A Probabilistic Model for Haustral Curvatures with Applications to Colon CAD

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Abstract. Among the many features used for classification in computer-aided detection (CAD) systems targeting colonic polyps, those based on differences between the shapes of polyps and folds are most common. We introduce here an explicit parametric model for the *haustra* or colon wall. The proposed model captures the overall shape of the haustra and we use it to derive the probability distribution of features relevant to polyp detection. The usefulness of the model is demonstrated through its application to a colon CAD algorithm.

1 Introduction

With over half a million obits, colorectal cancer was ranked as the fourth leading cause of cancer death worldwide in 2002 [3], and it is currently ranked as the second leading cause of cancer-related deaths in the United States [7]. Most colorectal cancers arise from benign colonic polyps, and their early detection can significantly increase survival rates [4]. Optical colonoscopy is part of the standard screening protocol for the detection of polyps in the colon [19], but the discomfort and long duration of this procedure has negative impact on patient compliance [6].

Virtual colonoscopy (VC) or computed tomography colonography (CTC) has shown promise as a less invasive method for detecting polyps, with performance at least as good as that of optical colonoscopy [18]. Once computed tomography (CT) imaging and advanced visualization tools were introduced, the natural second step is the use of computer-aided detection (CAD) systems for automating the search for colonic polyps, and a number of CAD techniques have been developed in recent years. Early examples include the work of Yoshida *et al* [28], in which principal curvatures were used to compute a shape index indicative of the roundness of polyps. Curvatures were also used by Vos *et al* [27] and by Summers *et al* [22,23]. More recently, modeling through spherical harmonics [10], surface normal overlap [17] and other curvature-based methods have been developed [1,26]. The use of small to moderate [5,8,26] and of large [25] feature sets followed by a more sophisticated classification mechanism have also been explored.

In the list above, curvature and curvature-based measures are the features most commonly used for classification. In particular, [22,28,27,24,1,26] built explicit models with different degrees of complexity for the ranges of curvatures observed in colonic polyps, folds, and, occasionally, the haustra (or colon wall) itself. However, whereas folds and

polyps are modeled through highly sophisticated schemes, the haustra is either altogether omitted [27,1,26] or simply mentioned as a region of low curvature [22,28]. This paper introduces a novel model for the haustra. We use the same assumptions as [22,28] regarding the shape of the haustra, but we augment the model with a component driven by recent results in the theory of Gaussian random fields [14]. This allows for an accurate estimation of the probability distribution of curvatures of isosurfaces of the haustra, which can be naturally fed into any curvature-based CAD system aimed at detecting colonic polyps. Results with real data show the usefulness of the proposed model as applied to colon CAD.

2 Haustra Model

Let the *volume image* I be defined as a twice-differentiable mapping from $\mathcal{V} \subset \mathbb{R}^3$ into $\mathbb{R}$. For any given c , we define an *isosurface* $M_c \subset \mathcal{V}$ at the *isovalue* c as the set of points $\mathbf{x}$ satisfying $I(\mathbf{x}) = c$ and $\nabla I(\mathbf{x}) \neq \mathbf{0}$. It can be shown that the principal curvatures of M_c at a point $\mathbf{x}$ can be computed in closed form as the eigenvalues of the matrix $\tilde{\mathbf{H}} = \mathbf{N}^T \mathbf{H} \mathbf{N} / \|\nabla I\|$ [20, pg. 138], where ∇I is the gradient of I , $\mathbf{H}$ is the Hessian of I , and columns of $\mathbf{N}$ form an orthonormal basis for the null space of ∇I . This computation can be carried out as the concatenation of a linear and a nonlinear step. As depicted in Fig. 1, the linear step comprises the computation of the gradient and the Hessian of the image input image. The nonlinear step involves the matrix multiplications $\mathbf{N}^T \mathbf{H} \mathbf{N}$, the computation of and division by the scalar $\|\nabla I\|$, and the actual computation of the eigenvalues of the resulting 2×2 matrix. In practice, the linear step also includes smoothing the volume image, which is done to control the noise associated with the computation of the derivatives ∇I and $\mathbf{H}$ but can also model the effects of the system point spread function.

