A Modified Quantum-Behaved Particle Swarm Optimization

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Abstract. Based on the previously introduced Quantum-behaved Particle Swarm Optimization (QPSO), a revised QPSO with Gaussian disturbance on the mean best position of the swarm is proposed. The reason for the introduction of this novel method is that the disturbance can effectively prevent the stagnation of the particles and therefore make them escape the local optima and sub-optima more easily. Before proposing the Revised QPSO (RQPSO), we introduce the origin and the development of the original PSO and QPSO. To evaluate the performance of the new method, the Revised QPSO, along with QPSO and Standard PSO, is tested on several well-known benchmark functions. The experimental results show that the Revised QPSO has better performance than QPSO and Standard PSO generally.

Keywords: Global Optimization, Swarm Intelligence, Particle Swarm, Quantum-behaved Particle Swarm and Gaussian distribution.

1 Introduction

The Particle Swarm Optimization (PSO), originally invented by J. Kennedy and R.C. Eberhart [7], is a member of a wider class of Swarm Intelligence methods used for solving Global Optimization (GO) problems. It was proposed as a population-based search technique simulating the knowledge evolvement of a social organism. In a PSO system, individuals (particles) representing the candidate solutions to the problem at hand, fly through a multidimensional search space to find out the optima or sub-optima. The particle evaluates its position to a goal (fitness) at every iteration, and particles in a local neighborhood share memories of their “best” positions. These memories are used to adjust particle velocities and their subsequent positions.

In the original PSO with M individuals, each individual is treated as an infinitesimal particle in D -dimensional space, with the position vector and velocity

vector of particle i , $X_i(t) = (X_{i1}(t), X_{i2}(t), \dots, X_{iD}(t))$ and $V_i(t) = (V_{i1}(t), V_{i2}(t), \dots, V_{iD}(t))$. The particle moves according to the following equations:

$$V_{ij}(t+1) = w \cdot V_{ij}(t) + c_1 \cdot r_1 \cdot (P_{ij}(t) - X_{ij}(t)) + c_2 \cdot r_2 \cdot (P_{gj} - X_{ij}(t)) \quad (1)$$

$$X_{ij}(t+1) = X_{ij}(t) + V_{ij}(t+1) \quad (2)$$

for $i=1,2,\dots,M; j=1,2,\dots,D$. The parameters c_1 and c_2 are called the acceleration coefficients. Vector $P_i = (P_{i1}, P_{i2}, \dots, P_{iD})$ known as the *personal best position*, is the best previous position (the position giving the best fitness value so far) of particle i ; vector $P_g = (P_{g1}, P_{g2}, \dots, P_{gD})$ is the position of the best particle among all the particles and is known as the *global best position*. The parameter w is the inertia weight and the parameters r_1 and r_2 are two random numbers distributed uniformly in $(0,1)$, that is $r_1, r_2 \sim U(0,1)$. Generally, the value of V_{ij} is restricted in the interval $[-V_{\max}, V_{\max}]$.

Many revised versions of PSO algorithm are proposed to improve the performance since its origin in 1995. For the detailed information about these variants of PSO, one may refer to the literature such as [1], [3], [8], [9], etc.

In our previous work, a Quantum-behaved Particle Swarm Optimization (QPSO), inspired by quantum mechanics was proposed in [10], [11] and [12]. In this paper, we introduce a method of improving QPSO by exerting a Gaussian disturbance on the mean best position of the swarm and therefore propose a revised version of QPSO.

The rest of the paper is structured as follows. In Section 2, the concept of QPSO is presented. The revised QPSO is proposed in Section 3. Section 4 gives the numerical results on some benchmark functions. Some concluding remarks and future work are presented in the last section.

2 Quantum-Behaved Particle Swarm Optimization

Trajectory analyses in [4] demonstrated the fact that convergence of PSO algorithm may be achieved if each particle converges to its local attractor $p_i = (p_{i1}, p_{i2}, \dots, p_{iD})$ with coordinates

$$p_{ij}(t) = (c_1 r_1 P_{ij}(t) + c_2 r_2 P_{gj}(t)) / (c_1 r_1 + c_2 r_2), \text{ or } p_{ij}(t) = \varphi \cdot P_{ij}(t) + (1 - \varphi) \cdot P_{gj}(t) \quad (3)$$

where $\varphi = c_1 r_1 / (c_1 r_1 + c_2 r_2)$. It can be seen that the local attractor is a stochastic attractor of particle i that lies in a hyper-rectangle with P_i and P_g being two ends of its diagonal. We introduce the concepts of QPSO as follows.

