An Improved CAViaR Model for Oil Price Risk

Dashan Huang¹, Baimin Yu², Lean Yu², Frank J. Fabozzi³,
and Masao Fukushima¹

¹ Department of Applied Mathematics and Physics, Graduate School of Informatics,
Kyoto University, Kyoto 606-8501, Japan
{dhuang,fuku}@amp.i.kyoto-u.ac.jp

² Academy of Mathematics and Systems Science, Chinese Academy of Sciences,
Beijing, 100080, China
{bmyu,yulean}@amss.ac.cn

³ School of Management, Yale University, New Haven, CT06520, USA
{frank.fabozzi}@yale.edu

Abstract. As a benchmark for measuring market risk, Value-at-Risk (VaR) reduces the risk associated with any kind of asset to just a number (amount in terms of a currency), which can be well understood by regulators, board members, and other interested parties. This paper employs a new kind of VaR approach due to Engle and Manganelli [4] to forecasting oil price risk. In doing so, we provide two original contributions: introducing a new exponentially weighted moving average CAViaR model and developing a least squares regression model for multi-period VaR prediction.

Keywords: CAViaR, oil price risk, exponentially weighted moving average.

1 Introduction

Because of the key role of oil in the economies throughout the world, predicting the future price of this commodity and managing the risks associated with future oil prices are critical for central governments and businesses. Therefore oil price prediction and its risk measurement has been a common research theme in the last decades. Some examples of the oil price prediction are (1) a semiparametric approach to short-term oil price forecasting developed by Morana [7], (2) a hybrid artificial intelligence system framework for crude oil price forecasting by integrating artificial neural networks and rule-based expert system with web-based text mining techniques proposed by Wang et al. [10], and (3) a support vector machine method (which is superior to the traditional ARIMA method and the Wang et al. proposal) suggested by Xie et al. [11]. Our focus in this paper is on oil price risk measurement. Previous work in this area includes (1) Cabedo et al. [2] who proposed an ARMA historical simulation approach with Value-at-Risk (VaR) ideology and (2) Kuper [6] who investigated (G)ARCH-based volatility performance with different historical data. A good literature review on oil price movements and their effects on economic and financial performance can be referred to Sauter and Awerbuch [9].

Our motivation for concentrating on quantifying oil price risk are twofold. First, as noted by Sauter and Awerbuch [9], while most oil price movements from 1948-1985 were price increases, since 1986 the pattern has changed; more specifically, large price decreases are now commonplace, implying a rise in the volatility/risk of oil prices. Second, price risk in investments is more important than price level which determines whether to invest or not. Compared with the more developed financial market risk measures, however, oil price risk management is still in its infancy. This is because, being a physical asset, oil has some additional attributes that are not associated with traditional financial assets. For example, as Morana [7] points out, the price of oil is strongly influenced by transportation, inventory levels, weather, short-term demand and supply imbalances, and political issues. Thus, volatile oil price movements motivate consideration of how to quantify price risk. As a benchmark for measuring market risk, Value-at-Risk (VaR) reduces the risk associated with any kind of asset to just a number (amount in terms of a currency), which can be well understood by regulators, board members, and other interested parties.

The main objective of this paper is to investigate the predictive performance of various types of the conditional autoregressive value-at-risk (CAViaR) specifications for oil price risk prediction. The choice of this particular parametric method is motivated by the fact that CAViaR does not require any assumption on the distribution of a time series and computes the VaR directly by quantile regression, implying that the model allows the time series to switch from one stochastic process to another. There are also two original theoretical contributions in this paper. First, we introduce a new exponentially weighted moving average CAViaR specification with only two parameters to be estimated, which in turn may reduce the model estimate risk. Second, we develop a least squares regression model for multi-period VaR prediction.

The rest of the paper is structured as follows. Section 2 briefly summarizes the CAViaR model and then proposes a new asymmetric CAViaR model. Section 3 provides some criteria for performance comparison. Section 4 presents some empirical results, followed by our concluding comments in Section 5.

2 Model

While the concept of VaR is intuitive, its modeling is a very challenging statistical problem. From a statistical point of view, VaR of a return series $\{r_t\}$, conditional on the information set $\mathcal{F}_{t-1}$, is defined as the negative θ -quantile, $q_t(\theta)$, for which $P(r_t \leq -q_t(\theta) | \mathcal{F}_{t-1}) = \theta$.

