A Novel DC Nodal Pricing Method Based on Linear Programming in Competitive Electricity Market

Bo Yang^{1,3}, Yun-ping Chen¹, Zun-lian Zhao², and Qi-ye Han³

¹ School of Electrical Engineering, Wuhan University, Wuhan 430072, China

yangbo@ccpg.com.cn

² State Grid Corporation of China, Beijing 100031, China

zl-zhao@sgcc.com.cn

³ Central China Grid Company Limited, Wuhan 430077, China

hanqy@ccpg.com.cn

Abstract. AC based nodal pricing method makes market rule less transparent and affects market decisions on risk and opportunity in short-run and long-run periods. Therefore, AC based nodal pricing formulation is simplified into DC based nodal pricing formulation, which maximizes total social welfare with the condition of satisfying Kirchhoff's laws and other operational constraints in modified DC optimal power flow model. Then, a simple but practical linear programming based nodal pricing method, which can be applied to solve market clearing mode with piecewise constant supply and demand curves, is proposed. The proposed method is tested on IEEE 30 bus system. Test results show that it can avoid premature and divergent phenomenon, which always occurs in the solution of AC based nodal pricing formulation, and has potential application in competitive electricity market.

Keywords: nodal price, linear programming, constrained optimization, optimal power flow, electricity market.

1 Introduction

Nowadays, electricity industry is undergoing deregulation and reconstruction. Spot pricing [1], real-time pricing for active and reactive power [2], and reserve pricing [3] have been proposed to determine price of energy and ancillary services in competitive electricity market. These pricing methods above are mainly based on nodal pricing principle which is supported by marginal cost theory. Mathematically, nodal pricing formulation is a large-scale, nonlinear, multi-variable, and constrained optimization problem. Its objective function is to maximize social welfare on condition that supply and demand curves are given and operational constraints are satisfied. But it has following disadvantages: (1) optimal dispatch schedule including active power and reactive power makes market rule less transparent and hence affects market decisions on risk and opportunity in short-run and long-run periods; (2) optimal price-quantity pairs in supply and demand sides at each node may not be obtained because optimization algorithm is trapped into local optimum or even diverges; (3) two totally

different dispatch schedules may have the same or similar social welfare value, which violates initial intension of market designer and confuses market participants. Therefore, a novel linear programming based DC nodal pricing method is proposed in the paper.

2 Nodal Pricing Formulation

Nodal pricing problem can be formulated as follows:

$$\text{Max } B = \sum_{i=1}^N \left[\int_0^{S_i^d} p_i^d(S) dS - \int_0^{S_i^s} p_i^s(S) dS \right] \quad (1)$$

subject to:

$$P_i^s - P_i^d - \sum_{j \in N} |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) = 0 \quad (2)$$

$$Q_i^s - Q_i^d - \sum_{j \in N} |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) = 0 \quad (3)$$

$$P_{i,\min}^s \leq P_i^s \leq P_{i,\max}^s, \quad Q_{i,\min}^s \leq Q_i^s \leq Q_{i,\max}^s \quad (4)$$

$$P_{i,\min}^d \leq P_i^d \leq P_{i,\max}^d, \quad Q_{i,\min}^d \leq Q_i^d \leq Q_{i,\max}^d \quad (5)$$

$$P_{ij,\min} \leq P_{ij} \leq P_{ij,\max}, \quad V_{i,\min} \leq V_i \leq V_{i,\max} \quad (6)$$

where S, P and Q are apparent power, active power, and reactive power, $Y_{ij} \angle \theta_{ij}$ and $V_i \angle \delta_i$ are the ij th element of bus admittance matrix and voltage at bus i , $p_i^d(S_i^d)$ and $p_i^s(S_i^s)$ are demand curve and supply curve at bus i , respectively. Superscripts d and s denote demand and supply, suffixes max and min denote upper and low limits, respectively. N is a bus set. P_{ij} is active power flow over $i \rightarrow j$.