Assume that each individual particle move in the search space with a δ potential on each dimension, of which the center is the point p_{ij} . For simplicity, we consider a particle in one-dimensional space, with point p the center of potential. Solving Schrödinger equation of one-dimensional δ potential well, we can get the probability distribution function D .

$$D(x) = e^{-2|p-x|/L} \quad (4)$$

Using Monte Carlo method, we obtain

$$x = p \pm \frac{L}{2} \ln(1/u), \quad u \sim U(0,1) \quad (5)$$

The above is the fundamental iterative equation of QPSO.

In [11], a global point called Mainstream Thought or Mean Best Position of the population is introduced into PSO. The global point, denoted as C , is defined as the mean of the personal best positions among all particles. That is

$$C(t) = (C_1(t), C_2(t), \dots, C_D(t)) = \left(\frac{1}{M} \sum_{i=1}^M P_{i1}(t), \frac{1}{M} \sum_{i=1}^M P_{i2}(t), \dots, \frac{1}{M} \sum_{i=1}^M P_{iD}(t) \right) \quad (6)$$

where M is the population size and P_i is the personal best position of particle i . Then the value of L is evaluated by $L = 2\alpha \cdot |C_j(t) - X_{ij}(t)|$ and the position are updated by

$$X_{ij}(t+1) = p_{ij}(t) \pm \alpha \cdot |C_j(t) - X_{ij}(t)| \cdot \ln(1/u) \quad (7)$$

where parameter α is called Contraction-Expansion (CE) Coefficient, which can be tuned to control the convergence speed of the algorithms. Generally, we always call the PSO with equation (7) Quantum-behaved Particle Swarm Optimization (QPSO), where parameter α must be set as $\alpha < 1.782$ to guarantee convergence of the particle [12]. In most cases, α can be controlled to decrease linearly from α_0 to α_1 ($\alpha_0 < \alpha_1$).

3 The Proposed Method

Although QPSO possesses better global convergence behavior than PSO, it may encounter premature convergence, a major problem with PSO and other evolutionary algorithms in multi-modal optimization, which results in great performance loss and sub-optimal solutions. In a PSO system, with the fast information flow between particles due to its collectiveness, diversity of the particle swarm declines rapidly, leaving the PSO algorithm with great difficulties of escaping local optima. Therefore, the collectiveness of particles leads to low diversity with fitness stagnation as an overall result. In QPSO, although the search space of an individual particle at each iteration is the whole feasible solution space of the problem, diversity loss of the whole population is also inevitable due to the collectiveness.

From the update equations of PSO or QPSO, we can infer that all particles in PSO or QPSO will converge to a common point, leaving the diversity of the population extremely low and particles stagnated without further search before the iterations is over. To overcome the problem, we introduce a Revised QPSO by exerting a Gaussian disturbance on the mean best position when the swarm is evolving. That is,

$$m_j(t) = C_j(t) + \varepsilon \cdot Rn, \quad j = 1, 2, \dots, D \quad (8)$$

where ε is a pre-specified parameter and Rn is random number with Gaussian distribution with mean 0 and standard deviation 1. Thus the value of L is calculated as $L = 2\alpha \cdot |m_j(t) - X_{ij}(t)|$ and equation (6) becomes

$$X_{ij}(t+1) = p_{ij}(t) \pm \alpha |m_j(t) - X_{ij}(t)| \cdot \ln(1/u) \quad (9)$$

The above iterative equation can effectively avoid the declination of the diversity and consequently the premature convergence. It is because that when the swarm is evolving, a consistent disturbance on mean best position can prevent the $L = 2\alpha \cdot |m_j(t) - X_{ij}(t)|$ decreasing to zero, maintaining the particle's vigor, which is particularly serviceable to make the particle escape local optima at the later stage of evolution, and able to result in a better performance of the algorithm overall. The revised QPSO is outlined as follows.

Revised QPSO Algorithm

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Initialize particles with random position Xi=X[i][:];
Initialize personal best position by set Pi=Xi;
while the stop criterion is not met do
  Compute the mean best position C[:] by equation (6);
  Get the disturbed point m by equation (8)
  for i = 1 to swarm size M
    If f(Xi)<f(Pi) then Pi=Xi; Endif
    Find the Pg=arg min f(P[g][:]);
    for j=1 to D
      =rand(0,1); u=rand(0,1);
      p=*P[i][j]+(1- )*P[g][j];
      if (rand(0,1)>0.5)
        X[i][j]=p+α*abs(m[j]- X[i][j])*ln(1/u);
      Else
        X[i][j]=p-α*abs(m[j]- X[i][j])*ln(1/u);
      Endif
    Endfor
  Endfor
Endwhile

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4 Numerical Experiments

In this section five benchmark functions listed in Table 1 are tested for performance comparisons of the Revised QPSO (RQPSO) with Standard PSO (SPSO) and QPSO algorithms. These functions are all minimization problems with minimum objective function values zeros. The initial range of the population is asymmetry, as used in [13], [14]. $V_{\max}$ for SPSO is set as the up-bound of the search domain.

