On the Assessment of Petroleum Corporation's Sustainability Based on Linguistic Fuzzy Method

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Abstract. By use of the relationship between uncertain fuzzy number and linguistic number, a linguistic fuzzy method to assess the sustainability of Petroleum Corporation as one kind of hybrid multi-attribute decision making problems is put forward. Simultaneously, the attribute weight of decision making problem is derived from the maximum entropy model. The *OWA* operator is used to aggregate the attribute information. Finally, it is illustrated by a numerical example that this method is feasible and effective.

Keywords: sustainability, hybrid multiple attribute decision making, linguistic fuzzy number, petroleum corporation.

1 Introduction

The assessment of Petroleum Corporation's sustainability involves economy, society and environment. The assessment attributes may be quantitative and qualitative. Other researchers usually assess it with crisp numbers. However, because of the complex geological structure, it would be practical to employ uncertain number to express some of the attributes. For the qualitative attributes, decision-makers sometimes use fuzzy linguistic terms ^[1] as the assessment information. Consequently, the assessment of Petroleum Corporation's sustainability is the hybrid attributes decision making issue. For solving the problem, a linguistic fuzzy method is proposed. Firstly, using the relationship between the uncertainty fuzzy number and the fuzzy linguistic terms, we transform the hybrid attributes into a uniform fuzzy linguistic attribute in order to make the assessment results being in good according with our understanding. Secondly, when selecting the weight of decision attributes, the attitude value is used for us to fully consider the risk attitude of decision makers. Finally, attribute values are aggregated by *OWA* (Ordered weighted averaging) operator.

2 The Maximum Entropy Model with the Attitude of Decision

2.1 OWA Operator

Let $X = \{x_1, x_2, \dots, x_m\}$ be a set of alternatives, $\{p_1, p_2, \dots, p_n\}$ be a set of attributes.

Definition 1 ^[2]: Let $OWA: L^n \mapsto L$, if

$$OWA_{\omega}(a_1, a_2, \dots, a_n) = \sum_{j=1}^n \omega_j b_j \quad (1)$$

Where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is a weighting vector associated with OWA operator, satisfied $\omega_j \in [0, 1]$, $\sum_{i=1}^n \omega_i = 1$; b_j is the j th largest element of $\{a_1, a_2, \dots, a_n\}$; L is a number set; OWA is called an order weighted averaging operator, that is OWA operator.

If the weight vector of attributes is taken as a kind of attitudinal-character probability, under a certain attitude, the attribute weight should be selected which can maximize the entropy as the following model (*):

$$\text{Max } E(v) = -\sum_{i=1}^n v_i \ln v_i \quad (2)$$

$$\alpha = \frac{1}{n-1} \sum_{i=1}^n (n-i) v_i \quad (3)$$

Where $\alpha \in [0, 1]$ is the degree of optimism and $\sum_{i=1}^n v_i = 1$, $v_i \in [0, 1]$, $i = 1, \dots, n$.

2.2 The 2-Tuple Linguistic Term and Uncertainty Fuzzy Number

Let $S = \{S_0, \dots, S_g\}$ be a linguistic term set, where $2 \leq g \leq 14$. The semantics of the terms is given by fuzzy numbers defined in $[0, 1]$ interval. A way to characterize a fuzzy number is to use a representation based on parameter of its membership function. Sometimes the linguistic terms can be representing by triangle fuzzy number, such as $S_i = (a_i, b_i, c_i)$, where $a_i = \frac{i-1}{g}$ ($1 \leq i \leq g$), $b_i = \frac{i}{g}$ ($0 \leq i \leq g$), $c_i = \frac{i+1}{g}$ ($0 \leq i \leq g-1$), $a_0 = 0, c_g = 1$.

The linguistic information is represented by a 2-tuple linguistic model^[3] (S_i, α_i) . The 2-tuple linguistic operator can be defined as follow:

Definition 2^[3]: Let $S = \{S_0, S_1, \dots, S_g\}$ be a linguistic term set and $\beta \in [0, g]$ is a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function Δ :

$$\Delta: [0, g] \rightarrow S \times [-0.5, 0.5]$$

$$\Delta(\beta) = (S_i, \alpha) \quad (4)$$

Where $i = \text{round}(\beta)$, $\alpha = \beta - i$, $\alpha \in [-0.5, 0.5]$ and "round" is the rounding operation.

Definition 3^[3]: Let I be an uncertain fuzzy number. Then I can be transforming into linguistic 2-tuple set by the function $\tau_{IS_T}: [0, 1] \rightarrow F(S_T)$.

$$\tau_{IS_T}(I) = \{(S_k, \omega_k) / k \in \{0, \dots, g\}\} \quad (5)$$

$$\omega_k = \max_y \min\{\mu_I(y), \mu_{S_k}(y)\} \quad (6)$$

Where $\mu_I(\cdot), \mu_{S_k}(\cdot)$ are the membership functions associated with the terms I and S_k .

Definition 4^[4]: Let $\tau(I) = \{(S_0, \omega_0), (S_1, \omega_1), \dots, (S_g, \omega_g)\}$ be a fuzzy set that represents a numerical value I over the linguistic term set S . we obtain a numerical value that represents the information from the fuzzy set by means of the function $\chi: F(S_T) \rightarrow [0, g]$.

$$\chi\tau(I) = \chi(F(S_T)) = \chi\{(S_j, \omega_j), j = 0, 1, \dots, g\} = \sum_{j=0}^g j\omega_j (\sum_{j=0}^g \omega_j)^{-1} = \beta \quad (7)$$

The value β is easily transformed into a linguistic 2-tuple using the definition 2.

