An Elastoplastic Hydro-mechanical Model for Unsaturated Soils

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Abstract. This paper presents a coupled elastoplastic constitutive model for predicting the hydraulic and stress-strain-strength behaviour of unsaturated soils. Hydraulic hysteresis in the water-retention behaviour is modeled as an elastoplastic process with the elastic region of the saturation degree. The effect of change in degree of saturation on the stress-strain-strength behaviour and the effect of change in void ratio on the water-retention behaviour are taken into consideration in the model, in addition to the effect of suction on the hydraulic and mechanical behaviour. Model predictions of the stress-strain and water-retention behaviour are compared with those obtained from triaxial tests on unsaturated soil along isotropic compression, triaxial stress paths with or without variation in suction.

Keywords: Unsaturated soil, elastoplastic model, water-retention curve, suction, triaxial test.

1 Introduction

Since the model was proposed by Alonso et al. [1], several elastoplastic constitutive models for unsaturated soils have been proposed, on both the experimental and theoretical studies. In the early models, the stress state variables employed in most models for unsaturated soils are the net stress (the difference between total stress and pore-air pressure) and the suction (the difference between pore-air pressure and pore-water pressure). In these models, the influence of unsaturation on the soil behaviour is considered through the variable of suction and the degree of saturation is computed from the suction using the soil-water characteristic curve. Hydraulic hysteresis which accounts for irreversible behaviour between the suction and the saturation is usually not considered in these constitutive models. Another shortcoming of these models is related to their implementation into existing finite element codes for saturated soils where constitutive relations are always described in terms of effective stresses. As discussed by Sheng et al [4], using the average skeleton stress and the suction as the stress variables leads to a convenient implementation of unsaturated soil models into finite element codes working with effective stresses, whereas using the net stress and the suction is more appropriate for finite element codes working with total stress.

In recent years, some conceptual models incorporating hydraulic hysteresis of unsaturated soils have been proposed [2][4][7]. However, most of these models can only describe the coupled hydro-mechanical behaviour in qualitative terms or are only formulated for isotropic stress conditions. This paper presents a complete mathematical formulation of an elastoplastic model for the hydraulic and mechanical behaviour of unsaturated soils, especially incorporating the effect of the deformation on the water-retention behaviour. The model performance is then validated quantitatively by experimental results obtained from suction-controlled triaxial tests on unsaturated compacted soil along various stress paths.

2 Coupled Hydro-mechanical Model for Unsaturated Soils

2.1 Stress-State Variable for Unsaturated Soils

To identify the hydraulic and mechanical behaviour of unsaturated soils properly, the stress-state variables employed in the model are the average skeleton stress σ'_{ij} and the suction s . The average skeleton stress σ'_{ij} is defined as:

$$\sigma'_{ij} = \sigma_{ij} - u_a \delta_{ij} + S_r s \delta_{ij} . \quad (1)$$

where σ_{ij} is the total stress tensor, S_r is the saturation degree, u_a is the pore-air pressure, and δ_{ij} is the Kronecker delta. Eq. (1) is similar to the effective stress proposed by Bishop [2], with S_r taking the place of the weighting factor χ . The average skeleton stress and the suction are not independent variables, but their work-conjugate strains (soil skeleton strains and the degree of saturation) are independent variables [4]. The average skeleton stress and suction variables adopted in the present study permit a general form of hydraulic behaviour to be represented and also provide a smooth transition between the saturated and unsaturated states.

2.2 Formulation of the Elastoplastic Model for Isotropic Stress States

In previous work, the author developed a so-called ‘load-collapse’ (LC) yield curve in the p - s plane for the isotropic stress [6]. A similar form is adopted in the following equation, except that the average skeleton stress is now defined as per Eq. (1).

$$p'_y = p'_n \left(\frac{p_{0y}}{p'_n} \right)^{\frac{\lambda(0) - \kappa}{\lambda(s) - \kappa}} . \quad (2)$$

where p_{0y} and p'_y are the yield stresses for saturated soil and unsaturated soil with suction s ; p'_n is an isotropic stress at which no collapse occurs when the suction is decreased; κ is a swelling index for unsaturated soils (including saturated soil in the e - $\ln p'$ plane); and $\lambda(0)$ and $\lambda(s)$ are the slopes of the normal compression lines of

saturated soil and unsaturated soil with suction s in the $e-\ln p'$ plane. The quantity $\lambda(s)$ is assumed as:

$$\lambda(s) = \lambda(0) + \frac{\lambda_s s}{s + p_a} \quad (3)$$

where λ_s is a material parameter for identifying the change of $\lambda(s)$ with suction s . $\lambda(s)$ in Eq.(3) has the following feature: when $s \rightarrow \infty$, $\lambda(s) = \lambda(0) + \lambda_s$.

Fig. 1. LC yield curve, SI and SD yield curves for compacted Pearl-clay under isotropic stress states

The initial LC yield curve CDEF and the subsequent LC yield curve GHIJ in Fig. 1 are depicted using Eqs (2) and (3), and values of the model parameters and the initial state for compacted Pearl-clay, which are given later. When the stress state (p' , s) is on the left side of CDEF, there is only elastic deformation occurring during the stress change.

