An Elastoplastic Model Considering Sand Crushing

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Abstract. The behavior of sand crushing will appear when the confining pressure is up to a certain value, which results in disappearing of the positive dilatancy of sand. Adopting a new hardening parameter with the crushing stress, an elastoplastic constitutive model considering sand crushing is proposed. Comparing the conventional triaxial compression test results with the model prediction, it shows that the proposed model can reasonably describe the dilatancy of sand from positive to negative.

Keywords: Crushing, Elastoplastic model, Hardening, Sand, Dilatancy.

1 Introduction

Many triaxial experiments^[1,2] indicate that sand presents crushing at a high confining stress, which results in positive dilatancy at low confining stress and negative dilatancy at high confining stress. The peak strength of sand also decreases with the confining stress increasing. Therefore, it is necessary to develop an elastoplastic constitutive model to describe the mechanical behavior of sand crushing.

2 The Model Considering Sand Crushing

Referencing the work by Sun et al.^[3], Nakai^[4], Yao and Luo^[5], the relationship between the elastoplastic volumetric strain ε_v^p (or ε_v^e) and the mean stress p under isotropic consolidation condition could be assumed as Eqs.(1) and (2):

$$\varepsilon_v^e = C^e \left[\left(p/p_a \right)^m - \left(p_0/p_a \right)^m \right] \quad (1)$$

$$\varepsilon_v^p = \left(C^{ep} - C^e \right) \left[\left(p/p_a \right)^m - \left(p_0/p_a \right)^m \right] \quad (2)$$

where p_0 is the initial mean stress, p_a is the atmospheric pressure, C^{ep} is the compression index, C^e is the swelling index and m is a coefficient for sand. There are two examples shown as Fig.1.

Fig. 1. The loading-unloading tests of isotropic compression, which of cement sand is shown in panel(a), Toyoura sand in panel(b)

Revising the hardening parameter^[6,7] developed by Yao et al. as

$$H = \int dH = \int \Theta d\varepsilon_v^p = \int \frac{M_{pt}^4}{M_f^4} \frac{M_f^4 - \eta^4}{M_{pt}^4 - \eta^4} d\varepsilon_v^p \quad (3)$$

$$M_f = M (p/p_c)^{-n} \quad (4)$$

$$M_{pt} = M (p/p_c)^n \quad (5)$$

in which M_{pt} is the stress ratio at characteristic point, M_f the stress ratio at shear failure, M the stress ratio at critical state, η the stress ratio ($\eta = q/p$), p_c the crushing stress, n a material parameter of sand ($0 < n < 1$). The parameters of proposed model are M , p_c and n . In the proposed model, the equations of the yield locus and plastic potential are similar to the modified Cam-clay model's^[8,9] on the p - q plane. Adopt the revised hardening parameter H with crushing stress p_c to replace the plastic volumetric strain ε_v^p in Cam-clay model. The yield function is given as

$$(C^{ep} - C^e) \left(\frac{p_0}{p_a} \right)^m \left[\left(\frac{p}{p_0} + \frac{q^2}{M^2 p p_0} \right)^m - 1 \right] - H = 0 \quad (6)$$

The stress-dilatancy equation is expressed as

$$d\varepsilon_v^p / d\varepsilon_d^p = (M_{pt}^2 - \eta^2) / 2\eta \quad (7)$$

3 Prediction Versus Experiments

The following seven soil parameters used in the proposed model: C^e , C^{ep} , m , M , p_c , n and the poisson coefficient ν , all of which can be determined via TC tests. The parameters C^e , C^{ep} and m are confirmed at isotropic consolidation conditions as in Fig.1. The parameters M , p_c and n are determined by drained triaxial compression tests at different initial stresses. The model parameters for Toyoura sand are listed in Tab.1 and the value of atmospheric pressure p_a is given as 0.1MPa.

Table 1. There are seven parameters required for the proposed model

Isotropic consolidation	Triaxial compression	Elastic poisson
$C^e = 0.0046$ $C^{ep} = 0.0068$ $m = 0.4$	$M = 1.35$ $p_c = 5.9\text{MPa}$ $n = 0.103$	$\nu = 0.3$

Fig. 2. The test results (data after Sun et al.^[10]) and the predicted stress-stain curves by proposed models in this paper

The test results and the predicted relationships between axial strain ε_a , radial strain ε_r , volumetric strain ε_v and principal stress ratio σ_a/σ_r are shown in Fig.2.

Analyzing the prediction results as shown in Fig.2, it can be seen that: ①The predicted curves by the proposed model agree well with the test results for Toyoura sand under conventional triaxial compression conditions except the ε_a - ε_v curves at $p=4$, 8MPa. ②When the confining stress of sand rises, the strength (peak principal stress ratio) decreases. ③When the confining stress of sand rises, the volumetric dilatant degree of sand decreases.

4 Conclusions

(1)Because of the behavior of sand crushing, the peak principal stress ratio and the degree of positive dilatancy for conventional triaxial compression tests both decrease when the confining stress rises. The dilatancy will disappear when the confining stress is larger than the crushing stress.

(2)The proposed elastoplastic constitutive model, adopting the new hardening parameter revised by the crushing stress, can reasonably describe the stress-strain behavior and the dilatancy of sand.

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