Online Estimation of the Target Registration Error for n -Ocular Optical Tracking Systems

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Abstract. For current surgical navigation systems optical tracking is state of the art. The accuracy of these tracking systems is currently determined statically for the case of full visibility of all tracking targets. We propose a dynamic determination of the accuracy based on the visibility and geometry of the tracking setup. This real time estimation of accuracy has a multitude of applications. For multiple camera systems it allows reducing line of sight problems and guaranteeing a certain accuracy. The visualization of these accuracies allows surgeons to perform the procedures taking to the tracking accuracy into account. It also allows engineers to design tracking setups interactively guaranteeing a certain accuracy.

Our model is an extension to the state of the art models of Fitzpatrick et al.[1] and Hoff et al. [2]. We model the error in the camera sensor plane. The error is propagated using the internal camera parameter, camera poses, tracking target poses, target geometry and marker visibility, in order to estimate the final accuracy of the tracked instrument.

1 Introduction

Intraoperative guidance systems (IGS) require the pose estimation of the patient and instruments. Most commercially available systems use optical tracking technology. The tracking system consists of n -ocular camera systems using linear CCD cameras or matrix cameras. The cameras detect the pose of the instruments by a rigidly attached set of fiducials. These fiducials are either light-emitting diodes or retroreflective objects reflecting light of flashes that are attached to the tracking cameras. In this paper we refer to a set of fiducials as tracking target.

The accuracy of such a system depends of a number of variables that interact in a non obvious manner. The accuracy of the tracking system strongly depends on the geometry of the system. Also the measurement technology as the cameras, computer vision and optimization algorithms, light conditions, and physical properties of fiducials play an important role. The geometry includes the number of visible fiducials, the spatial arrangement of these fiducials, and the pose of the tracking cameras.

We model the tracking system error as follows:

- IPE.** The image plane error (IPE) is the measurement error on the camera sensor. In our previous work we have shown that it is valid to assume that a variety of systematic and random errors can be modeled by a Gaussian error distribution in the image plane [1].
- FLE.** The fiducial location error (FLE) represents the three-dimensional error of a single fiducial. It depends on the IPE and the geometrical setup of the cameras.
- MTE.** The mean target error (MTE) represents the spatial error of a rigid tracking target in the centroid of the fiducials. It comprises of rotational and translational parts. It depends on the FLE of each fiducial and the spatial arrangement of the tracking target.
- TRE.** The target registration error (TRE) represents the actual error of a point in the tracking target coordinate system. The point of interest is generally not the centroid of the fiducial, but a point on the attached object. The error is amplified due to the rotational part of the MTE.

The error of a system can be described as the root-mean-square (RMS) error, which is the standard deviation of a measurement. By definition the RMS cannot provide information if there is one direction with a higher variance than the others. Therefore it implicitly models isotropic and independent error. If the error is expected to be anisotropic it can be described with the covariance matrix of the distribution. These descriptions of the error implicitly model the error with a zero-mean Gaussian distributions.

In current systems the accuracy of a tracking system is specified statistically by moving fiducials through the specified measurement volume and computing the root-mean-square error [3]. This error is propagated to the point of interest according to the spatial arrangement of the markers. The point of interest may be the tip of an instrument. Fitzpatrick et al. [1] describe a simple formula to predict the target registration error (TRE) from the fiducial location errors (FLE) given as a one-dimensional RMS. The RMS is provided by the tracking system manufacturer.

However, it is known that due to the underlying geometrical situation, the tracking errors have non-uniform error distributions throughout their measurement volumes [4,1]. According to Khadem et al. [4] the errors of different commercial systems show a significant anisotropy.

Hoff et al. [2] model the TRE with covariances. They measure the root-mean-square deviation of the FLE and propagate it. In our previous work we have proposed a model for the estimation of the fiducial location error from the camera geometry [5]. The FLE is modeled using covariances, since the error is anisotropic (see figure 1).

