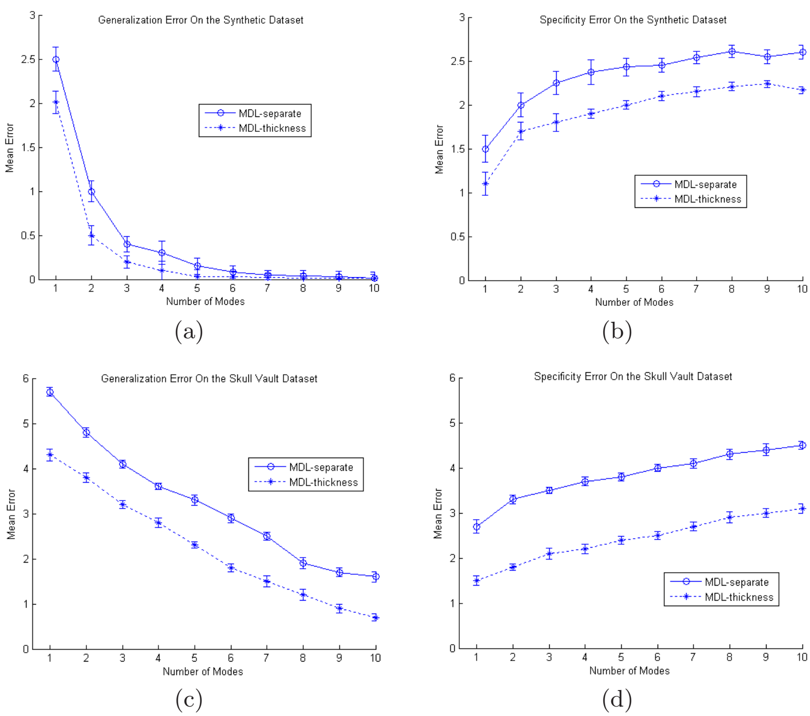


Fig. 5. The generalization error and specificity error of MDL-separate and MDL-thickness on the synthetic dataset and the real skull vault dataset

4 Conclusion

This paper describes a generic automatic landmarking method for structures with coupled surfaces by minimizing the description length. In this method, the local thickness gradient is treated as an extra property of each landmark, and thus the positions with group-wise consistent thickness changes are implicitly favored. Once the landmark on one surface is determined, its counterpart on the other surface can be found directly. The optimization converges fast as the gradient descent method is used. The quality of the models constructed from our proposed method are evaluated and compared with those obtained by treating the coupled surfaces as independent. The evaluation results show the advantage of considering thickness information in landmarking volumetric layers.

Acknowledgement

The work described in this paper was supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region (Project no. CUHK4453/06M) and CUHK Shun Hing Institute of Advanced Engineering.

This work is also affiliated with the Virtual Reality, Visualization and Imaging Research Center at The Chinese University of Hong Kong as well as the Microsoft-CUHK Joint Laboratory for Human-Centric Computing and Interface Technologies.

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