Setting the fitness value as function value, we had 50 trial runs for every instance and recorded mean best fitness and standard deviation. In order to investigate the scalability of the algorithm, different population sizes M are used for each function with different dimensions. The population sizes are 20, 40 and 80. The maximum generation (iteration) is set as 1000, 1500 and 2000 corresponding to the dimensions 10, 20 and 30 for first four functions, respectively. The maximum generation for the last function is 2000. For SPSO, the acceleration coefficients are set to be $c_1=c_2=2$ and the inertia weight is decreasing linearly from 0.9 to 0.4 as in [13], [14].

Table 1. Expression of the five tested benchmark functions

	Function Expression	Search Domain	Initial Range
Sphere	$f_1(X) = \sum_{i=1}^n x_i^2$	(-100, 100)	(50, 100)
Rosenbrock	$f_2(X) = \sum_{i=1}^{n-1} (100 \cdot (x_{i+1} - x_i^2)^2 + (x_i - 1)^2)$	(-100, 100)	(15, 30)
Rastrigrin	$f_3(X) = \sum_{i=1}^n (x_i^2 - 10 \cdot \cos(2\pi x_i) - 10)$	(-10, 10)	(2.56, 5.12)
Griewank	$f_4(X) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	(-600, 600)	(300, 600)
Shaffer's	$f_5(X) = 0.5 + \frac{(\sin(\sqrt{x_1^2 + x_2^2}))^2}{(1.0 + 0.001(x_1^2 + x_2^2))^2}$	(-100, 100)	(30, 100)

In experiments for QPSO and RQPSO, the value of CE Coefficient α varies from 1.0 to 0.5 linearly over the running of the algorithm as in [11], [12]. For Revised QPSO (RQPSO), the parameter ϵ is fixed at 0.001. The mean values and standard deviations of best fitness values for 50 runs of each function are recorded in Table 2 to Table 5.

The numerical results show that both QPSO and RQPSO are superior to SPSO except on most of the function. On Shpere Function the RQPSO has worse performance than QPSO and SPSO because maintaining the diversity by consistent disturbance on the mean best position leads to poor local convergence of the algorithm. On Rosenbrock function, the RQPSO outperforms the QPSO when the swarm size is 20, but it does not show the better performance than QPSO when the swarm size is 40 and 80. On Rastrigrin function, it is shown that the RQPSO generated better results than QPSO when the number of particles is 20, but its advantages over the QPSO are not remarkable if the sampling errors are considered. On Griewank function, RQPSO is superior to the QPSO in most cases, particularly when the dimension of the problem is high. On Shaffer's function, the RQPSO shows no improvements for QPSO. Generally speaking, the Revised QPSO has better global search ability than QPSO when the problem is complex.

The figure 1 shows the convergence process of the RQPSO and QPSO on the first four benchmark functions with dimension 30 and swarm size 20. It is shown that the RQPSO has comparable convergence speed with QPSO and both of them can converge more rapidly than SPSO.

Table 2. Numerical results on Sphere function

M	Dim.	Gmax	SPSO		QPSO		RQPSO	
			Mean Best	St. Dev.	Mean Best	St. Dev.	Mean Best	St. Dev.
20	10	1000	3.16E-20	6.23E-20	2.29E-41	1.49E-40	1.7412E-007	4.594E-008
	20	1500	5.29E-11	1.56E-10	1.68E-20	7.99E-20	1.8517E-006	3.387E-007
	30	2000	2.45E-06	7.72E-06	1.34E-13	3.32E-13	6.0118E-006	1.086E-006
40	10	1000	3.12E-23	8.01E-23	8.26E-72	5.83E-71	1.5035E-007	4.627E-008
	20	1500	4.16E-14	9.73E-14	1.53E-41	7.48E-41	1.3454E-006	2.572E-007
	30	2000	2.26E-10	5.10E-10	1.87E-28	6.73E-28	4.9908E-006	7.878E-007
80	10	1000	6.15E-28	2.63E-27	3.10E-100	2.10E-99	1.0779E-007	3.592E-008
	20	1500	2.68E-17	5.24E-17	1.56E-67	9.24E-67	1.3454E-006	2.572E-007
	30	2000	2.47E-12	7.16E-12	1.10E-48	2.67E-48	4.1723E-006	6.179E-007