From Eq. (2) we have

$$dp'_y = \frac{\partial p'_y}{\partial p_{0y}} dp_{0y} + \frac{\partial p'_y}{\partial s} ds \quad (4)$$

When the stress state is inside the LC yield curve, the elastic volumetric strain increment is given by:

$$d\epsilon_v^e = \frac{\kappa dp'}{(1+e)p'} \quad (5)$$

When the stress state is on the LC yield curve and its increment vector points to the right side of the LC yield curve, the plastic volumetric strain increment is given by:

$$d\epsilon_v^p = \frac{(\lambda(0) - \kappa)dp_{0y}}{(1 + e)p_{0y}} \quad (6)$$

Or, from Eqs (4) and (6):

$$d\epsilon_v^p = \frac{\lambda(0) - \kappa}{(1 + e)p_{0y}} (dp_y - \frac{\partial p_y}{\partial s} ds) / \frac{\partial p_y}{\partial p_{0y}} \quad (7)$$

In addition to the LC yield curve, two more yield curves are needed to model hydraulic behaviour as an elastoplastic process, as shown in Fig. 1. The water-retention behaviour is represented by a suction increase (SI) yield curve and a suction decrease (SD) yield curve. The water-retention curve is assumed as Fig. 2.

Fig. 2. Model for water retention behaviour at different void ratios

The change in the degree of saturation due to the change in void ratio is attributed to the elastic part of the increment of the degree of saturation. When the change in the degree of saturation is elastic, i.e. along the scanning curves, the elastic increment of the degree of saturation can be derived from Eq. (1) as follows:

$$dS_r^e = -\lambda_{sc} de - k_s \frac{ds}{s} \quad (8)$$

The plastic change in the degree of saturation can expressed as:

$$dS_r^p = dS - dS_r^e = (\lambda_{sr} - k_s) \frac{ds}{s} \quad (9)$$

From the condition $dS_r^p = 0$ we can obtain the following SI and SD yield curves as

$$f_{SI} = \ln(s_i) + c_1 \quad (\text{for SI}) \quad (10)$$

$$f_{SD} = \ln(s_D) + c_2 \quad (\text{for SD}) \quad (11)$$

where c_1 and c_2 are integration constants.

For a given plastic change in S_r , the SI and SD curves can be drawn as shown in Fig. 1. The SI curve AK stems from Eq. (12), while the SD curve BL follows from Eq. (13). The plastic volumetric strain ϵ_v^p is equivalent along the same LC yield curve, with the plastic change in S_r being equivalent along the same SI curve or the same SD curve. Therefore, as shown in Fig. 1, there are six zones with different yielding characteristics for ϵ_v^p and dS_r^p .

2.3 Formulation of the Elastoplastic Model for General Stress States

The Modified Cam-clay model has been used extensively for saturated soils, and gives reasonably good predictions for clays. This model is adopted here because it is simple. Assuming an associated flow rule, the yield function (f) and the plastic potential function (g) are proposed to have the following form:

$$f = g = q'^2 + M^2 p' (p' - p'_y) = 0 \quad (12)$$

The associated flow rule is obeyed in the 'effective stress' space; that is:

$$d\epsilon_{ij}^p = \Lambda \frac{\partial f}{\partial \sigma'_{ij}} \quad (13)$$

where the proportionality constant Λ can be determined from the consistency condition. Eq. (12) can be rewritten as $f = f(p', q', p'_y) = 0$, leading to:

$$df = \frac{\partial f}{\partial p'} dp' + \frac{\partial f}{\partial q'} dq' + \frac{\partial f}{\partial p'_y} dp'_y = 0 \quad (14)$$

Substituting Eq. (4) into Eq. (14) and re-arranging gives:

$$df = \frac{\partial f}{\partial p'} dp' + \frac{\partial f}{\partial q'} dq' + \frac{\partial f}{\partial p'_y} \frac{\partial p'_y}{\partial p_{oy}} dp_{oy} + \frac{\partial f}{\partial p'_y} \frac{\partial p'_y}{\partial s} ds \quad (15)$$

where the isotropic yielding stress p_{oy} for saturated soil is related to the volumetric strain ϵ_v^p and is the same as that used in the traditional Cam-clay model. Because the plastic volumetric strain ϵ_v^p is a hardening parameter in the present model, the

volumetric plastic strains $d\epsilon_v^p$ caused by dp_{oy} in a saturated soil are the same as those in an unsaturated soil which are caused by dp_y' and/or ds . Allowing for Eq. (13), the following is obtained:

$$dp_{oy} = \frac{1+e}{\lambda(0)-\kappa} p_{oy} d\epsilon_v^p = \frac{1+e}{\lambda(0)-\kappa} p_{oy} \Lambda \frac{\partial f}{\partial p'}. \quad (16)$$

Substituting Eq. (16) into Eq. (15) and solving for Λ gives:

$$\Lambda = - \frac{\frac{\partial f}{\partial p'} dp' + \frac{\partial f}{\partial q'} dq' + \frac{\partial f}{\partial p_y'} \frac{\partial p_y'}{\partial s} ds}{\frac{\partial f}{\partial p_y'} \frac{\partial p_y}{\partial p_{oy}} p_{oy} \frac{1+e}{\lambda(0)-\kappa} \frac{\partial f}{\partial p'}}. \quad (17)$$

From Eqs (13) and (17), it is possible to calculate the plastic strain increments caused by the increment in the 'effective stress' and/or the decrement in suction.