3 The Aggregation Approach

Consider a multi-attribute decision making problem with m alternatives $x_1, x_2, \dots, x_m$, and n decision attributes $p_1, p_2, \dots, p_n$. Each alternative is assessed with respect to each attribute. The assessment scores assigned to the attributes are the components of decision matrix denoted by $R = (\tilde{r}_{ij})_{m \times n}$, where $\tilde{r}_{ij}$ is a numerical value or uncertainty fuzzy number. The decision making steps are given as follows:

Step 1. Construct the decision matrix.

Step 2. Normalize the decision matrix $R = (\tilde{r}_{ij})_{m \times n}$ to matrix $R' = (r'_{ij})_{m \times n}$. For instance, the elements $\tilde{r}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ of the decision matrix $R = (\tilde{r}_{ij})_{m \times n}$ are triangular fuzzy numbers, then $r'_{ij} = (\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+})$, $j \in B$, and $c_j^+ = \max_i c_{ij}$, $j \in B$. Any decision matrix with entries being numerical values can be normalized using the same method mentioned above.

Step 3. Using equations (4-7), transform the attribute values into 2-tuple linguistic terms. We get a decision matrix $P = (p_{ij})_{m \times n}$ with entries being 2-tuple linguistic values.

Step 4. Calculate the comprehensive attribute values. The weight vector $\omega^* = (\omega_1^*, \dots, \omega_k^*, \dots, \omega_n^*)^T$, $k = 1, \dots, n$ can be got by model(*). Then the comprehensive attributes value t_i , $i = 1, 2, \dots, m$ of each alternative can be aggregated by the OWA operator:

$$t_i = \phi_Q(p_{i1}, \dots, p_{in}) = \Delta(\sum_{k=1}^n \omega_k^* \Delta^{-1}(p'_{ik})) \quad (8)$$

Where p'_{ik} is the k th largest value in the set $\{p_{i1}, \dots, p_{in}\}$.

Step 5. Rank the alternatives $x_1, x_2, \dots, x_m$.

4 Numerical Example

Let $X = \{x_1, \dots, x_9\}$ denotes nine Petroleum Corporations set, which are BP, Exxon-Mobil, Shell, Anni, CNPC, SINOPEC, CNOOC, Statoil, Petrobras. The attributes of sustainability of Petroleum Corporation are given in table 1.

The values of sub-attributes are given according to the following principle: The numerical values are given by 2004 annually report directly or through simple calculated. Especially, the data of oil and gas surplus workable reserves attributes take the form of the triangle fuzzy numbers. These two triangles fuzzy numbers sets are constructed according to the reserves data in 2003 annals and in 2004 annals and the reserves in 2003 annals go through 3%'s fluctuation. The society benefit and environment harmonious attributes gain from investigation.

Table 1. The sustainability attributes of Petroleum Corporation

Attributes	Scale of resource	Acquire the resource	Capital increment	Society benefit	Environment harmonious
Sub-attributes	Total assets	Oil output	Net profit	Satisfied product need	Provide new energy ability
	Oil surplus workable reserves	Gas output	Return on asset	Provide jobs ability	Avoid venture ability
	Gas surplus workable reserves	Refine ability	Return on equity	Guarantee safe ability	Protect the environment

Suppose that the decision makers hold the normal risk attitude, then attributes weight can be derived from the model(*), which is $\omega^* = (.1620, .1791, .1980, .2190, .2421)^T$. By (8), the ranking order of nine petroleum corporations is $x_2 \succ x_1 \succ x_3 \succ x_5 \succ x_6 \succ x_4 \succ x_8 \succ x_7 \succ x_9$.

The appraisal result is reasonable. We know the sustainability of Petroleum Corporation is close connecting with its comprehensive ability. This ranking order is the same as the order of their comprehensive ability that US "petroleum intelligence weekly" magazine given in 2003. It can be conclude that the assessment method given is objective and valid.

5 Conclusions

This paper develops an assessment approach of Petroleum Corporation's sustainability, and it is illustrated by a numerical example that this method is feasible and effective. Using the relationship between the uncertainty fuzzy number and the linguistic fuzzy term, we transform the hybrid attribute values into the 2-tuple linguistic values which makes the assessment results being more intuitionist and more understandable. In the process of selecting the weight of decision attributes, we consider the attitudes of decision-makers, and in the meantime, we aggregated the attribute values by utilizing the OWA operator which makes the assessment results being more objective and scientific.

References

1. Yongqi, Xia, Qizong, Wu: A Technique of Order Preference by Similarity to Ideal Solution for Hybrid Multiple Attribute Decision Making Problems. Journal of Systems Engineering, Vol 19. (2004) 630-634.

2. Yager, R. R.: Applications and Extensions of OWA Aggregations. *International Journal of Man-Machine Studies*, Vol 37. (1992) 103-132.
3. Herrera, F, Martinez, L.: Sanchez Managing Non-Homogeneous Information in Group Decision Making. *European Journal of Operational Research*, Vol 166. (2005) 115-132.
4. Herrera, F, Martinez, L.: An Approach for Combing Linguistic and Numerical Information Based on the 2-tuple Fuzzy Linguistic Representation Model in Decision Making. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, Vol 8. (2000) 539-562.