Table 3. Numerical results on Rosenbrock function

M	Dim.	Gmax	SPSO		QPSO		RQPSO	
			Mean Best	St. Dev.	Mean Best	St. Dev.	Mean Best	St. Dev.
20	10	1000	94.1276	194.3648	59.4764	153.0842	47.6904	99.3668
	20	1500	204.337	293.4544	110.664	149.5483	70.7450	116.1039
	30	2000	313.734	547.2635	147.609	210.3262	103.7322	166.9141
40	10	1000	71.0239	174.1108	16.2338	24.46731	20.1872	33.3523
	20	1500	179.291	377.4305	46.5957	39.536	46.8270	51.7003
	30	2000	289.593	478.6273	59.0291	63.494	78.5551	71.2005
80	10	1000	37.3747	57.4734	8.63638	16.6746	13.4288	14.5301
	20	1500	83.6931	137.2637	35.8947	36.4702	29.8257	33.2984
	30	2000	202.672	289.9728	51.5479	40.849	54.1137	38.2911

Table 4. Numerical results on Rastrigrin function

M	Dim.	Gmax	SPSO		QPSO		RQPSO	
			Mean Best	St. Dev.	Mean Best	St. Dev.	Mean Best	St. Dev.
20	10	1000	5.5382	3.0477	5.2543	2.8952	4.4489	2.2451
	20	1500	23.1544	10.4739	16.2673	5.9771	15.9715	6.4180
	30	2000	47.4168	17.1595	31.4576	7.6882	27.6414	6.1376
40	10	1000	3.5778	2.1384	3.5685	2.0678	3.2081	1.4512
	20	1500	16.4337	5.4811	11.1351	3.6046	10.5817	3.6035
	30	2000	37.2796	14.2838	22.9594	7.2455	20.9748	5.2488
80	10	1000	2.5646	1.5728	2.1245	2.2353	2.0922	1.5245
	20	1500	13.3826	8.5137	10.2759	6.6244	8.4794	2.7922
	30	2000	28.6293	10.3431	16.7768	4.4858	16.4016	4.4312

Table 5. Numerical results on Griewank function

M	Dim.	Gmax	SPSO		QPSO		RQPSO	
			Mean Best	St. Dev.	Mean Best	St. Dev.	Mean Best	St. Dev.
20	10	1000	0.09217	0.0833	0.08331	0.06805	0.0694	0.0641
	20	1500	0.03002	0.03255	0.02033	0.02257	0.0201	0.0212
	30	2000	0.01811	0.02477	0.01119	0.01462	0.0091	0.0129
40	10	1000	0.08496	0.0726	0.06912	0.05093	0.0531	0.0509
	20	1500	0.02719	0.02517	0.01666	0.01755	0.0180	0.0165
	30	2000	0.01267	0.01479	0.01161	0.01246	0.0107	0.0137
80	10	1000	0.07484	0.07107	0.03508	0.02086	0.0342	0.0423
	20	1500	0.02854	0.0268	0.01463	0.01279	0.0141	0.0158
	30	2000	0.01258	0.01396	0.01136	0.01139	0.0066	0.0105

Table 6. Numerical results on Shaffer's f6 function

M	Dim.	Gmax	SPSO		QPSO		RQPSO	
			Mean Best	St. Dev.	Mean Best	St. Dev.	Mean Best	St. Dev.
20	2	2000	2.782E-04	0.001284	0.001361	0.003405	0.001433	0.0036
40	2	2000	4.744E-05	3.593E-05	3.891E-04	0.001923	2.1303E-008	6.726E-008
80	2	2000	2.568E-10	3.134E-10	1.723E-09	3.303E-09	3.5979E-009	1.251E-008

Fig. 1. Convergence process of the RQPSO and QPSO on the first four benchmark functions with dimension 30 and swarm size 20 averaged on 50 trail runs

5 Conclusions

In this paper, a Revised QPSO with the mean best position exerted by Gaussian disturbance is proposed. The reason for introduction of the disturbance is that it can avoid the decline of the diversity effectively, which is particularly serviceable for the particle's escaping local optima at later stage of the evolution. The iterative equation derives from the hybrid of exponential and normal distributions. The numerical results on benchmark functions show that the Revised QPSO enhance the global search ability of QPSO efficiently.

Our future work will focus on find out more efficient methods of improving QPSO. A promising method may be introducing another type of probability distribution into QPSO to replace exponential distribution. Moreover, we will also be devoted to applying the novel QPSO to many real world problems.

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