2 Methods

In our approach the error and its propagation is modeled as a Gaussian distribution. The error is defined by its expected value and its covariance. The error

propagation follows the formula $cov(\mathbf{A}\mathbf{x}) = \mathbf{A}cov(\mathbf{x})\mathbf{A}^T$ for affine transformations $\mathbf{A}$. The propagation of the expected value is straightforward. Thus we focus on propagating the covariances. In the case of a non-linear propagation of the error we can use the first order approximation $cov(f(\mathbf{x})) = \mathbf{J}_f cov(\mathbf{x})\mathbf{J}_f^T$ with $\mathbf{J}_f$ as the Jacobian of f at $\mathbf{x}$, if the function is locally linear around $\mathbf{x}$.

The following steps have to be performed:

- The estimation of IPE is described in section 2.1. We create a 2D covariance matrix for each camera.
- The propagation from IPE to FLE is described in section 2.2. We create a 3D covariance matrix for each fiducial.
- The propagation from FLE to MTE is described in section 2.3. We create a 6D covariance matrix for each target.
- The propagation from MTE to TRE is computed with the error propagation formula for affine transformations. We create a 3D covariance matrix for the point of interest.

2.1 Measurement of the Image Error

The error of an optical tracking system have different sources: Internal calibration of cameras, imperfect lenses, computer vision algorithm, and image blur. All of these can be modeled as a 2D error in the plane of the camera sensor.

In an approximation we regard the error to be constant on the sensor of the camera for a certain size of the fiducial. For the estimation of errors in the sensor plane we used covariance measurements of 3D fiducial in the tracking space.

We calculated the noise in the sensor plane of each camera by finding the inverse propagation described in section 2.2 by a numerical optimization using the Levenberg-Marquardt method. The covariance has been constrained to be isotropic and uniform for a numerically stable solution. The measured standard deviation in our setup is approximately $\frac{1}{52}$ pixels for each fiducial.

2.2 From Image Error to Fiducial Error

We model this transformation as in our previous work [5] using the geometry of the cameras.

If we use an n -ocular stereo system with pinhole cameras with intrinsic parameters $\mathbf{K}_i$ and extrinsic parameters $\mathbf{T}_i$ of the i -th camera detecting the same point $\mathbf{x}$ at the location $\mathbf{u}$. We get the measurement function for the *Triangulation*, a set of nonlinear camera equations p :

$$p : \quad \begin{array}{l} \mathbf{u}_1 = \frac{1}{\rho_1} \mathbf{K}_1 \mathbf{T}_1 \mathbf{x} \\ \vdots \\ \mathbf{u}_n = \frac{1}{\rho_n} \mathbf{K}_n \mathbf{T}_n \mathbf{x} \end{array} \quad \text{using } \rho \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \mathbf{K} \mathbf{T} \mathbf{x}$$

as the projection function. ρ_i denotes the normalization factor needed for homogeneous coordinates. We use linear models throughout the error propagation, so we can safely assume a pinhole camera model here.

Fig. 1. Predicted FLE accuracy of a two camera setup. Covariance ellipsoids (left) and RMS error (right). The RMS is close to constant while the covariances reveal significantly changing directions of the estimated errors. Coordinate systems depict the camera positions.

In order to compute the FLE, we build the Jacobian $\mathbf{J}_p = \frac{\delta p}{\delta \mathbf{x}}$ and apply the backward propagation formula.

$$\Sigma_x = \left(\mathbf{J}_p^T \begin{bmatrix} \Sigma_{u_1} & \mathbf{0} \\ & \ddots \\ \mathbf{0} & \Sigma_{u_n} \end{bmatrix}^{-1} \mathbf{J}_p \right)^+ \quad (1)$$

The resulting equations are analytically computed using a computer algebra system and exported to C code.