Following the formulation in plasticity, the plastic increment of S_r is written as

$$dS_r^p = \Lambda_\alpha \frac{\partial f_\alpha}{\partial s}. \quad (\alpha = SI, SD) \quad (18)$$

where Λ_α are plastic multipliers. From Eqs (11) and (12), we have

$$\frac{\partial f_{SI}}{\partial s} = \frac{1}{s_i}; \quad \frac{\partial f_{SD}}{\partial s} = \frac{1}{s_D} \quad (19)$$

Substituting Eq (19) into Eq (18) and then comparing with Eq (9) gives

$$\Lambda_{SI} = \Lambda_{SD} = (\lambda_{sr} - \kappa_s) ds \quad (20)$$

3 Model Predictions Versus Experimental Results

The model requires five parameters to describe the stress-strain behaviour ($\lambda(0)$, λ_s , κ , p_n' and M) and three parameters to describe the water-retention behaviour (λ_{sr} , κ_s and λ_{sc}).

We have performed a series of triaxial tests on unsaturated Pearl clay with suction control. Details of the Pearl clay properties, specimen preparation, and testing procedure can be found in the previous works of Sun et al. [5], which provides information about the stress-strain behaviour of Pearl clay, but do not give details of the measured water-retention behaviour. These are given below together with the model predictions. The model parameters for compacted Pearl clay are as follows: $\lambda(0) = 0.12$, $\kappa = 0.03$, $\lambda_s = 0.12$, $p_n' = 2\text{MPa}$, $M = 1.15$ and $\lambda_{sc} = 0.35$, $\lambda_{sr} = 0.1$, $\kappa_s = 0.01$.

Fig. 3. Predicted and measured results of triaxial compression test under $s=147\text{kPa}$, wetting ($s=147\text{kPa} \rightarrow 0\text{kPa}$) and zero suction ($s=0\text{kPa}$) and $p=196\text{kPa}$

Figure 3 shows predicted and measured results of isotropic compression, triaxial shear and wetting tests. The stress paths consist of CC'D (isotropic compression with constant suction s of 147kPa), DE (triaxial compression with constant mean net stress p of 196kPa and $s=147\text{kPa}$), EF (wetting from $s=147\text{kPa}$ to 0kPa under $p=196\text{kPa}$ and principal stress ratio of 2.0), and FG (triaxial compression under $p=196\text{kPa}$ and $s=0\text{kPa}$). It can be seen the model predicts well not only the stress-strain relationship but also hydraulic characteristics of unsaturated compacted clay.

Figure 4 shows predicted and measured results of triaxial tests on unsaturated compacted Pearl clay with almost the same initial void ratio but different degrees of

Fig. 4. Predicted and measured results of triaxial compression tests with different degrees of saturation

saturation (S_{r0}) at the beginning of triaxial shear tests. The measured stress-strain relationship and strength are different due to different degrees of saturation before loading even if the initial density and imposed net stress path and suction are the same. Hence, the mechanical behaviour of unsaturated soils depend not only the net stress and suction but also the degree of saturation. It can be seen from Fig. 4 that our model can predict the unsaturated soil behaviour.

4 Concluding Remarks

A coupled elastoplastic constitutive model is proposed for predicting the hydraulic and stress-strain-strength behaviour of unsaturated soils, and the predicted and measured results of triaxial tests were compared to validate the model.

References

1. Alonso F.E., Gens A., Josa A.: A constitutive model for partially saturated soils, *Geotechnique* 1990; 40(3): 405-430.
2. Bishop A. W. : The principal of effective stress, *Teknisk Ukeblad*, 1959, 39, 859-863.
3. Buisson M.S.R., Wheeler S.J.: Inclusion of hydraulic hysteresis in a new elastoplastic framework for unsaturated soils. In: Tarantino A. and Mancuso C. (eds): *Experimental Evidence and Theoretical Approaches in Unsaturated Soils*, Balkema (2000) 109-1119.
4. Sheng D.C., Sloan S.W., Gens A.: A constitutive model for unsaturated soils: thermomechanical and algorithmic aspects. *Computational Mechanics* 2004; 33: 453-465.
5. Sun D.A., Matsuoka H, Xu Y.F.: Collapse behavior of compacted clays by suction-controlled triaxial tests. *ASTM Geotechnical Testing Journal* 2004; 27(4): 1-9.
6. Sun D.A., Matsuoka H., Yao Y.P., Ichihara W.: An elastoplastic model for unsaturated soil in three-dimensional stresses. *Soils and Foundations* 2000; 40(3): 17-28.
7. Wheeler S.J., Sharma R.S., Buisson M.S.R.: Coupling of hydraulic hysteresis and stress-strain behaviour in unsaturated soils, *Geotechnique* 2003; 53(1):41-54.