Surface-to-Surface Registration Using Level Sets

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Abstract. This paper presents a general approach for surface-to-surface registration (S2SR) with the Euclidean metric using signed distance maps. In addition, the method is symmetric such that the registration of a shape A to a shape B is identical to the registration of the shape B to the shape A.

The S2SR problem can be approximated by the image registration (IR) problem of the signed distance maps (SDMs) of the surfaces confined to some narrow band. By shrinking the narrow bands around the zero level sets the solution to the IR problem converges towards the S2SR problem. It is our hypothesis that this approach is more robust and less prone to fall into local minima than ordinary surface-to-surface registration. The IR problem is solved using the inverse compositional algorithm.

In this paper, a set of 40 pelvic bones of Duroc pigs are registered to each other w.r.t. the Euclidean transformation with both the S2SR approach and iterative closest point approach, and the results are compared.

1 Introduction

This paper addresses the problem of shape registration or alignment which plays an essential role in shape analysis. Many registration procedures such as generalized Procrustes analysis [9,7] rely on a prior manual annotation of landmarks. The main drawback with these approaches is the reliance on manual annotation which becomes cumbersome and infeasible for larger 2D datasets and for 3D data.

Methods for explicitly deriving landmarks from training curves/surfaces based on information theoretic theory has been published [6]. Unfortunately these often suffer from exceeding use of computation time.

The iterative closest point (ICP) algorithm by Besl et al. [2] solves the problem of landmark dependence by iteratively updating the point correspondence after the closest point criterium. Since the introduction in 1992 many extensions and improvements of original ICP have been proposed in literature [8,11,10]. Most of these methods still require a good initial estimate in order not to converge to a local minimum. Furthermore, common for these methods are that they do not utilize the knowledge of the connectedness of the point cloud, which is available in many cases.

The approach described in this paper is in many ways related to the approach presented by Darkner et al. [5], which aligns two point clouds by minimizing the sum of squared difference between the distance functions of the point clouds in some rectangular box domain. The problem with the scheme by Darkner et al. is that it is likely produce a suboptimal result when applied to concave shapes. That is, the concave parts of a shape will not propagate as far out in a distance map as the convex parts. As a consequence points placed on the convex parts of a shape are given more weight than points on concave parts. Our approach differs from the approach presented in [5], as it uses *signed* distance maps and minimizes the squared difference between the signed distance maps restricted to a shrinking narrow band. Thus, it does not suffer from the same defect as [5].

2 Theory

The registration of a surface S_x to a surface S_y w.r.t the Euclidean metric can be expressed as the minimization of the functional

$$F_1(\mathbf{p}) = \oint_{S_x} d(W(\mathbf{x}; \mathbf{p}), S_y)^2 d\mathbf{x}, \quad (1)$$

where $W(\cdot; \mathbf{p})$ is the warp function.

A minor flaw with this approach is that the registration of S_x to S_y is not necessarily equivalent to the registration S_y to S_x . If $W(\cdot; \mathbf{p})$ is invertible (1) can be extended to

$$\begin{aligned} F_2(\mathbf{p}) &= \oint_{S_x} d(W(\mathbf{x}; \mathbf{p}), S_y)^2 d\mathbf{x} + \oint_{S_y} d(W(\mathbf{y}; \mathbf{p})^{-1}, S_x)^2 d\mathbf{y} \\ &= \oint_{S_x} d_{S_y}(W(\mathbf{x}; \mathbf{p}))^2 d\mathbf{x} + \oint_{S_y} d_{S_x}(W(\mathbf{y}; \mathbf{p})^{-1})^2 d\mathbf{y}, \end{aligned} \quad (2)$$

where d_{S_x} and d_{S_y} are the distance maps of the surfaces S_x and S_y , respectively. This energy functional ensures a symmetric registration.