2.3 From Fiducial Location Error to Tracking Target Error

We assume that all coordinates and covariances are given in target coordinates, so $[R|t] = I$. Without loss of generality the origin is at the centroid of the fiducial positions. This function maps the six-dimensional marker target error $[\Delta_R|\Delta_t]$ to the three-dimensional fiducial location error Δ_{t_c} at the point $q = (q_x, q_y, q_z)$.

$$\Delta_{t_c} = \Delta_R q + \Delta_t \quad (2)$$

We linearize this function around the zero-mean error Δ_t :

$$J_f(q) = \frac{\delta}{\delta(\Delta_t)} \Delta_{t_c} \Big|_{\Delta_t=0} = \begin{bmatrix} 1 & 0 & 0 & 0 & q_z - q_y \\ 0 & 1 & 0 & -q_z & 0 & q_x \\ 0 & 0 & 1 & q_y & -q_x & 0 \end{bmatrix} \quad (3)$$

We can now stack the equations for all fiducials together in a single matrix

$$M = [J_f(q_1), \dots, J_f(q_n)]^T \quad (4)$$

and apply the backward propagation formula:

$$\mathbf{J}_{f-1} \text{cov}(\mathbf{x}) \mathbf{J}_{f-1}^T \quad (5)$$

which can be easier calculated as according to Hartley et al.[6]:

$$\text{cov}(f^{-1}(\mathbf{x})) = (\mathbf{J}_f^T \text{cov}(\mathbf{x})^{-1} \mathbf{J}_f)^{-1} \quad (6)$$

$$\Sigma_c = \left(M^T \begin{bmatrix} \Sigma_{p_1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \Sigma_{p_n} \end{bmatrix}^{-1} M \right)^{-1} = M^+ \begin{bmatrix} \Sigma_{p_1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \Sigma_{p_n} \end{bmatrix} (M^T)^+ \quad (7)$$

where Σ_{p_i} is the covariance of the i -th fiducial in target coordinates and Σ_c represents the covariance matrix of the MTE at the centroid.

2.4 Visualization Setup

The estimated error are visualized in the video and in the following images in an augmented reality system using a second, independent tracking system. We visualize only the tracking error. The overall application error could be estimated by additional error propagation using our model, but is not the scope of this work. Addition of errors and further propagation is well described in the work of Hoff et al.[2].

In the images the tracking error is visualized as the 95% confidence level of the estimated error amplified by 33.

3 Results

The error estimation in this paper is an extension of the estimation by Fitzpatrick et al. [1] and Hoff et al. [2]. Using the same assumptions as in their work, our model provides mathematically the same results. However, these assumptions are too strict according to our computations, because the FLE has a non uniform and anisotropic error.

Figure 2 shows the behavior of the different error propagations. This figure depicts the expected error of a four camera setup where the fiducial error is nearly isotropic. Therefore the middle and the right image show almost the same expected error. The difference between the corresponding becomes obvious when the one or more fiducials or cameras are occluded (see section 3.3 and 3.2).

3.1 Inclusion of Visibility Data

Optical tracking systems have the inherent property, that the cameras need to see the fiducials to be able to compute a pose estimation. If this line of sight is blocked between one camera and a target the accuracy of measurement degrades. Figure 3 depicts the RMS in a plane of the same four camera setup using with one camera occluded in each image. The depicted plane is parallel to the plane including the four cameras at 1 meter distance. The error has been computed according to the proposed model in section 2.2. The images clearly show that assuming a constant RMS of fiducials in the volume is not sufficient. It also shows that the visibility plays an important role in the accuracy of each fiducial.

Fig. 2. Estimation of RMS at point of interest using target RMS [1] (left), estimation of covariance using fiducial RMS [2] (middle), estimation of covariance using fiducial covariance (right) - covariance of fiducial (a), covariance at point of interest (b), covariance at center of fiducials (c)

Fig. 3. Occlusion of cameras changes the expected fiducial location error significantly: The coordinate systems depict the camera positions. The grey level indicates the estimated root mean square error of a fiducial in a plane of 1m distance. The asymmetry is due to asymmetric camera orientation.