2.1 CAViaR

Building on the fact that the volatility of returns is highly autocorrelated (clustering), Engle and Manganelli [4] proposed the CAViaR model which computes $q_t(\theta)$ directly. In this paper, we will focus on the following four specifications:

Adaptive CAViaR: $q_t(\theta) = q_{t-1}(\theta) + \beta_1 \{ [1 + \exp(G[r_{t-1} + q_{t-1}(\theta)])]^{-1} - \theta \}$,
 Symmetric CAViaR: $q_t(\theta) = \beta_0 + \beta_1 q_{t-1}(\theta) + \beta_2 |r_{t-1}|$,
 Asymmetric CAViaR: $q_t(\theta) = \beta_0 + \beta_1 q_{t-1}(\theta) + \beta_2 (r_{t-1})^+ + \beta_3 (r_{t-1})^-$,
 GARCH(1,1) CAViaR: $q_t(\theta) = (\beta_0 + \beta_1 q_{t-1}^2(\theta) + \beta_2 r_{t-1}^2)^{1/2}$.

In the above, G is a positive number, and hence as $G \rightarrow \infty$, the adaptive CAViaR converges to $q_t(\theta) = q_{t-1}(\theta) + \beta_1 [I(r_{t-1} \leq -q_{t-1}(\theta)) - \theta]$, where $I(\cdot)$ is the indicator function. As noted by Engle and Manganelli [4], the CAViaR specifications generalize the traditional GARCH-based VaR models by allowing for different stochastic processes in the tail of the return series, and can also deal with non-i.i.d. processes.

2.2 Improved CAViaR

Instead of estimating the entire distribution of a time series, the CAViaR model computes the conditional quantile directly, thereby exhibiting some theoretical advantages over the traditional VaR approach. However, in practice, CAViaR may face some implementation challenges. For example, to investigate the different effects of positive and negative returns on the prediction from a VaR model, Engle and Manganelli [4] suggested using the asymmetric CAViaR specification. This requires estimating four parameters in the optimization framework. Unfortunately, as an econometric model, more parameters may lead to more estimation error, which will in turn make the model unstable or unrobust.

Based on the exponentially weighted moving average volatility prediction idea, we propose a new asymmetric CAViaR model:

$$q_t(\theta) = \beta_0 q_{t-1}(\theta) + (1 - \beta_0) \left(\frac{v}{1 - \beta_1} I(r_{t-1} > 0) + \frac{v}{\beta_1} I(r_{t-1} \leq 0) \right) |r_{t-1} - \mu|,$$

where $v = \sqrt{\beta_1^2 + (1 - \beta_1)^2}$ ($0 < \beta_1 < 1$) and μ is the sample mean. This CAViaR specification allows the VaR prediction to respond asymmetrically to positive and negative returns. For example, when $\beta_1 < 0.5$, negative returns have a larger influence on the VaR prediction. Moreover, in contrast to the asymmetric CAViaR model in [4], our model has only two parameters to be estimated. In Section 4, we show that our model can yield performance at least as good as the model of Engle and Manganelli (2004).

For parameter estimation, we follow Engle and Manganelli [4] and employ the linear and non-linear quantile techniques:

$$\hat{\beta}(\theta) = \operatorname{argmin}_{\beta} \left\{ \sum_{\{t: r_t \geq -q_t(\theta)\}} \theta |r_t + q_t(\theta)| + \sum_{\{t: r_t < -q_t(\theta)\}} (1 - \theta) |r_t + q_t(\theta)| \right\}.$$

This regression technique has a tractable property in that it does not require any assumption about the entire distribution of the error terms. Moreover, we can easily conduct various hypothesis tests on the quantile models [4,5].

2.3 Multi-period CAViaR

The simplest approach to generating multi-period VaR forecasts is to multiply the one step-ahead prediction by the square root of the holding period k , the procedure usually used in RiskMetrics [8]. However, this approach implicitly requires that the variance of the time series is constant in the holding period, thereby offsetting advantage that CAViaR can accommodate different stochastic processes in the holding period. But as a simple alternative, we provide an intuitive least squares regression model for multi-period VaR estimates:

$$q_{t+k}(\theta) = \alpha_0 + \alpha_1 q_t(\theta) + \alpha_2 [I(r_t \leq -q_t(\theta)) - \theta],$$

where k is the holding period and the parameters α_0 , α_1 , and α_2 are estimated by the least squares regression. This regression model incorporates the following rules: When the time series follows a unique stochastic process in the holding period, α_1 will approach $\sqrt{k}$, and α_0 and α_2 will converge to 0. Moreover, in a similar spirit to the adaptive CAViaR model, whenever the loss exceeds VaR (underestimate), we increase it immediately, but when the loss does not exceed VaR (overestimate), we just decrease it slightly.

Note that while the multi-period VaR prediction approach examined here has two steps (estimate the one step-ahead VaR and estimate multi-period VaR) it may be combined into one step and thereby estimating all the parameters simultaneously.

3 Evaluation of Performance

To assess the predictive performance of the models investigated, in this section we employ tests designed for evaluating the predictive quantile performance as proposed by Bao et al. [1] and the references therein.