A minimum of F_2 can be obtained by any gradient or Newton based optimization scheme. However, such schemes may very well get stuck in a local minimum instead of the global minimum. To overcome this problem we introduce a slightly different energy functional

$$F_3(\mathbf{p}) = \int_{U_x^r} (\Phi_y(W(\mathbf{x}; \mathbf{p})) - \Phi_x(x))^2 d\mathbf{x} + \int_{U_y^r} (\Phi_x(W(\mathbf{y}; \mathbf{p})^{-1}) - \Phi_y(y))^2 d\mathbf{y}, \quad (3)$$

where $\Phi_x(\mathbf{x})$ and $\Phi_y(\mathbf{y})$ are the signed distance maps (SDMs) of the surfaces S_x and S_y , and $U^r = \{\mathbf{x} \mid \mathbf{x} \in \mathbb{R}^d, |\Phi(\mathbf{x})| < r\}$. And we note that

$$F_3 \rightarrow F_2 \text{ for } r \rightarrow 0. \quad (4)$$

(a) A simple shape.

(b) F_2 cost functional.

(c) F_3 cost functional.

Fig. 1. Cost as a function of translation in x and y direction

Now, consider the shape in Figure 1(a) consisting of two identical rectangles. If we translate the shape in both the x and the y direction between -25 and 25 pixels and calculate the energy in each position using F_2 and F_3 with $r = 25$ pixels, we get energy landscapes shown in Figure 1(b,c). In this case, the F_2 cost function produces three minima while the F_3 cost function produces only the global minimum.

3 Method

The energy functional defined in F_3 can be viewed as a image registration problem between two SDMs Φ_x and Φ_y , where the points to be warped are those inside the narrow bands U_x^r and U_y^r . The problem is solved using an extended version of the inverse compositional algorithm presented by Baker et al [1]. To preserve the same notation as in [1], we assume that Φ_x and Φ_y are discretized SDMs. Thus, F_3 becomes

$$F_4(\mathbf{p}) = \sum_{\mathbf{x} \in U_x^r} (\Phi_y(W(\mathbf{x}; \mathbf{p})) - \Phi_x(\mathbf{x}))^2 + \sum_{\mathbf{y} \in U_y^r} (\Phi_x(W(\mathbf{y}; \mathbf{p})^{-1}) - \Phi_y(\mathbf{y}))^2. \quad (5)$$

If the set of warps forms a *group* the minimization of F_4 is equivalent to the minimization of

$$F_5(\mathbf{p}) = \sum_{\mathbf{x} \in U_x^r} (\Phi_y(W(\mathbf{x}; \mathbf{p})) - \Phi_x(W(\mathbf{x}; \Delta \mathbf{p})))^2 + \sum_{\mathbf{y} \in U_y^r} (\Phi_x(W(\mathbf{y}; \mathbf{p})^{-1}) - \Phi_y(W(\mathbf{y}; \Delta \mathbf{p})^{-1}))^2. \quad (6)$$

with the update rule $W(\mathbf{x}; \mathbf{p}) \leftarrow W(\mathbf{x}; \mathbf{p}) \circ W(\mathbf{x}; \Delta \mathbf{p})^{-1}$. By applying the first order Taylor expansion to (6) we get

$$F_5(\mathbf{p}) \approx \sum_{\mathbf{x} \in U_x^r} \left(\Phi_y(W(\mathbf{x}; \mathbf{p})) - \Phi_x(W(\mathbf{x}; \mathbf{0})) - \nabla \Phi_x \frac{\partial W(\mathbf{x}; \mathbf{0})}{\partial \mathbf{p}} \Delta \mathbf{p} \right)^2 + \sum_{\mathbf{y} \in U_y^r} \left(\Phi_x(W(\mathbf{y}; \mathbf{p})^{-1}) - \Phi_y(W(\mathbf{y}; \mathbf{0})^{-1}) - \nabla \Phi_y \frac{\partial W(\mathbf{y}; \mathbf{0})^{-1}}{\partial \mathbf{p}} \Delta \mathbf{p} \right)^2 \quad (7)$$

By taking the derivatives of F_5 w.r.t. $\Delta \mathbf{p}$ and setting them equal to zero we get the update equation

$$\Delta \mathbf{p} = -\mathbf{H}^{-1} \left(\sum_{\mathbf{x} \in U_x^r} \mathbf{S}_x^\top \mathbf{E}_x + \sum_{\mathbf{y} \in U_y^r} \mathbf{S}_y^\top \mathbf{E}_y \right), \quad (8)$$