3 LP-Based Method for DC Nodal Pricing

3.1 Modifications to Nodal Pricing Formulation

Given that branch resistances and charging capacitances are negligible, bus voltage magnitudes are close to 1 p.u., and voltage angle differences are small, AC based nodal pricing formulation can be modified into DC based nodal pricing formulation which is given as follows:

$$\text{Min } B = \sum_{i=1}^N \left[\int_0^{P_i^s} P_i^s(P) dP - \int_0^{P_i^d} P_i^d(P) dP \right] \quad (7)$$

subject to:

$$\sum_{i=1}^N (P_i^s - P_i^d) = 0, P_{i,\min}^s \leq P_i^s \leq P_{i,\max}^s, P_{i,\min}^d \leq P_i^d \leq P_{i,\max}^d, P_{ij,\min} \leq P_{ij} \leq P_{ij,\max} \quad (8)$$

3.2 Nodal Price of Active Power

The Kuhn-Tucker conditions for the DC nodal pricing formulation can be solved to obtain nodal price of active power at each bus. λ , $\lambda_{i,\max}^s$, $\lambda_{i,\min}^s$, $\lambda_{i,\max}^d$, $\lambda_{i,\min}^d$, $\lambda_{ij,\max}$ and $\lambda_{ij,\min}$ are Lagrangian multipliers of corresponding constraints.

$$P_i^s(P_i^s) - \lambda + \lambda_{i,\max}^s - \lambda_{i,\min}^s + \sum_{i,j \in N} \lambda_{ij,\max} \frac{\partial P_{ij}}{\partial P_i^s} - \sum_{i,j \in N} \lambda_{ij,\min} \frac{\partial P_{ij}}{\partial P_i^s} = 0 \quad (9)$$

$$-P_i^d(P_i^d) + \lambda + \lambda_{i,\max}^d - \lambda_{i,\min}^d + \sum_{i,j \in N} \lambda_{ij,\max} \frac{\partial P_{ij}}{\partial P_i^d} - \sum_{i,j \in N} \lambda_{ij,\min} \frac{\partial P_{ij}}{\partial P_i^d} = 0 \quad (10)$$

$$\sum_{i=1}^N (P_i^s - P_i^d) = 0 \quad (11)$$

$$\lambda_{i,\max}^s (P_{i,\max}^s - P_i^s) = 0, \lambda_{i,\min}^s (P_i^s - P_{i,\min}^s) = 0 \quad (12)$$

$$\lambda_{i,\max}^d (P_{i,\max}^d - P_i^d) = 0, \lambda_{i,\min}^d (P_i^d - P_{i,\min}^d) = 0 \quad (13)$$

$$\lambda_{ij,\max} (P_{ij,\max} - P_{ij}) = 0, \lambda_{ij,\min} (P_{ij} - P_{ij,\min}) = 0 \quad (14)$$

3.3 Solution Method

In many existing electricity markets, market rules set strict restrictions on the form of demand and supply curves. For example, demand and supply curves are required to be constant or piecewise constant functions. Since all constraints are linear in our proposed DC nodal pricing formulation, linear programming can be used to solve nodal price of active power at each bus if the restrictions above are imposed on demand and supply curves. It can avoid premature and divergent phenomenon, which always occurs in the solution of AC based nodal pricing formulation, and give certain price signal for market participants in competitive electricity market.

4 Simulation Results

To demonstrate the feasibility of the proposed method, we test it on IEEE 30 bus system [4]. In the experiment, demand price elasticity is zero and supply curve is piecewise constant function. Two cases are studied. In case 1, power flow limit over each transmission line is not attained; in case 2, power flow limits over transmission line $1 \rightarrow 2$ and $1 \rightarrow 3$ are decreased from 130 MW to 16MW. After computation by the proposed method, nodal price at each bus is 44.000\$/MW in case 1, but is different in case 2. Nodal prices of case 2 are given in Fig. 1. Apparently, because of power flow limits over transmission line $1 \rightarrow 2$ and $1 \rightarrow 3$, nodal price dramatically decreases at bus 1, greatly increases at bus 2, slightly changes at bus 13,22,23 and 27. Besides, we can draw a conclusion that power flow transmission limit has greater influence on nodal price at neighboring bus than at remote bus.

Fig. 1. Nodal price at each bus in case 2

5 Conclusions

A novel DC nodal pricing method based on linear programming is proposed in the paper. It can provide useful price signal for demand and supply sides. Because of its simplicity and practicability, it can be applied to day-ahead and balance markets.

References

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