When assessing the accuracy of a VaR forecast model, there is always a general hypothesis required to be satisfied. That is,

$$E[H_t(\theta) | \mathcal{F}_{t-1}] = \theta, \tag{1}$$

where $H_t(\theta) = I(r_t < -q_t(\theta))$. Below we denote $H_t = H_t(\theta)$. If (1) holds, we say the VaR prediction is efficient with respect to $\mathcal{F}_{t-1}$. This means H_t is uncorrelated with any information in $\mathcal{F}_{t-1}$ up to time $t - 1$. In this case, the forecast for VaR at time t and H_t cannot be improved. There are two implications in condition (1): correct unconditional coverage and independence of VaR forecasts.

With respect to the out-of-sample prediction, we split the sample into two parts, T_{in} and T_{out} , and use T_{in} to estimate the parameters and T_{out} for out-of-sample performance testing. Thus, we can compute the empirical coverage probability resulting from the VaR prediction series $\{\hat{q}_t(\theta)\}_{t \in T_{\text{out}}}$, that is,

$$\hat{\theta} = \frac{1}{|T_{\text{out}}|} \sum_{t \in T_{\text{out}}} \hat{H}_t,$$

Where $|T_{\text{out}}|$ denotes the cardinality of T_{out} . A VaR prediction model is preferred if its $\hat{\theta}$ is closer to the nominal value of θ .

3.1 Unconditional Coverage Test

Suppose VaR estimates are accurate. Then, the event of failure $\hat{H}_t = 1$ can be modeled as independent draws from a binomial distribution with a probability of occurrence equal to the significance level ($\hat{\theta} = \theta$). Thus, the likelihood-ratio test statistic (LR) is

$$LR = -2\ln[(1 - \theta)^{|T_{out}|*(1-\hat{\theta})}\theta^{|T_{out}|*\hat{\theta}}] + 2\ln[(1 - \hat{\theta})^{|T_{out}|*(1-\hat{\theta})}\hat{\theta}^{|T_{out}|*\hat{\theta}}].$$

Under the null hypothesis, LR follows a χ^2 distribution with 1 degree of freedom.

3.2 Independence Test

A good VaR model must generate independent forecasts, i.e., violation ($\hat{H}_t = 1$) today has no effect on the probability of violation tomorrow.

In an approach similar to Engle and Manganelli [4], we investigate the regression of H_t on H_{t-i} ($i = 1, \dots, M$) and $\hat{q}_t(\theta)$, that is,

$$H_t = \theta_0 + \sum_{i=1}^M w_i H_{t-i} + w_{M+1} \hat{q}_t(\theta) + u_t, \quad (2)$$

where w_i , $i = 1, \dots, M + 1$, are regression parameters and

$$u_t = \begin{cases} -\theta, & \text{with probability } 1 - \theta, \\ 1 - \theta, & \text{with probability } \theta. \end{cases}$$

Under the null hypothesis, $\theta_0 = \theta$ and $w_i = 0$, $i = 1, \dots, M + 1$. Denoting $w_0 = \theta_0 - \theta$, we rewrite (2) in vector notation as

$$\mathbf{H} - \theta \mathbf{1} = \mathbf{X}\mathbf{w} + \mathbf{u},$$

where $\mathbf{1}$ denotes the vector of ones. Thus, under the null hypothesis, $\Theta_0 : \mathbf{w} = 0$, Engle and Manganelli [4] proposed the following test statistic:

$$DQ = \frac{\hat{\mathbf{w}}' \mathbf{X}' \mathbf{X} \hat{\mathbf{w}}}{\hat{\theta}(1 - \hat{\theta})} \rightarrow \chi_{M+2}^2 \text{ (asymptotically).}$$

In the empirical application, we follow Engle and Manganelli (2004) to explore the dynamic quantile test: DQ in which $\mathbf{X}$ contains a constant, four lagged hits, $H_{t-1}, \dots, H_{t-4}$, and a contemporaneous VaR estimate, $q_t(\theta)$.