3.2 Occlusion of Single Fiducials

Current navigation systems request full visibility of each fiducial in each camera for the maintenance of the required accuracy. Some tracking systems already provide error estimates based on the work of Fitzpatrick et al. [1]. Using our proposed method, we can now provide more accurate error estimates. We can show that in typical configurations, the expected error will be actually larger than expected, while the average error estimate over the whole tracking space still stays the same. Figure 5 shows the effect of occluding one fiducial on the estimation the instrument tip.

Fig. 4. Effects of occlusion: No occlusion (left), fiducial occlusion of four cameras (middle), several fiducials occluded (right), each circles depicts the visibility status of the fiducial in each of the four cameras

Fig. 5. Effects of camera occlusion: no camera occluded (left), one camera occluded (middle), two camera occluded (right)

3.3 Occlusion of Cameras

Occlusion of cameras is a problem that appears only in multi-camera setups. In current tracking systems in surgical navigation the pose estimation is stopped in case of camera occlusion. The occlusion of cameras has a significant influence on the tracking accuracy. (see figure 3 and 5).

Tracking systems in commercial medical applications need to be accuracy-certified. In order to achieve this certification, the manufacturer has to guarantee a certain level of accuracy. Right now, this is done by mounting the cameras in a mechanically rigid configuration and then by calibrating and measuring the error in a predefined working volume. For this reason, up to now all commercially used tracking systems use a configuration with two matrix cameras or three linear cameras. Using multi-camera setups would provide redundant cameras to avoid occlusions.

Unfortunately, guaranteeing accuracy for flexible setups is a complex task without taking the visibility into account.

The online error estimation presented in this work provides an important step towards accuracy-certified multi-camera setups. If we can guarantee a certain accuracy on the image plane this accuracy can be propagated to provide accuracy estimates at a point of interest. By providing many distributed tracking cameras in the operating room thanks to the proposed method one would be able to allow tracking and navigation even if one or more camera views are occluded. The navigation could continue if the system could guarantee the required level of accuracy.

4 Conclusion

We have proposed a novel way of estimating the error in optical tracking systems. We model the FLE and the TRE as anisotropic non-uniform error. We deduce it from the error in the tracking camera plane. The error is computed online based on the visibility of each fiducial. The effects can be appreciated best in the supplementary material.

The interactive propagation of errors in real time based on the geometry allows for a multitude of applications. Engineers designing tracking systems [7] have the opportunity to get direct feedback regarding of the final accuracy. Physicians may benefit from the online generated information in intraoperative visualization.

The accuracy can be visualized in the navigation rather than current approach of not visualizing any pose information in case of an accuracy below a certain threshold.

The online estimation of error based on visibility of fiducials allows for creating a multiple camera tracking setup for surgical navigation. In order to guarantee a certain accuracy, which is necessary for navigation systems, we can compute the expected error based on the visibility. The proposed method allows on-line estimation of the final accuracy while one or more fiducials or camera views are occluded. This could allow the computer aided solutions to gain considerable flexibility. They could in fact continue functioning as far as such occlusions do not cause the level of accuracy to decrease below the acceptable limits.

5 Future Work

The model of the error estimation has been improved. However, there are some approximations that we are going to investigate. We estimate the accuracy in the image plane to be constant for a certain fiducial size. The accuracy depends on the fiducial size in the image. We have to investigate whether this approximation is valid for arbitrary setups and how we can integrate the fiducial size into our model. Furthermore we model the computation of the target pose as a least square optimization of 3D fiducials while the fiducials positions are triangulated from the camera. This linear model is not used in tracking system software, but a non-linear optimization of a reprojection of the target pose in the image. This allows for a special weighting favoring rotational values for translational ones. We are going to investigate these effects in our further research.

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