Fig. 1. Step-by-step computation of curvatures on isosurfaces of volume images

2.1 The Geometric Model

In [21], the haustra of the colon is defined as: “the sacculations of the colon, caused by the teniae, or longitudinal bands, which are slightly shorter than the gut so that the latter is thrown into tucks or pouches”. These sacculations are the curved segments of the colon wall. The morphological models in Langer *et al.* [11] justify the adoption

by [22,28] of a low curvature surface representation for the haustra. However, such a model, by itself, is not enough to specify the distribution of curvatures that one expects at the colon wall. In order to achieve this, we consider a volume image $I_0(\mathbf{x})$ representing any particular low curvature geometric model of the haustra. Now let $h_0(\mathbf{x})$ be a corruption of $I_0(\mathbf{x})$ by additive white noise $\eta_0(\mathbf{x})$. Following the pipeline described in Fig. 1 with $h_0(\mathbf{x})$ as the input image, we obtain a smoothed volume $h(\mathbf{x}) = I(\mathbf{x}) + \eta(\mathbf{x})$, where $I(\mathbf{x})$ and $\eta(\mathbf{x})$ are smooth versions of $I_0(\mathbf{x})$ and $\eta_0(\mathbf{x})$. In particular, $\eta(\mathbf{x})$ is still Gaussian noise, but no longer white. Assuming that the variance of the input (zero-mean) white noise is σ_η^2 , the autocorrelation function $R(\mathbf{x})$ of the filtered noise will be $R(\mathbf{x}) = \sigma_\eta^2 \exp(-\mathbf{x}^T \mathbf{\Sigma} \mathbf{x} / 4)$ [2], where $\mathbf{\Sigma}$ is the covariance matrix of the smoothing kernel. For an isotropic kernel, we have $\mathbf{\Sigma} = \sigma^2 \mathbb{I}$.

Denoting the function that maps a matrix $\mathbf{A}$ to its eigenvalues by $\boldsymbol{\lambda}(\mathbf{A})$, the nonlinear step of the curvature computation yields

$$\boldsymbol{\kappa} = \boldsymbol{\lambda} \left(\frac{-\mathbf{N}^T \mathbf{H}_h \mathbf{N}}{\|\nabla h\|} \right) = \boldsymbol{\lambda} \left(\frac{-\mathbf{N}^T (\mathbf{H}_I + \mathbf{H}_\eta) \mathbf{N}}{\|\nabla I + \nabla \eta\|} \right), \quad (1)$$

where $\boldsymbol{\kappa} = (\kappa_1, \kappa_2)$ are the principal curvatures of $h(\mathbf{x})$ at the point $\mathbf{x}$.

2.2 Distribution of Curvatures

The stochastic differential equation in (1) can be simplified by making reasonable assumptions about the shape and appearance of the haustra as observed in CT images. First, the matrix $\mathbf{N}^T \mathbf{H}_I \mathbf{N}$ must have, on average, a small Frobenius norm compared to that of $\mathbf{N}^T \mathbf{H}_\eta \mathbf{N}$. To demonstrate this, observe that, per the discussion in section 2.1, the magnitude of the eigenvalues of $\mathbf{N}^T \mathbf{H}_I \mathbf{N}$ is small, reflecting the low curvature of the haustra. Since $\mathbf{N}^T \mathbf{H}_I \mathbf{N}$ is a symmetric matrix, its singular values must also be small. This last observation indicates that a Taylor expansion of (1) around $\mathbf{N}^T \mathbf{H}_I \mathbf{N} = \mathbf{0}$ yields a good approximation for $\boldsymbol{\kappa}$, i.e.,

$$\boldsymbol{\kappa} \approx \boldsymbol{\lambda} \left(\frac{-\mathbf{N}^T (\mathbf{H}_\eta) \mathbf{N}}{\|\nabla I + \nabla \eta\|} \right) + \text{vec}(\mathbf{N}^T \mathbf{H}_I \mathbf{N})^T \frac{\partial \boldsymbol{\kappa}}{\partial \text{vec}(\mathbf{N}^T \mathbf{H}_I \mathbf{N})^T} \Big|_{\mathbf{H}_I=0} \quad (2)$$

$$\approx \boldsymbol{\lambda} \left(\frac{-\mathbf{N}^T (\mathbf{H}_\eta) \mathbf{N}}{\|\nabla I + \nabla \eta\|} \right) + \boldsymbol{\lambda} \left(\frac{-\mathbf{N}^T \mathbf{H}_I \mathbf{N}}{\|\nabla I + \nabla \eta\|} \right), \quad (3)$$

where $\text{vec}(\mathbf{A})$ indicates the vector built out the matrix $\mathbf{A}$ by stacking its columns. Indicating the Kronecker product by $\otimes$ and the i -th eigenvalue of the (symmetric) matrix $\mathbf{A}$ by λ_i , the equality of (2) and (3) can be verified by using the relations $\text{vec}(ABC) = (C^T \otimes A) \text{vec}(B)$ and $\partial \lambda_i(\mathbf{A}) / \partial \text{vec}(\mathbf{A})^T = \mathbf{v}_i \otimes \mathbf{v}_i$, where $\mathbf{v}_i$ is the eigenvector associated to λ_i [13].

