A New Fault-Tolerant Routing Algorithm for m-ary n-cube Multi-computers and Its Performance Analysis

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Abstract. A new algorithm for fault-tolerant routing based on detour and backtracking techniques is developed for m-ary n-cube multi-computer networks. We analyzed its performance under the condition that when an arbitrary number of components have been damaged and derived some exact expressions for the probability of routing messages via optimal paths from the source node to obstructed node. The probability of routing messages via an optimal path between any two nodes is a special case of our results, and can be obtained by replacing the obstructed node with the destination node.

1 Introduction

m-ary n-cube usually denoted by $Q_n(m)$ is a kind of generalized hypercube and has drawn many attention. [1] showed that the restricted connectivity is $2n(m-1)-m$ and the restricted fault diameter can be controlled to be less than $n+3$. [2] and [3] concluded that there are $n(m-1)$ disjoint paths linking any two nodes and the $n(m-1)-$wide diameter is $n+1$. Two deterministic shortest and fastest routing algorithm have been proposed in [3]. [4]and [5]designed the fault-tolerant routing schemes for $Q_n(m)$. [6] discussed its fault tolerance and transmitting delay. Other parameters can be found in [7].

We shall develop a routing scheme for $Q_n(m)$, in which each message is accompanied with a stack which keeps track of the history of the path travelled as it is routed toward its destination, and tries to avoid visiting a node more than once unless a backtracking is enforced, each node is required to know only the condition (faulty or not) of its adjacent components. This routing algorithm is analyzed rigorously. Similar discussion has been made for hypercube ([8]).

The first node in the message's route that is aware of the nonexistence of an optimal path from itself to the destination is called obstructed node. At the obstructed node, the message has to take a detour. In this paper, we derive exact expressions for the probabilities of optimal path routing from the source node to a given obstructed node in the presence of components failures. Note that determination of the probability for optimal path routing between any two nodes can be viewed as an obstructed node that is 0 hop away from the destination node.

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$Q_n(m)$ has vertex set $V(Q_n(m)) = \{x_1x_2\ldots x_n,\ x_i \in \{0, 1\ldots, m-1\}\}$, x and y are adjacent if and only if they differ by exactly one bit. For $x = x_1x_2\ldots x_n$, the leftmost coordinate of the address will be referred to as 1-dimension, and the second to the leftmost coordinate as 2-dimension, and so on.

Suppose $x = x_1x_2x_3\ldots x_n$ and $y = y_1y_2y_3\ldots y_n$ be two nodes of $Q_n(m)$, xy is an edge of $i-$dimension if $x_j = y_j$ for $j \neq i$ and $x_i \neq y_i$. From the definition, $Q_n(m)$ contains m^n vertices and $m^n n(m-1)/2$ edges and $Q_n(m)$ is $n(m-1)$-regular with diameter n.

An optimal path is a path whose length is equal to the Hamming distance between the source and destination. We call the routing via an optimal path the optimal path routing. An incident link of node x is said to be toward another node y if the link belongs to one of the optimal path from x to y and call y the forward node of x.

A given path of length k between x and y in $Q_n(m)$ can be described by a coordinate sequence $C = [c_1, c_2, \ldots, c_k]$ where $1 \leq c_i \leq n$, the coordinate sequence is a sequence of ordered pairs. A coordinate sequence is said to be simple if any dimension does not occur more than once in that sequence. It is easy to see that a path is optimal if and only if its coordinate sequence is simple. For example, [0002, 0000, 0010, 2010] is an optimal path from 0002 to 2010, and can be represented by a coordinate sequence [4, 3, 1].

The number of inversions of a simple coordinate sequence $C = [c_1, c_2, \ldots, c_k]$ denoted by $V(C)$, is the number of pairs (c_i, c_j) such that $1 \leq i < j \leq k$ but $c_i > c_j$. For example $V([4, 3, 1]) = 3$.

2 Routing Algorithm

Algorithm A: Fault-tolerant Routing Algorithm

Step 1. If $u = d$, the message is reached destination, Stop.
Step 2. If the forward adjacent node v of u is normal and the link uv is normal and $v \notin TD$, select such a vertex v satisfied $i = \min\{i : uv \text{ is an edge of} i-\text{dimension}\}$, then

send (message TD) to v, $TD = TD \cup \{u\}$, $u = v$.