where $\mathbf{S}_x = \nabla \Phi_x \frac{\partial W(\mathbf{x}; \mathbf{0})}{\partial \mathbf{p}}$, $\mathbf{S}_y = \nabla \Phi_y \frac{\partial W(\mathbf{y}; \mathbf{0})^{-1}}{\partial \mathbf{p}}$, $\mathbf{E}_x = \Phi_y(W(\mathbf{x}; \mathbf{p})) - \Phi_x(\mathbf{x})$, $\mathbf{E}_y = \Phi_x(W(\mathbf{y}; \mathbf{p})^{-1}) - \Phi_y(\mathbf{y})$ and $\mathbf{H} = \sum_{\mathbf{x} \in U_x^r} \mathbf{S}_x^\top \mathbf{S}_x + \sum_{\mathbf{y} \in U_y^r} \mathbf{S}_y^\top \mathbf{S}_y$. Note that $\mathbf{H}^{-1}$, $\mathbf{S}_x$ and $\mathbf{S}_y$ only have to be computed once. A S2SR can be obtained with Algorithm 1.

The best sequence of r_i 's is properly highly depended on the problem. We have applied the following scheme with success:

$$r_{i+1} \approx \frac{r_i}{2}. \quad (9)$$

Selecting a suitable initial narrow band r_0 is however not entirely straight forward. If the choice of r_0 is too small the algorithm may get stuck in a local

Algorithm 1. S2SR

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1:  $\mathbf{r} = [r_1 \dots r_n]; \{ r_i > r_{i+1} \}$ 
2: for each  $r_i \in \mathbf{r}$  do
3:    $k = 0$ ;
4:   repeat
5:     update  $\mathbf{p}$  using (8) with  $U_x^{r_i}$  and  $U_y^{r_i}$ ;
6:      $k = k + 1$ ;
7:   until convergence or  $k > k_{max}$ 
8: end for
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minima, and if it is too large the algorithm will use an unnecessary amount of computational power. Note, that the computation time is much more depended on the radius of the initial narrow band than the number of narrow bands as the global minimum of F_3 for the narrow band r_i properly is relatively close to the global minimum of F_3 for r_{i+1} . In general, r_0 should be larger than the width of largest structure or feature in the image which might introduce a local minimum in F_3 .

3.1 Extending Approach to Open Surfaces

SDMs are in principal only defined for closed surfaces as it is impossible to label the inside and outside of an open surface. Thus, our approach can only be applied to closed surfaces. To overcome this problem we introduce the notion of a pseudo SDM.

The pseudo SDM of triangle mesh of an open surface is computed using the following recipe:

1. Close the surface by triangulating all the holes in the triangle mesh.
2. Compute the SDM of the closed triangle mesh. Bærentzen et al. [3,4] describe how to compute the SDM of a closed triangle mesh.
3. Set all voxels in the discretized SDMs with distances to the added faces to an undefined value.

Under the registration, voxels from one SDM may be warped to an undefined volume of the other SDM. In such cases, it is reasonable to assume that the distance in the undefined volume is 0, as we have no way of knowing whether the point is on the outside or inside of the shape. This hack allows for a bit of slack around the open areas of a surface. In many cases a surface is only open as it has been chosen to disregard a part of the shape - cutting away part of a shape in the exact *same place* is impossible. Furthermore, the gradient of a SDM at the borders between the defined and undefined volumes is likewise assumed to be equal to 0.

Sometimes, it is impossible to close an open surface with triangulation without introducing intersections between the new faces and the existing faces. Also, it might not be reasonable to close a surface if the hole is very large. In such cases, it might be more advisable simply to use unsigned distance maps instead of SDMs. The question of how large a hole in a surface can be, before the registration algorithm fails or produces suboptimal result with pseudo SDMs, needs to be investigated in the future.

4 Experiments

Two experiments were conducted to test the surface registration approach; (i) a toy example where the outline of the right and left hand of one of the authors were registered to each other, and (ii) a real example where 40 pelvic bones of Duroc pigs were registered with the ICP algorithm by Fitzgibbon [8] and with our S2SR algorithm.

4.1 Hand Example

To test the robustness of the S2SR algorithm a left and a right hand were traced on a piece of paper and scanned into a computer. The left hand was flipped horizontally, displaced 100 pixels in the x -direction and -25 pixels in the y -direction, and rotated 5 degrees counter clockwise. Figure 2 shows the initial position of the hands, the final position with regular S2SR¹ and the final position with our S2SR algorithm with the narrow bands $r = 30, 15, 7, 3, 1$. Evidently, the regular S2SR approach gets stuck in a local minimum or saddle point, while the shrinking narrow band S2SR approach registers the left and right hand perfectly.