We further simplify (1) by assuming that the magnitude of ∇I is, on average, large compared to that of $\nabla \eta$. This assumption simply reflects the fact that the haustra corresponds to a sharp interface between air and soft tissue. Hence, (1) becomes,

$$\boldsymbol{\kappa} \approx \frac{\boldsymbol{\lambda}(\mathbf{N}^T \mathbf{H}_\eta \mathbf{N})}{\|\nabla I\|} + \boldsymbol{\kappa}_0, \quad (4)$$

where $\mathbf{\kappa}_0 = \boldsymbol{\lambda} \left(\frac{-\mathbf{N}^T \mathbf{H}_\eta \mathbf{N}}{\|\nabla I + \nabla \eta\|} \right)$. Note that $\mathbf{\kappa}$ is a random variable, and, from (4), we can see that its probability density $p(\mathbf{\kappa})$ is given by

$$p(\mathbf{\kappa}) = p_{\lambda_\eta}((\mathbf{\kappa} - \mathbf{\kappa}_0) \|\nabla I\|) \|\nabla I\|^2, \quad (5)$$

where p_{λ_η} is the probability density of the random variable $\lambda_\eta = \boldsymbol{\lambda}(\mathbf{N}^T \mathbf{H}_\eta \mathbf{N})$.

In order to derive $p(\mathbf{\kappa})$ we adapt recent results in the theory of Gaussian random fields [14], which establish that λ_η is distributed according to a linear combination of Gaussian and chi-distributed independent random variables. More precisely, for an isotropic smoothing kernel with covariance matrix $\boldsymbol{\Sigma} = \sigma^2 \mathbb{I}$, $\lambda_\eta \sim \alpha' [N(0, 2) \mp \mathcal{X}(2)]$, with $\alpha' = \sigma_\eta / (2\sigma^2)$, resulting in

$$\begin{aligned} p_{\lambda_\eta}(\lambda) &= \frac{1}{2\alpha'^2} p_{N(0,2)}((\lambda_2 + \lambda_1)/(2\alpha')) p_{\mathcal{X}(2)}((\lambda_2 - \lambda_1)/(2\alpha')) \\ &= \frac{1}{8\sqrt{2\pi}\alpha'^3} (\lambda_2 - \lambda_1) e^{-\frac{5\lambda_1^2 - 6\lambda_2\lambda_1 + 5\lambda_2^2}{32\alpha'^2}} \end{aligned} \quad (6)$$

Assuming a simple spherical model of radius R for the input image I , we have $\mathbf{\kappa}_0 = (-1/R, -1/R)$. Therefore, the final probability distribution of the haustral curvatures is given by

$$p(\mathbf{\kappa}) = \frac{1}{8\sqrt{2\pi}\alpha^3} (\kappa_2 - \kappa_1) e^{-\frac{(5\kappa_1^2 - 6\kappa_2\kappa_1 + 5\kappa_2^2)R^2 + 4(\kappa_1 + \kappa_2)R + 4}{32R^2\alpha^2}}, \quad (7)$$

with $\alpha = \sigma_\eta / (2\sigma^2 \|\nabla I\|)$.

3 Experiments and Results

As a first step to demonstrate the usefulness of the proposed model we captured principal curvature data from random samples of pedunculated, sessile and flat polyps, haustral folds, and haustra. In Fig. 2, we display this scatter data over the decision boundaries generated from our haustra model (yellow) and models for pedunculated (red), sessile and flat polyp (green), and haustral fold (blue), described in [1]. The overlaid yellow scatter data points provide visual validation for the haustra model decision boundary, as shown in Fig. 2a. In Fig. 2b, we show the decision boundaries obtained by considering only two model categories: for (polyps) and against (haustra and folds).