4 Oil Price VaR

To implement the Engle and Manganelli approach and our proposed CAViaR model on real oil price risk forecasting, we took a sample of 2,475 daily data from Energy Information Administration for Brent and WTI spot prices, and computed the daily returns as 100 times the difference of the log of the prices,

Table 1. Performance of one-day-ahead VaR Prediction

		Adaptive	SCAV	ACAV	GARCH	Improved
1%	$\hat{\theta}$ Brent	0.0100	0.0100	0.0100	0.0080	0.0060
	WTI	0.0060	0.0040	0.0040	0.0040	0.0100
	p-value Brent	0.0016	0.9940	0.9694	0.9627	0.9908
	WTI	0.9907	0.8639	0.8636	0.9063	0.0002
5%	$\hat{\theta}$ Brent	0.0560	0.0440	0.0440	0.0420	0.0440
	WTI	0.0500	0.0360	0.0280	0.0320	0.0560
	p-value Brent	0.6001	0.9722	0.7462	0.8389	0.4582
	WTI	0.3393	0.7609	0.3035	0.5889	0.0481

Note: SCAV: symmetric CAViaR; ACAV: asymmetric CAViaR.

thereby generating 2,474 return scenarios. The samples range from November 29, 1996 to November 28, 2006. We used the first 1,974 observations for the estimation of the model parameters and the remaining 500 for out-of-sample testing. For the optimization routines, we employed a similar procedure to that described by Engle and Manganelli (2004). Because the objective function of the quantile regression model is designed to obtain the exact sample quantile with a given level, it is not surprising that the in-sample empirical coverage is quite precise (see Engle and Manganelli (2004) and Huang et al. (2006)). In this paper, we concentrated on the one-day-ahead and five-day-ahead out-of-sample VaR predictions. In the experiment, we use the empirical θ -quantile of the first 300 observations as the initialization to compute the VaR series.

For the one day-ahead VaR forecasting, in Table 1 we report the values of the empirical unconditional coverage probability and the p-value of the dynamic quantile test. As described in Section 3, the empirical coverage probability $\hat{\theta}$ represents the percentage of observations falling below the VaR predictions. Ideally, with a sufficiently large sample, the percentage should converge to θ . In this sense, all the models (the Engle and Manganelli model and our proposed model) pass the unconditional coverage test in general with only one exception (the confidence interval is [0.2%, 2%] at the 1% level and it is [3%, 7%] at the 5% level). Especially, all models perform well with Brent since the percentage of empirical coverage seems reasonably close to 1% or 5%. There is a surprising result that the asymmetric CAViaR specification does not pass the unconditional coverage test at the 5% level (the empirical coverage 0.028 is out of the confidence interval). This is in sharp contrast to the results reported by Engle and Manganelli (2004) which show that the asymmetric CAViaR model may do the best job.

With respect to the independence test, the adaptive model is rejected by Brent and the improved CAViaR is rejected by WTI at the 1% level. However, all the models survive in the independence test at the 5% level, producing rather large p-values.

Figure 1 plots the Brent return and one-day-ahead VaR forecast curves, which shows that although our proposed model has only two parameter, it can produce

Fig. 1. Brent return and one-day-ahead estimated CAViaR plots at the 1% level

an estimate as good as the asymmetric CAViaR model with four parameters. Moreover, the superiority of our model will be more evident in the case of multi-period prediction.

In Table 2, for both Brent and WTI, we present the least squares regression parameters, α_0 , α_1 , and α_2 , estimated for five-day-ahead VaR prediction. Interestingly, all the estimates of α_1 are far from $\sqrt{5}$, implying that both the Brent and WTI return processes do not follow a certain stochastic process in the holding period. Conversely, they may switch from one process to another; that is, the oil price process has structural breakpoints resulting from natural or politic events (Sauter and Awerbuch (2002) and Wang et al. (2004) identify important events affecting the price of oil). We also report the out-of-sample forecasting performance from the different models. The basic measure reported is the coef-

Table 2. Performance of five-day-ahead VaR Prediction at the 1&5% respective levels

	1%				5%			
	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	R^2	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	R^2
Adaptive Brent	0.0806	0.9891	0.9341	0.9841	0.0687	0.9826	0.1771	0.9680
WTI	0.1044	0.9871	0.9880	0.9823	0.1536	0.9636	0.4742	0.9313
SCAV Brent	3.0473	0.4894	2.0959	0.2710	1.0678	0.7259	0.3763	0.5589
WTI	4.9158	0.3476	1.5650	0.1301	2.3295	0.4049	0.3305	0.1820
ACAV Brent	3.2865	0.4730	2.4061	0.2486	2.3161	0.3984	0.5588	0.1934
WTI	4.8500	0.3585	1.6729	0.1383	2.4744	0.3772	0.3808	0.1614
GARCH Brent	2.9499	0.5344	3.0928	0.3182	1.0172	0.7378	0.4619	0.5770
WTI	5.9880	0.1595	0.5159	0.0263	1.6245	0.5942	0.4757	0.3767
Improved Brent	0.0788	0.9896	0.2663	0.9939	0.0167	0.9959	0.0387	0.9947
WTI	0.5929	0.9174	0.6988	0.8424	0.1461	0.9642	0.1300	0.9256

ficient of determination, R^2 , from the least squares regression. The results show that both the adaptive and the proposed CAViaR specifications outperform the other three specifications (all the R^2 values exceed 0.9).

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