Fig. 2. Rigid registration of left (green) and right hand (red)

4.2 Pelvic Bones

Half pig skeletons were automatic extracted from CT scans of half pig carcasses and fitted with implicit surfaces. From the implicit surfaces triangle meshes were created, and the pelvic bones were manually removed from the triangulated skeletons. An example of a pelvic bone can be found in Figure 3.

Fig. 3. Example of a pelvic bone from a Duroc pig

From the set of pelvic bone shapes a shape was selected to be the reference, and the remaining shapes were registered to the reference shape with ICP and our level set based S2SR algorithm with $r = 20, 10, 5, 2.5, 1 \text{ mm}$. To compare

¹ Simulated with the small narrow band $r=1$.

the two registration approaches we use the mean squared error (MSE) and the maximum error (ME). As ICP minimizes the point-to-closest-point (CP) distance and our algorithm minimizes the surface-to-surface distance², we evaluate the performance of the registration algorithms using both distance concepts. Furthermore, as our registration algorithm does symmetric minimization of the squared distances and ICP does not, the MSE and the ME are calculated in the same direction as the ICP registration, in the other direction and in both directions combined. The registration results for ICP and S2SR can be found in Table 1 and 2, respectively. As no surprise, the ICP registration has a lower MSE and ME in the same direction as the registration, when we are using the CP distance. It is neither a surprise that our S2SR algorithm has lower MSEs and MEs in the opposite direction of the ICP registration and in both directions. It is however a bit of a surprise, that our S2SR algorithm has a smaller MSE than ICP when using the SDMs to extract distances. A possible explanation for this result is that our algorithm allows for a bit of slack around the open regions of the surface and is therefore better at fitting the remaining regions of the surface. Figure 4 illustrates this by color-coding the surfaces of two registered pelvic bones with the shortest distance.

Table 1. The MSE and ME averaged over the 39 registrations after ICP registration

Method	ICP ($A \rightarrow B$)					
Direction	$A \rightarrow B$		$A \leftarrow B$		$A \leftrightarrow B$	
Measure	$\sqrt{MSE}$	ME	$\sqrt{MSE}$	ME	$\sqrt{MSE}$	ME
SDM	11.92	20.13	12.70	27.04	12.34	27.24
CP	12.32	20.85	14.40	29.41	13.44	29.48

Table 2. The MSE and ME averaged over the 39 registrations after S2S registration

Method	S2SR ($A \leftrightarrow B$)					
Direction	$A \rightarrow B$		$A \leftarrow B$		$A \leftrightarrow B$	
Measure	$\sqrt{MSE}$	ME	$\sqrt{MSE}$	ME	$\sqrt{MSE}$	ME
SDM	11.69	24.11	12.18	24.03	11.90	26.28
CP	12.77	25.70	13.11	26.34	12.95	28.30

W.r.t. computation time, it can be mentioned that it took approximately 20 minutes to run the 39 registrations with ICP and approximately 50 minutes to run 39 registrations with S2SR on a standard Dell laptop with a 1.6Ghz Centrino CPU and 2Gb ram. It is difficult to compare the computation time of the two algorithm as the computation time for the S2SR algorithm is vastly depended

² The distances are found by interpolating the SDMs. To ensure fairness, when evaluating the MSE and ME, points, which are warped to an undefined area of a SDM, are ignored instead of receiving the distance 0.

Fig. 4. Distance color-coded surfaces after registration. Black areas are areas where the distance could not be interpolated in the SDM because the point is situated in a undefined area.

on the chosen parameters, e.g. the chosen narrow bands and the resolution of the discretized SDMs.

5 Conclusion

This paper has presented a method for S2SR. The registration algorithm was tested on two examples, where its properties were highlighted; (i) it is less prone to fall into local minima than ordinary S2SR, (ii) and it does symmetric registration. As the method relies on SDMs it only works in theory on surfaces. Nevertheless, this paper has demonstrated that it can work on open surfaces by introducing a pseudo SDM, where distances are not defined in volumes *close* to the open regions of the surface.

In the future, we will use non-rigid transformations with the registration approach.

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