Step 3. If v is a adjacent node of u and satisfies the following condition:

1. $v \notin TD$
2. v is not a forward node node of u to d
3. v and uv are normal components

select such a vertex v satisfied that $j = \min\{j : uv \text{ is an edge of } j-\text{dimension}\}$, then send (message TD) to v, $TD = TD \cup \{v\}$, $u = v$. Go to Step 1.
Step 4. If the algorithm is not terminated yet, then Backtracking is taken, the message must be returned to the node from which this message was originally received. Go to Step 3.

3 Performance Analysis of Routing Algorithm

Theorem 1. *Suppose x and y are respectively the source and destination in $Q_n(m)$, $H(x,y) = n$. Then the number of fault components required for the simple coordinate sequence $C = [c_1, c_2, \ldots, c_t]$ to be the path chosen by algorithm A to an obstructed node located j hops away from y is $V(C)+W(c_1, c_2, \ldots, c_t) - \sum_{i=1}^{t} i + j$, where $t = n - j$,$W(c_1, c_2, \ldots, c_t) = \sum_{i=1}^{t} c_i$.*

Let $S(n,r)$ be the set of combinations of r different numbers out of $\{1, \ldots, n\}$ and $I_n(r)$ denote the number of permutations of n numbers with exactly r inversions.

Theorem 2. *Suppose there are f fault links in a $m-$ary $n-$cube computer network, and a message is routed by A from node x to node y where $H(x,y) = n$. Let h_L be the Hamming distance between obstructed node and the destination node. Then*

$$P(h_L = j) = \frac{1}{C_L^f} \sum_{\sigma \in S(n,t)} \sum_{k=0}^{\min\{\frac{n(n-1)}{2}, f-j\}} I_t(\alpha) C_{L-n-k}^{f-j-k}$$

where $\alpha = k - W(\sigma) + \frac{t(t+1)}{2}$ and $P(A)$ is the probability of event A, $L = n(m-1)m^n/2$ and $t + j = n$.

The probability of an optimal path routing can be viewed as a special case of Theorem 2 by setting the obstructed node to the destination node, namely, $P(h_L = 0)$.

Corollary 1. *The probability for a message to be routed in an $Q_n(m)$ with f fault links via an optimal path to a destination node which is n hops away can be expressed as*

$$P(h_L = 0) = \frac{1}{C_{\frac{n(m-1)m^n}{2}}^f} \sum_{k=0}^{\min\{\frac{n(n-1)}{2}, f\}} I_n(k) C_{\frac{n(m-1)m^n}{2}-n-k}^{f-k}$$

Theorem 3. *Suppose there exist h faulty nodes in a $Q_n(m)$, and a message is routed by A from x to y where $H(x,y) = n$. Let h_N be the Hamming distance between obstructed node and the destination node. Then for $2 \leq j \leq \min\{h, n\}$, we have,*

$$P(h_N = j) = \frac{1}{C_{m^n-2}^h} \sum_{\sigma \in S(n,t)} \sum_{k=0}^{\min\{\frac{n(n-1)}{2}, h-j\}} I_t(k - W(\sigma) + \frac{t(t+1)}{2}) C_{m^n-2-n-k}^{h-j-k}$$

where $t + j = n$.

Corollary 2. *Under algorithm A, the probability for a message to be routed in a $Q_n(m)$ with h faulty nodes via an optimal path to a destination located n hops away is*

$$P(h_N = 0) = \frac{1}{C^h_{m^n-2}} \sum_{k=0}^{\min\{\frac{n(n-1)}{2},h\}} I_n(k) C^{h-k}_{L=m^n-1-n-k}.$$

4 Conclusion

This paper proposed a new fault-tolerant routing algorithm for m-ary n-cube. This algorithm is based on the detour and backtracking technique. The knowledge on the number os inversions of a given permutation is used to analyze the performance of this routing. The number of faulty components required for a coordinate sequence to become the coordinate sequence of a path toward a given obstructed node is determined.Probability for routing messages via optimal path to given obstructed node location are determined.

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