In Fig. 3, we provide another depiction of the benefit derived from the addition of the haustra model to the colon CAD system. Figure 3d shows an image region centered at an actual polyp. Figure 3a shows the result of a Bayesian competition [9] between the combined polyp model (pedunculated, sessile and flat) against the fold model, without inclusion of the haustra model in the same region. The strongest responses (magenta) are indeed in the polyp, but there are weaker responses (blue) scattered all around the colon wall. Even though most of those can be discarded through thresholding, such a “clean-up” algorithm can remove true positive detections. Figure 3b shows the same region as in Fig. 3a, but now with the haustra model included. The haustra model clearly plays a significant role in reducing the polyp responses on the haustra without affecting

Fig. 2. Partitioning of κ_1 - κ_2 space with overlying scatter plot data: (a) Model-based partitioning of κ_1 - κ_2 space, red = pedunculated polyps, green = sessile and flat polyps, blue = haustral folds, yellow = haustra, (b) Binary partitioning of κ_1 - κ_2 space, red = polyp responses, blue = non-polyp responses

the response at the polyp itself. In effect, the haustra model eliminates many potential false positives. In Fig. 3c we see the haustra responses alone, obtained by competing the haustra against all the other models, and the reason for the differences between Fig. 3a and Fig. 3b become clear.

Finally, we show the application of our model to a the colon CAD system described in [1]. The test data consisted of a subset of 36 CT volumes from the WRAMC dataset, from which 23 polyps with diameter above 6 mm were marked by expert radiologists and confirmed by optical colonoscopy.¹ The protocol for patient preparation consisted of oral administration of 90 ml of sodium phosphate and 10 mg of bisacodyl, with a clear-liquid diet that included 500 ml of barium for stool tagging and 120 ml of diatrizoate meglumine and diatrizoate sodium for fluid tagging [18]. The complete WRAMC dataset comprises many more images, but unfortunately ground truth is provided as a distance from the rectum along the colon centerline only, and the precise image location of polyps in this dataset must still be carried out by an expert.

It is important to note that this dataset is significantly more challenging to CAD than the “fully prepped” data commonly used throughout the literature (e.g., all of the CAD work mentioned in the introduction with the exception of the work of Summers *et al.*, such as in [24]). However, if the use of CTC demands full cleansing of the colon, patient compliance may still be an issue, since the strongest factor affecting acceptance of colonoscopy is the extent of bowel preparation [6]. Minimally invasive protocols such as the one applied to the collection of the WRAMC data mitigate this problem [12], but pose new difficulties for the interpretation of the images [4]. In Fig. 4, we present a free-response receiver operating characteristic (FROC) curve demonstrating the performance of a lung CAD system that makes use of the proposed haustra model.

¹ This data has been provided courtesy of Dr. Richard Choi, Virtual Colonoscopy Center, Walter Reed Army Medical Center.

Fig.3. Visual Hausstra Results: (a) polyp responses without the haustra model in a window centered at an actual polyp, (b) polyp responses with the haustra model, (c) haustra responses alone, and (d) Raw image data

Fig.4. FROC curve for the performance of a colon CAD system using the proposed haustra model. A sensitivity of 83% is achieved ad a cost of 6.25 false positive detections per case.

A sensitivity of 83% is achieved at a cost of 6.2 false positive detections per case, and running time is in the order of 10 min for a $512 \times 512 \times 700$ -voxel CT volume with a research prototype implemented with ITK.

4 Conclusions and Future Work

In this work, we have introduced a novel probabilistic model for the curvature of isosurfaces of the haustra. An expression of the probability density function of such curvatures

was provided by considering Gaussian random perturbation to a geometric abstraction of the colon wall. The model augments the set of models developed in [1] for applications in colon CAD, and was demonstrated in a specific colon CAD application.

In the current formulation the radius R of the haustra is a fixed parameter representative of the expected radius of the insufflated colon. However, we could account for colonic haustra variations in size and shape by marginalizing over the parameter R . The prior for this marginalization will depend upon either training data of prior clinical knowledge of insufflated haustra radii. In [15] we have shown how to compute the probability distribution of curvatures for a class of ellipsoidal surfaces, suggesting a mechanism to achieve such generalization.

An interesting debate is presented in [16], which, although in the context of lung CAD, is relevant to this work. From that discussion it is clear that many radiologist see CAD not necessarily as a tool to improve upon the performance of the best radiologists, but as a means to standardize or regularize results of radiologists with varying degrees of experience. To validate such expectation, however, it is necessary to have data read by multiple radiologists, which is not the case with the WRAMC data set.

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